CRITICAL REVIEW ON VISCOUS FINGERING OF NON-NEWTONIAN FLUID DEVELOPED IN HELE-SHAW CELL

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***Abstract*— The development of the viscous fingering instabilities in Hele-Shaw cell has great practical and scientific importance. The displacement of a high-viscosity non-Newtonian fluid by a low-viscosity Newtonian fluid in a Hele-Shaw cell is capable of producing ramified viscous-fingering patterns exhibiting fractal characteristics. In this paper, we briefly overview some theoretical approaches and empirical modeling approaches of the fingering instabilities of non-Newtonian fluid in Hele-Shaw cell, which have been developed over recent decades.**

***Index Terms*— Viscous fingering, Hele-Shaw cell, Non-Newtonian fluid, Micro fractals, Interfacial instability**

I. INTRODUCTION

The study of the fingering instabilities and the different attributes under which viscous fingerings are formed has long been the area of interest in the field of engineering. When the Non-Newtonian fluid is compressed by Newtonian fluid in the porous media or in two smooth flat plates squeeze flow is occurred. These plates are separated by small distance and excited by motion of one of the plate in direction normal to flat surface [1]. At the time of separation of plates the fingering instabilities are formed. These fingering instabilities have great practical and scientific importance. The behavior of generating fingering instabilities is studied with help of Darcy's law. Darcy's law is phenomenologically derived constitutive equation that describes the flow of a fluid through a porous medium. Darcy's law at constant elevation is a simple proportional relationship between the instantaneous discharge rate through a porous medium, the viscosity of the fluid and the pressure drop over a given distance. These fingering instabilities are changed according to different input conditions like quantity of fluid, fluid properties, velocity of lifting cell.

II. LITURATURE REVIEW

The purpose of this literature review is to give an overview of the knowledge on viscous fingering, physics of its formation and their instabilities. The work done towards modeling, simulation and controlling fingering formation is discussed exhaustively in this section. Assumptions made in modeling the phenomenon are also discussed in the section. At the end of the review the possible directions to be explored are identified and are summarized. The research papers which were studied to understand the above parameters are as follows.

Anke Lindner et. al. (2002) [1] had studied the viscous fingering or Saffman-Taylor instability in two different polymer solutions each exhibiting in essence only a single non-Newtonian flow property. Authors stated that the viscosity of solutions of stiff polymers has strong shear rate dependence. Narrower fingers are found for rigid polymers for non-Newtonian fluid with respect to Newtonian fluid. Solutions of rigid polymers show a strong shear-thinning behavior, but have negligible elastic effects. Flexible polymer solutions, on the other hand, show negligible shear thinning but strong elastic effects, notably normal stresses that can easily exceed the viscous stresses and a large elongational viscosity.

M. Dutta Choudhury et. al. (2014) [2] reported in their paper that fingering instability at the interface separating different fluids may develop under different conditions. Authors used potato starch gel of different concentrations in Hele-Shaw cell and taken 'n' numbers of readings starting with a fresh substrate. When potato gel is squeezed between Hele-Shaw cell average radius of blob increases with time, shown in Figure 1. It must be noted that the radius does not increase monotonically with time in the latter section due to oscillations of fluid. Figure 2 gives demonstrate the time development with radius in chronological order. This graph looks like a random walk, but here the back and forth motion shows clearly the non-monotonic variation of the wavelength as the radius increases and decreases repeatedly with time.

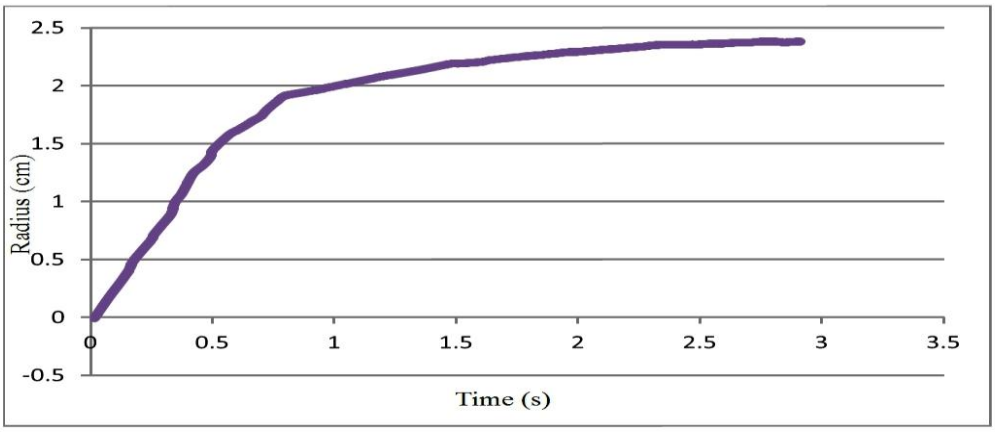

Fig 1: Variation of finger radius with time. [2]

