Mathematical Modelling of Threshold Voltage based Resistive Memory

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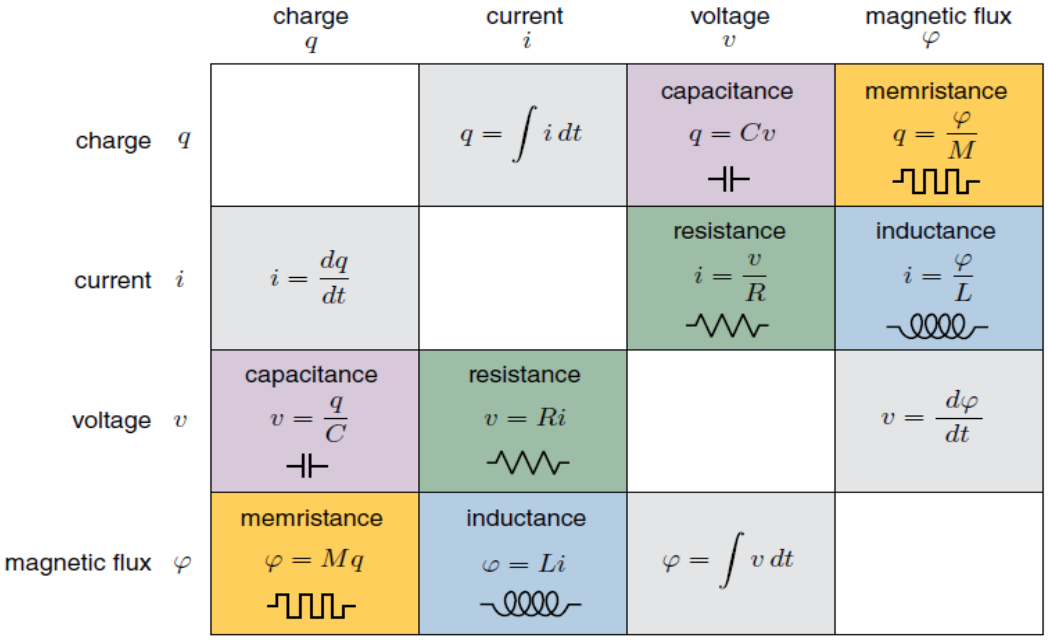
Abstract: The fourth fundamental electrical circuit element, memristor was implemented by HP Labs in 2008 that had been hypothesized by Leon Chua in 1971. The memristive technological innovation turns into a potentially great innovative technology these days that makes it possible for the consistent progressiveness of electrical power, area, and performance and as a consequence continuing to keep Moore's law alive. It provides excellent scalability at very low-level technology, superior utilization whenever put to use as memory, alleviates overall electrical consumption. This paper impersonates comprehensive study of memristor behavior together with the modeling of memristor analysis is carried out with the assistance of Mathcad and Matlab Simulink simulator. We created Matlab Simulink algorithm for the model and afterward analysis the functionality; it is well-matched with the functionality of HP memristor and in addition, viability to be used in memory circuits. Within this paper, the primary goal ought to analyses the mathematical relationship among voltage, current, memristance along with the length of device furthermore demonstrated graphically. The memristor comprises memory with elastic resistance and additionally used as the high-speed switch. It is usually used in non-volatile memory in consideration of its high-speed switching operation.

