Impact of viscoelastic coefficient on shearing stress of oscillatory walls during electroosmotic flow

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***Abstract*— Of concern the paper is a review of electro-osmotic flow of a viscoelastic bio-fluid in the light of wall shear stress developed due to the velocity gradient across the channel. The channels are in a state of oscillatory motion and the flow is driven by an alternating electric field. Analytical solutions are derived from the set of governing equations under boundary layer approximations. For an illustrative purpose, a case study is made by analyzing the flow behavior, taking blood sample in which the viscoelastic properties are modeled as a determining factor for the case of fully developed flow in a micro channel. It is revealed that a higher viscoelastic coefficient results in a stronger wall shear stress in the time periodic profile, characterized by second grade viscoelastic model. The results given here seems to be worthy of attention, for an accurate interpretation of wall shear stress during arterial blood flow through a micro-vessel.**

***IndexTerms*— Wall shear stress; Viscoelastic co-efficient, Electro-osmotic flow** *(keywords)*.

I. INTRODUCTION

Human blood contents platelets, red blood cells, thrombocytes, plasma and other microparticles. The deformation of microorganisms under shearing forces causes storage and release of energy due to its elastic behavior. This non-Newtonian concept of blood is applied during the investigation by many researchers in the past [1-5]. The non-Newtonian aspect of blood is worthy of consideration to interpret several cardiovascular incidents that includes heart diseases and strokes. To have a better understanding to the fact above, blood is considered as a second grade viscoelastic fluid.

Wall shear stress is developed across the channel for the changes in velocity from the wall to the centre of the vessel. The velocity gradient near the wall may be defined as wall shear rate. A basic knowledge about the dynamics of blood flow and its consequent development of wall shear stress, is essential to detect any arterial disease that arises from the stress of the vessel wall is concerned. Recent advancement of blood sample carrier as bio microfluidic system also needs to have a better understanding of the stresses developed, to design a particular device in an accurate manner. Several researchers took interest to determine the shearing stresses during blood flow [6-8].

Thurston [9] explained that for any velocity gradient which is lesser than 1-2 per second, the components of both the viscosity and elasticity are related to the velocity gradient and for particle density of more than 20% the elastic component of viscosity supersedes the viscous component. The deformation of micro-particles that are suspended thus develops a shear stress which causes a pressure in the intravascular channel wall. The reason why, study of wall shear stress with the variation in viscoelastic coefficient, is needed to infer the causes of certain pathogenic factors relating to growth of cerebral aneurysms.

Considering blood as a second grade viscoelastic fluid, any researchers [10,11] have carried out their studies. Chandra [12] studied blood as a second grade viscoelastic fluid that is driven by an electric field. Electro-osmotically driven viscoelastic fluid flow is also found in many literatures (cf. [13]-[14]). But, in the literatures as mentioned above, calculation of wall shear stress was not made. Motivating towards that, a mathematical model under the purview of second grade viscoelastic fluid, is built to determine the wall shear stress taking into account the pulsatile flow of blood in a porous medium, actuated by an alternating electric field. Wall shear stress is estimated with the variation in viscoelastic coefficient and for a more realistic approach, the channel wall is taken as an one to add up the unsteadiness of blood flow.

II. PROBLEM FORMULATION AND SOLUTION

Micro channel is considered and the boundary of channel wall is determined by the plates located at $y = \pm h$. The velocity of the plates are given by u= $u_S e^{iw_1 t}$ and for the flow it is taken as $q = (u(y,t), 0, 0)$, so that the continuity equation is satisfied, automatically. The time periodic electric field of angular velocity ω_e is applied externally across the plates that are parallel to xz-plane. The fluid is moving towards x- direction. The flow is considered to be symmetric and so our analysis is limited to limited to the region $0 \le y \le h$, for $x \in [0, L]$, where L is the length of the channel.