***IndexTerms* – Moore's law, Scalability, Memristor, Memristance, Non -volatile.**

I. INTRODUCTION

In 1971, Leon Chua formulated the fourth elementary electrical element at the time of endeavoring to establish a missing constitutive working relationship between the electrical charge and magnetic flux [1]. Chua called this speculative nonlinear gadget as memristor (memory together with resistor). It is far from energy storage element exhibits hysteresis characteristics pinched at origin furthermore its nonlinear resistance is generally remembered inconclusively by curbing the flow of the electrical charge and the magnetic flux. It is a two terminal circuit element in which the flux between the two terminals is a function of the amount of charge that transferred through it. Assuming the charge flows via the memristor in one of the terminal consequently the memristance is going to be increases considering that the charge flows from another terminal of memristor in it follows that the memristance can certainly decreases. If perhaps the charge is abstained from the applied voltage, the element can certainly remember the very last resistance that it acquired, once the flow of charge initiates again, the resistance of the element is going to be exactly what it was when it was last active. Memristor exhibits pinched hysteresis functionality for voltage-current curve. Whenever excited by applying a periodic signal at zero voltage the current also zero. Thus voltage and current have identical zero crossing. If any device produce hysteresis curve then possibly that has been either memristor or memristive systems.



	charge q	current i	voltage v	magnetic flux φ
charge q		$q = \int i\,dt$	capacitance $q = Cv$	memristance $q = \frac{\varphi}{M}$
current i	$i = \frac{dq}{dt}$		resistance $i = \frac{v}{R}$	inductance $i = \frac{\varphi}{L}$
voltage v	capacitance $v = \frac{q}{C}$	resistance $v = Ri$		$v = \frac{d\varphi}{dt}$
magnetic flux φ	memristance $\varphi = Mq$	inductance $\varphi = Li$	$\varphi = \int v\,dt$	

Figure 1: Physical quantities and their relation with fundamental electrical circuit elements.

II. THE MEMRISTOR

The Two terminal element memristor has something to do with the two physical measurements of electric charge coupled with the flux. The fallowing mathematical formula might be realize by presuming the flux (φ) is a function of charge (q).

$$\varphi = f(q) \tag{1}$$

We are familiar with the differential equations of the electric charge and flux with respect to the time are given in equations (2) and (3)

$$dq = idt \tag{2}$$

$$d\varphi = vdt \tag{3}$$

From the above equations

$$v = \frac{d\varphi}{dt} = \frac{d\varphi}{dq}\frac{dq}{dt} = \frac{d\varphi}{dq}i \tag{4}$$

As similar to the ohms law, it has constant memristance if the ratio between flux and charge is constant. If the relationship between is nonlinear then it exhibits nonlinear memristance as the property of memristance.

$$v = M(q)\mathrm{i} \tag{5}$$

$$M(q) = \frac{\mathrm{d}\varphi}{dq} \tag{6}$$

The electric charge and flux are time integrals of the current and voltage respectively as shown in equations (7) and (8).

$$q(t) = \int_{-\infty}^{\tau} i(\tau)d\tau \tag{7}$$

$$\varphi(t) = \int_{-\infty}^{\tau} v(\tau)d\tau \tag{8}$$

From the equations (1), (2) and (8) we derive as

$$\int_{-\infty}^{\tau} i(\tau)d\tau = f(\int_{-\infty}^{\tau} v(\tau)d\tau) \tag{9}$$

This mathematical equation signifies that the memristor is element whose relationship between the voltage and current is nonlinear.

III. H.P. LABS MEMRISTOR

Thirty seven years later, electronics have found small enough to explore the secrets of the fourth element [2]. In 2008 HP labs publicized the physical realization of a memristor-based on a thin film of titanium dioxide. They specifically opted for titanium dioxide because of the equations of drift of oxygen vacancies in titanium dioxide and additionally their impact on the electronic conduction in the component which had been very much the same with the equivalent circuit model [3]. A two-layer titanium dioxide layer is sandwiched between a pair of layers of platinum electrodes. Right there with the layers is TiO_2, typically which is insulator (resistance (R_{OFF})) and also the one other layer is TiO_{2-x} , which function as conductor (width (w) and its resistance(R_{ON})) because of its oxygen vacancies are donors of electrons, helping to make the vacancies themselves positively energized to be able to be pushed back and forth at will in the titanium dioxide layer. The total memristance of the memristor (total width D) is given by

$$M(w) = R_{ON}\frac{w}{D} + R_{OFF}(1 - \frac{w}{D}) \tag{10}$$

The mathematical modelling of HP memristor is defined through the two equations as given below

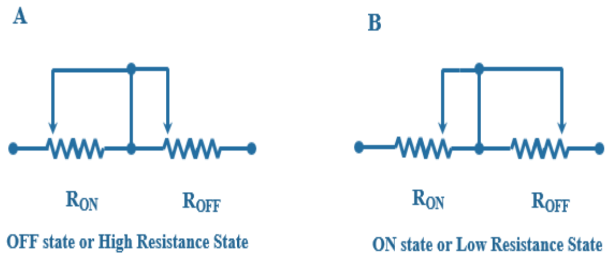
$$v(t) = M(x, i, t)i(t) \tag{11}$$

$$\hat{x} = f(x, i, t) \tag{12}$$

In which v is exerted voltage, i is the electric current, M is the total memristance and x is the internal state variable of memristor which represents the state of the device, that is basically the drift equation of the vacancies. The simplest linear drift mathematical formula is usually defined as

$$\hat{x} = \mu\frac{R_{ON}}{D^2}i \tag{13}$$

On which μ is the drift mobility of the oxygen vacancy, this model is not realistic model mainly because it is unable to acquire the significantly nonlinear responses at large-boned positions inside the memristor and also it may not be met the boundary conditions. And as a consequence new and innovative models are designed with nonlinear drift equations, for this function some analog circuit designers are introduced the window function to fulfill boundary conditions. Even so many limitations such as the dead state of boundaries and discontinuities [4], [9]. The pre-existing models still is unable to engage in nonlinear effects [5] while other models establish asymmetric characteristics between turn on and turn off, on the other hand, turning off a memristor is significantly slower when compared with turn on operation [6].



OFF state or High Resistance State

ON state or Low Resistance State

Figure 2: OFF state and ON state internal resistance of memristor

The memristor can store information in form of resistance. High resistance state is considered as logic 0 or OFF state and low resistance state is considered as logic 1 or ON state. The switching operation depend on the boundary drift. The V-I characteristics of memristor form hysteresis curve, it represent the storage of information. At origin the curve is pinched i.e. at zero voltage it has no current flow. When the device is initially in off state and applying the positive voltage there is no current flow until a particular voltage then it change its state from ON to OFF state, afterwards the current is directly proportional to the applied voltage. When the applied voltage reaches negative threshold then only the memristor changes it state from ON state to OFF, and it hold in this stage until the applied voltage reaches the positive threshold, the OFF state of memristor. Based on this switching characteristics, the memristor is used as switch. The ON state or High Resistance State (HRS) and OFF state or Low Resistance State internal resistance are observed as shown in figure 2(A) and 2(B) respectively.