Now, under Boussinesq boundary layer approximations the set of equations that governs the present problem in case of a second grade viscoelastic fluid under the action of electric field is given by,

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} + K\frac{\partial^3 y}{\partial t\, \partial y^2} - \frac{\nu}{k_p}u + \frac{\rho_e}{\rho}(E_x e^{i\omega_e t}) \quad \ldots\ldots(1)$$

and

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} \quad \ldots\ldots(2)$$

In which, u, ρ, ν, K, k_p ,p represents the velocity, the density of fluid, the kinematic viscosity, the viscoelastic coefficient, the co-efficient of porous medium permeability, and the pressure respectively.

Gauss's law of charge distribution can be written as,

$$\frac{\partial^2 \psi}{\partial y^2} = -\frac{\rho_e}{\varepsilon}, \quad \ldots\ldots(3)$$

Where ψ, ρ_e, w_e, E_x , t denotes the induced potential field in the transverse direction, the distribution of net electric charge density in equilibrium near the charged walls, the angular frequency of the AC electric field, the amplitude of the electric field and time, respectively.

Introducing a set of non-dimensional variables as :

$$x^* = \frac{x}{h}\ ;\ y^* = \frac{y}{h}\ ;\ u^* = \frac{u}{U_{HS}}\ ;\ \omega^* = \frac{\omega}{\frac{U_{HS}}{h}}\ ;\ \omega_e^* = \frac{\omega_e}{\frac{U_{HS}}{h}}\ ; \quad \ldots\ldots(4)$$

$$R = \frac{U_{HS} h}{\nu}\ ;\ K^* = \frac{K U_{HS}}{\nu h}\ ;\ p^* = \frac{p h}{\rho\nu\, U_{HS}}\ ;\ \psi^* = \frac{\psi}{\zeta}\ ;\ t^* = \frac{t}{\frac{h}{U_{HS}}} \quad \ldots\ldots(5)$$

Where U_{HS} represents Helmholtz-Smoluchowski electro-osmotic velocity, which is defined by,

$$U_{HS} = -ME_x = -\frac{\zeta\varepsilon E_x}{\mu}\ , \quad \ldots\ldots(6)$$

In which M, ζ , ε stands for the mobility, the zeta potential and the dielectric constant of the medium, respectively. u = ρν , denotes the dynamic viscosity. Equations (1)-(3) reduces to

$$R\frac{\partial u^*}{\partial t^*} = -\frac{\partial p^*}{\partial x^*} + \frac{\partial^2 u^*}{\partial y^{*2}} + K^*\frac{\partial^3 u^*}{\partial t^* \partial y^{*2}} - \frac{1}{D^*}u^* + \frac{\partial^2 \psi^*}{\partial y^{*2}}(e^{i\omega_e^* t^*}) \quad \ldots\ldots(7)$$

$$\frac{\partial p^*}{\partial y^*} = 0 \quad \ldots\ldots(8)$$

and

$$\frac{\partial^2 \psi^*}{\partial y^{*2}} = -\frac{2n_0 ez\, h^2}{\zeta\varepsilon}\sinh\left(\frac{ez\zeta}{K_B T}\psi^*\right), \quad \ldots\ldots(9)$$

R represents the Reynolds number and is given R = $\frac{U_{HS} h}{\nu}$, where ν and h are respectively the kinematic viscosity and the half channel width, D^* represents porous medium shape factor parameter and is given by $D^* = \frac{k_p^*}{h^2}$, where k_p^* denotes the permeability coefficient. Considering, $|ez\zeta\psi^*| < |K_B T|$, and making use of boundary conditions $\psi^*(0) = 0$, $\psi^*(\pm 1) = 1$ and $(\frac{\partial \psi^*}{\partial y^*})_{y^*=0} = 0$, we obtain from equation (9),

$$\psi^*(y^*) = -\frac{\cosh(m y^*)}{\cosh(m)} \quad \ldots\ldots(10)$$

Where, m is the Debye-Hückel parameter, in non-dimensional form.

Dropping the superscript * and considering the velocity as, u= $u_S e^{iw_1 t}$ (where us represents the steady part of the velocity) and with the boundary conditions as,

$\frac{\partial u}{\partial y} = 0\ at\ y = 0\ (owing\ to\ flow\ symmetry)$ and $u = 0\ at\ y = \pm 1\ (usual\ no - slip\ condition)$

the wall shear stress $\tau_w = \mu \frac{\partial u}{\partial y}$ $at\ y = 0$, ($where$,$\mu = \rho\nu$, is the dynamic viscosity.) is calculated analytically and then plotted its profile with the variation in viscoelastic co-efficient.