IV. PROPOSED MODEL MATHEMATICAL EQUATIONS

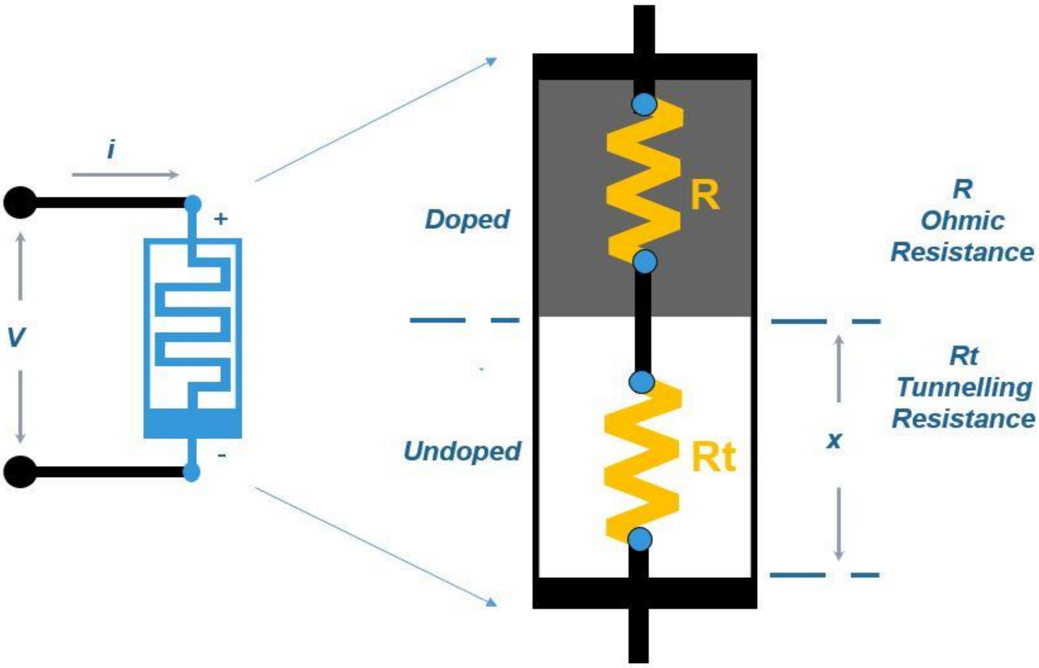


Figure 2: Memristor symbol and physical internal structure

Characterizing the memristor we must have a minimum of two equations i.e port and state equations [10], [11]. The port equation interact with external environment and state equation defined the internal state of device. We design the voltage controlled memristor model as shown in figure. 3 and its state equation and port equations are described in the equation (14) and (15) respectively.

$$i = G_M(x, v, t)v \qquad (\because v = iR_M \;\Rightarrow\; i = \frac{v}{R_M} \;\Rightarrow\; i = G_M v) \tag{14}$$

The current that passes through the device model is the product of memductance of device and the voltage across the two terminals of the device.

$$\hat{x} = f(x, v, t) \tag{15}$$

The quantities G_M and R_M represent memductance (state controlled conductance) and memristance respectively. The individual physical measurement units are Siemens and Ohms. The function G_M performing the non-linear characteristics as well as function ‘f’ governs the way in which state variable x can change with time t. the v and i are the voltage and current which pass through the use of the memristor in the time (t) period. x depicts the single state variable (in-built memristor state), which is actually the thickness of tunnel barrier of the undoped oxide layer . The time derivative of state variable ($\hat{x}$) of the voltage influenced memristive method is a result of voltage that passes by way of the memristive port. It corresponds the motion of barrier between the two deposits. The state variable is proportional to tunneling resistance. Once the state variable enhances then tunneling resistance is greater than before, which means that its value is estimated with boundary under adopted polarity of the voltage. The derivative of the state is controlled by voltage, in spite of this, such type of memristor is represented as Flux controlled device $\left(G_M(\varphi) = \frac{dq}{d\varphi}\right)$.

The port voltage is usually masked in our design to function within the boundary limits at hard switching circumstance by a function H such a way that flux always seems to be within limitations. The masking functionality of the voltage controlled memristor is defined as

$$H(v) = \begin{cases} v, & if\ R_M \in (R_{min}, R_{max}) \\ 0, & else\ if\ v\ does\ not\ pass\ zero. \end{cases} \tag{16}$$

The above mathematical equation eradicated any an excessive amount of applied voltage applied to the memristor. Once it has reached the value of boundary limit resistance, it hold on to that resistance regardless of input amplitude. It bring back to the resistive region.

The G_M and ‘f’ are generally the functions the associated with distinct constants pertaining to memristor model initial state and in addition physical properties of memristor. This pretty much all is subjected to the physical realization of memristor in addition to prove how the device is going to be used to design hardware synapses.

$$x(v,t) = x_0\left[1 - \frac{p}{y(v,t)}\right] \tag{17}$$

The parameter ‘x’ represents the at the most value that ‘x’ may be able to grow to in the oxide layer. Tunneling barrier width (x) is within the bound range only, for that reason fitting constant m and function y(v, t) is going to be used to evaluate the boundaries such that the function x(v ,t) shows expected response under the applied voltage. The function y(v, t) will provide the dynamics and current position of the barrier within the predetermined boundaries of initial values of y_{min} and y_{max} particularly as that $y_{max} > y > y_{min}$. To refrain from the backing problem, in the mathematical equation (17) the data set up as $y_{min} \neq 0$ and (p/y_{min}) < 1. The parameter ‘y’ corresponds to ‘x’ in the oxide region, henceforward the tunneling resistance is placed as maximum such that the memristor is within OFF condition (R_{OFF}) otherwise its value is defined to the very least such that the memristor is basically in ON state (R_{ON}).

Sigmoid function plays switching processes at SET and RESET threshold voltages (v_{ths}, v_{thr}) and moreover involved in nonlinear functioning (‘s’ shaped curve). The derivative of sigmoid task works by using quotient rule. This function adds or subtracts from the numerator, as a result it plays the role of memory feed-forward activation function. This separate the voltage-current plot characteristics section into linear as well as nonlinear, below and above the $|v_{th}|$ respectively.

$$\hat{y}(v,t) = \frac{\alpha(v+v_{ths})}{\gamma|v+v_{ths}|}(-v_0 \le v \le v_{thr}) + \beta v(v_{thr} \le v \le v_{ths}) + \frac{\alpha(v+v_{thr})}{\gamma|v+Vthr|}(v_{ths} < v \le v_0) \tag{18}$$

The fitting constant parameter α, β and γ are utilized to alter the speed of memristance by configuring α >> β and γ between ‘0’ and ‘1’. In accordance with the material properties this model variables are re-structured for switching rates (rate of change of tunnel barrier width) and different thresholds. Whenever β = 0, state switch not happens until the exerted voltage attains threshold. The memristance of device can certainly be measured as

$$R(v,t) = z\frac{e^{2x(v,t)}}{x(v,t)} \tag{15}$$

V. RESULTS AND DISCUSSION

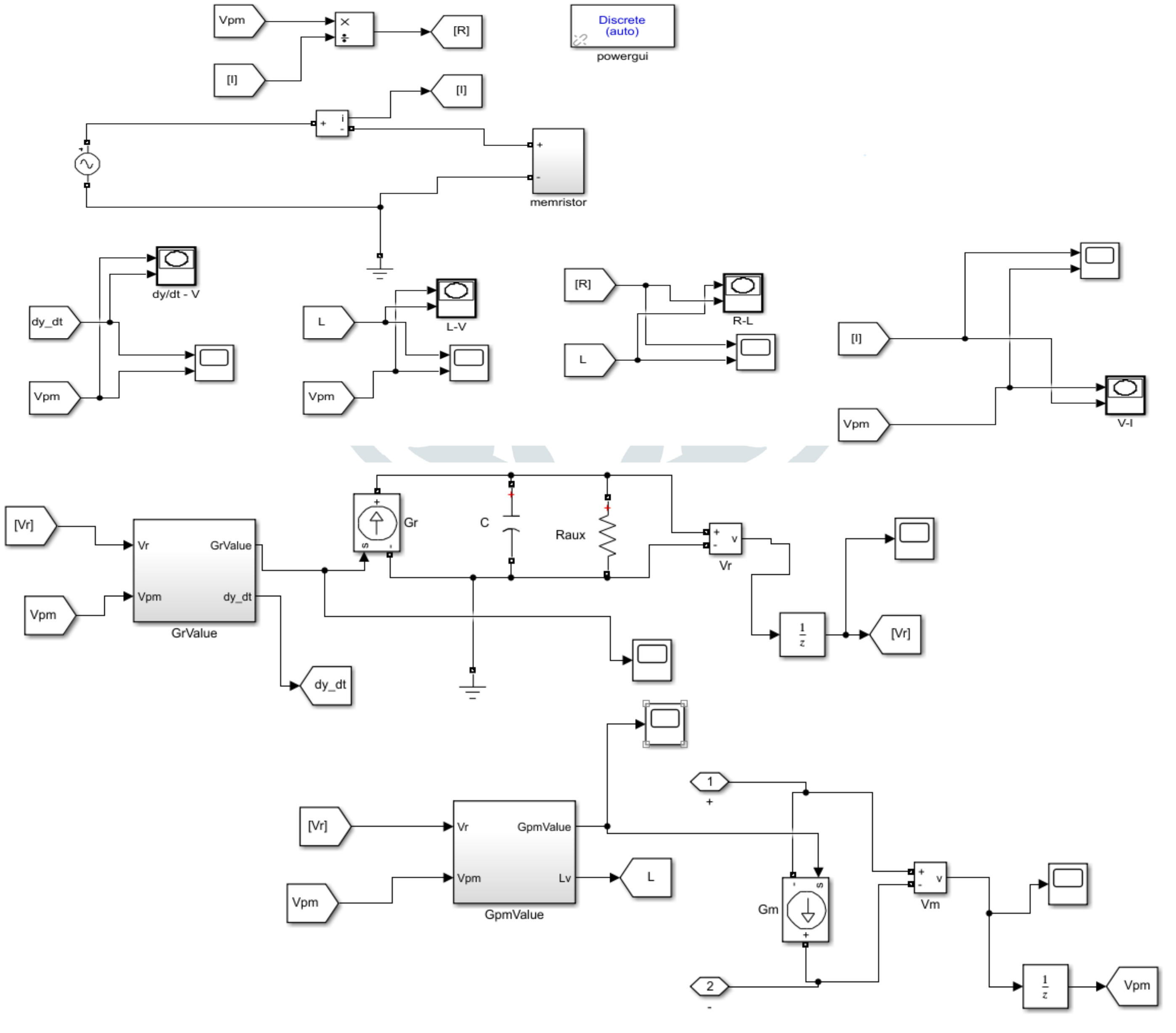


Figure 4: Simulink model of threshold voltage based memristor model

In this section we present the Simulink model of threshold model to describe the functionality of memristor. As shown in Figure 4 we design the most reliable model that match with the design of SPICE simulation with threshold voltage functionality. Here we apply the sinusoidal signal of amplitude of 3v and frequency of 100Hz as shown in figure 5. When the applied voltage exceeds the threshold voltage it starts to flow current through it i.e. it turns on and once the applied voltage reaches the the negative threshold then it stops the flow of current i.e. it turn off as shows in figure 5(b).

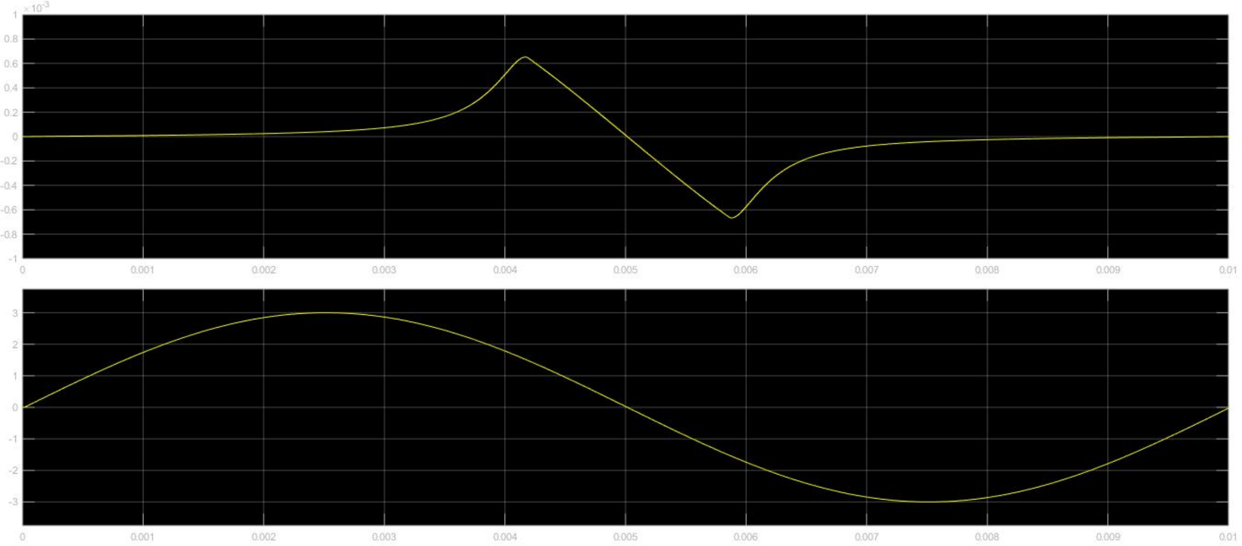


Figure 5: Current passes through for corresponding voltage across the memristor.