III. RESULTS

The present study is valid for variety of fluids. But, as a special case, here the parameters are taken in commensurate with that of the blood. The accuracy of the solution is validated for any fully developed flow inside the vessel. The wall shear stress distribution profile is plotted in Figure 1 taking into consideration the variation in viscoelastic coefficient.

Figure 1 reveals that the magnitude of wall shear stress τ_w is increased with the increase in viscoelastic coefficient. Hematocrit density is an important factor of determining viscoelastic coefficient of blood. Naturally, the value of that viscoelastic coefficient may change and here, it is observed that with the rise in viscoelastic coefficient wall shear stress increases. So, for a blood with less viscoelastic coefficient has the ability to bear only lower value of stresses. It is also found that the there is also changes of amplitude as we go in a time scale. This shows the variation in stress with the variation in time, during pulsatile flow of blood that is actuated by an alternating electric field of different angular frequency.

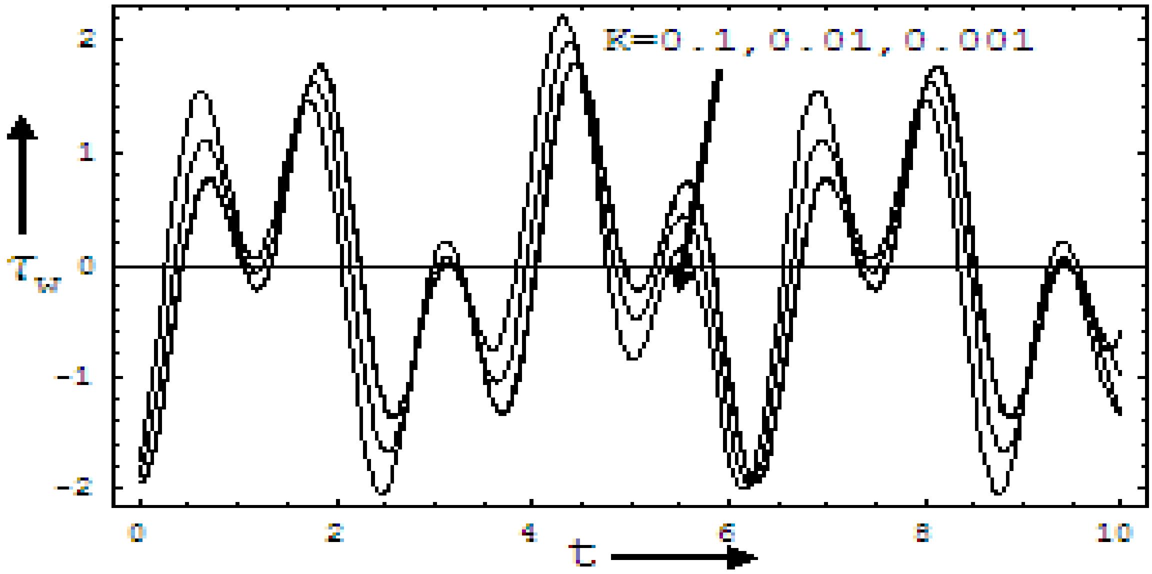

Figure 1 Time periodic change in Wall shear stress with viscoelastic co-efficient K where R=0.1, y=0, m=100, B=30, D=0.05, w_e = 500, w_1 = 200.

IV CONCLUSION

The present paper aims to build an idea about wall shear stress developed during blood flow in an oscillatory micro channel. The flow is driven by a time periodic electric field with different angular frequency to approach towards the reality of the situation. The problem is formulated taking into account, blood as a second-grade viscoelastic fluid which is flowing through a porous medium. The impact of viscoelastic coefficient onto the wall shear stress has also importance into the process of designing any kind of bio sensing and bio-fluidic devices.

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