The memristor model exhibits the hysteresis characteristics for the V-I characteristics as shown in figure 6(V-I) as same as [12], it change its state at threshold voltage as shown in V-I characteristics. Figure 6(L-V) shows how the barrier changes as the voltage varies. As the length varies the resistance change is depicted in figure 6(R-L). The sigmoid function is shown in figure 6(dy/dt-v).

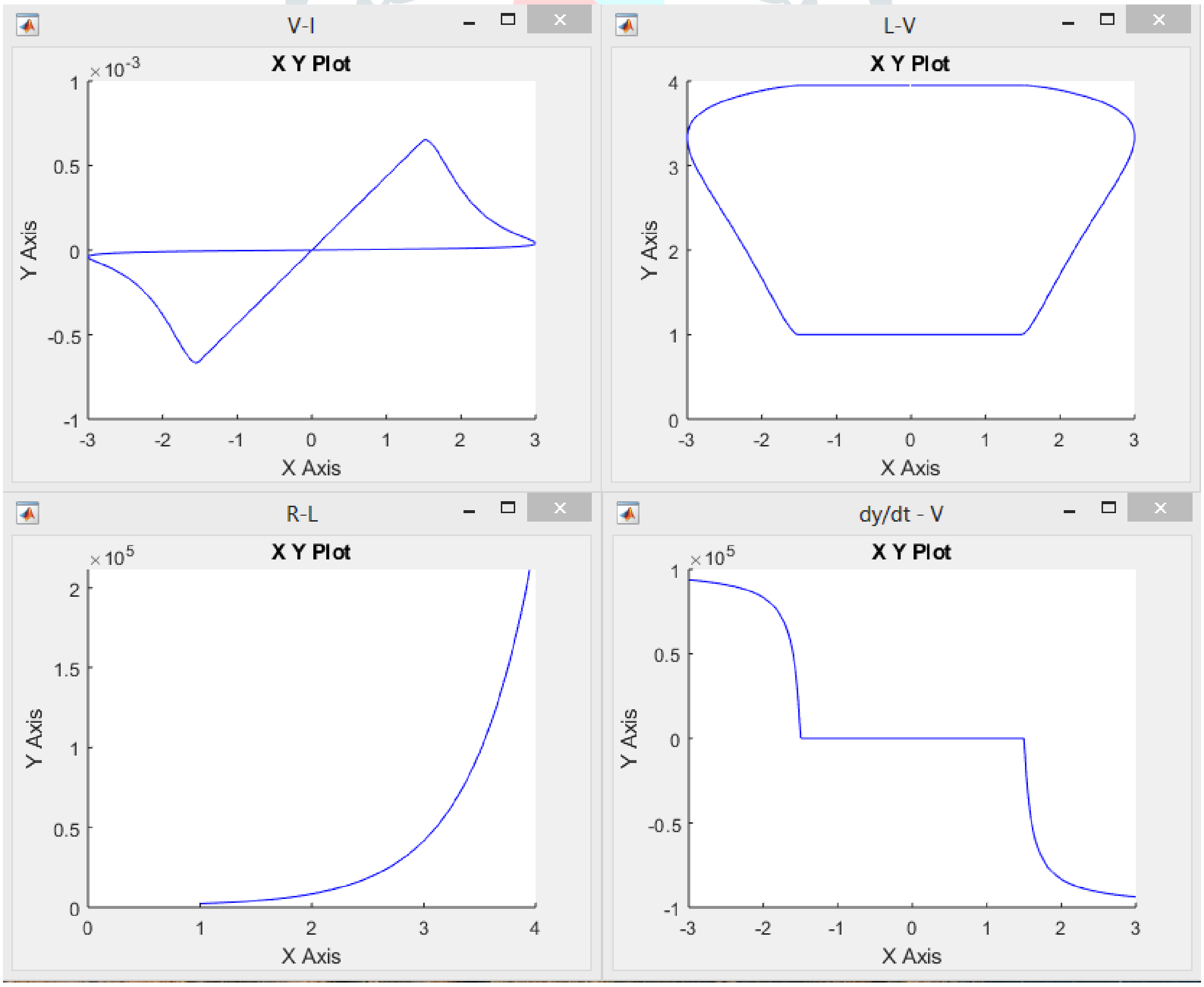


Figure 6: Simulation results of V-I characteristics, length vs voltage, length vs memristance, sigmoid function vs voltage.

VI. CONCLUSION

In this paper we analyze the threshold model of memristor, this model exhibit good functionality from different perspectives. The Simulink results represented to show how the physical properties of device functions by using mathlab Simulink simulator. This model is used to design the SPICE model of memristor. in this paper we analysis the all physical parameters how the change with respect to the electrical parameters in such way that may be used to implement physical model of the memristor. This model is also verified by using Mathcad simulator. This model has no backing problems and works with in the limits of boundary. This model can satisfied all conditions of the memristor in such a way that in may be used as the memory applications.

REFERENCES

[1] L. O. Chua, 1971. Memristor – the missing circuit element. IEEE transaction on Circuit Theory, 18(5):507-519.

[2] D. B. Strukov, G. S. Snider, D. R. Stewart, and R. S. Williams. 2008 . The missing memristor found. Nature, vol. 453. 80–83.

[3] T. Prodromakis, B. P. Peh, C. Papavassiliou, and C. Toumazou. 2011. A versatile memristor model with nonlinear dopant kinetics. IEEE Trans. Electron Devices. 58(9): 3099–3105.

[4] O. Kavehei, A. Iqbal, Y.S. Kim, K. Eshraghiam, S.F. Al-Sarawi and D. Abbott. 2010. the fourth element: characteristics, modelling and electromagnetic theory of the memristor. Proceeding of the royal society a mathematical physical and engineering sciences, vol. 466:2175-2202.

[5] D. B. Struckov and R. S. Williams. 2009. Exponential ionic drift: fast switching and low volatility of thin-film memristors. Applied Physics a materials science & processing. vol.94:515-519.

[6] D.B. Strukov, J. L. Borghetti and R.S. Williams 2009. Coupled ionic and electronic transport model of thin-film semiconductor memristive behavior. Small, vol. 5:1058-1063.

[7] Yao-Feng Chang ; Yen-Ting Chen ; Fei Xue ; Yanzhen Wang ; Fei Zhou ; Burt Fowler ; Jack C. Lee. 2012. Study of SiOx-based complementary resistive switching memristor. Device Research Conference.1-7.

[8] A. Ascoli ; R. Tetzlaff . 2015. Analytical model for ideal generic memristor circuits based on the theory of Volterra. IEEE Proceedings of Reliability by Design; 21:36-42.

[9] Ascoli, A., Corinto, F., Tetzlaff, R. 2015.: Generalized boundary condition memristor model. Int. J. Circ. Theory.1-25.

[10] X. Guan, S. Yu, H.-S. P. Wong. 2012. A SPICE compact model of metal oxide resistive switching memory with variations. IEEE Electron Device Lett., 33:1405-1407.

[11] E. Gale, A. Adamatzky, B. De Lacy Costello. 2015. Slime mould memristors. *BioNanoScience.*, 5(1):1-8.

[12] Chua, L. O. 2014. If It's Pinched, It's a Memristor. Special Issue on Memristive Devices, Semiconductor Science and Technology. 29(10):42-48.

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