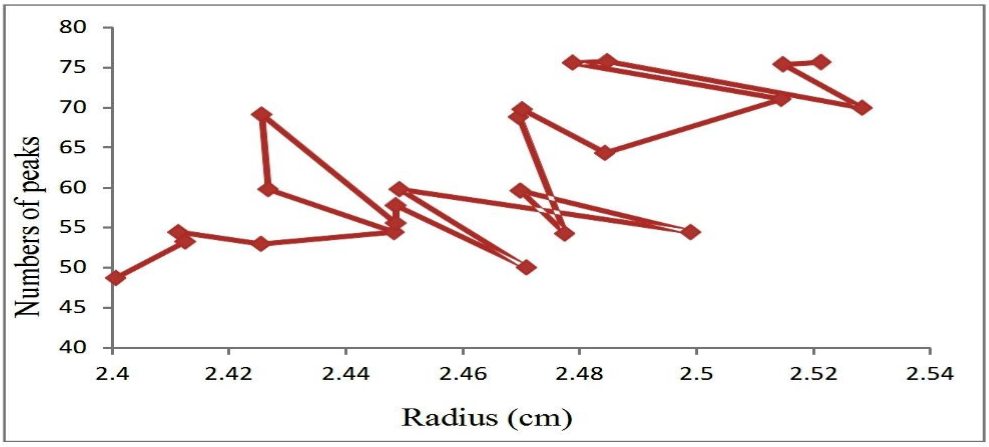

Fig 2: Chronological development of the number of peaks with average radius of the blob. [2]

Authors also reported reverse effect when a non-Newtonian fluid is squeezed between two plane surfaces by applying a force and it was observe that a wave-like irregularity develops on the interface, though the viscosity of the air surrounding the fluid is negligible compared to the apparent viscosity of the thick potato starch gel under study. Authors had experimentally demonstrated a reverse fingering, the 'reverse' fingering is defined as instability in a fluid-fluid interface with high pressure on the higher viscosity side. Reverse fingering demonstrated on a higher viscosity (thick potato starch gel in this case) fluid penetrating into a lower viscosity fluid, (air in this case) in the form of undulating fingers

Ljubinko Kondic et. al. [3, 4] studied the Saffman-Taylor instability of a non-Newtonian fluid in a Hele-Shaw cell. The Saffman-Taylor instability occurs when a fluid is pushed by another one of lower viscosity in a confined geometry, such as porous media or a Hele-Shaw cell. Saffman & Taylor (1958) [5] gave the first theory and performed simulated experiments in a Hele-Shaw cell. In the process of finger formation gas bubble may be trapped between the Hele- Shaw cells. Authors explored the Saffman-Taylor instability of a gas bubble expanding into a shear thinning liquid in a radial Hele-Shaw cell. For a weakly shear-thinning liquid, author calculated corrections to the Newtonian instability of an expanding bubble in a radial cell. Author performs simulations of bubble expanding into a strongly shear thinning liquid using Darcy's law generalized for non-Newtonian fluids. The simulations showed that shear thinning significantly influences the developing interfacial patterns. Shear thinning can suppress tip splitting, and produce fingers which oscillate during growth and shed side branches [4]. Emergent length scales showed reasonable agreement with a general linear stability analysis.

Takehiro Yamamoto et. al. (2001) [6] used the finite element method together with the volume of fluid (VOF) method for the representation of an interface between two viscous fluids. VOF is a numerical technique for tracking and locating the free surface (or fluid-fluid interface). Authors treated interfacial tension as a volumetric force using the continuum surface force (CSF) model. They compare viscous fingering in a Newtonian fluid and non-Newtonian fluid. The computation was performed under various conditions of initial finger shape, interfacial tension, and mean velocity. The numerical results qualitatively predicted the typical phenomena such as shielding, spreading, splitting and the effect of shear- thinning viscosity.

Ekkehard Holzbecher [7] focused on the classical constellation of miscible displacement, as it has been investigated in Hele-Shaw cells. Using COMSOL Multiphysics Authors investigate the solution of a 2D generic set-up in an Eulerian system and explored the fingering solutions in terms of various numerical parameters, i.e. for meshes of various type and refinement. Execution time, i.e. the performance of the model was examined.

Christophe Chevalier et. al. (2006) [8] presented in his study that the inertia of the fluid may become important for high finger speeds for the Saffman–Taylor instability. Authors investigated the effects of inertia on the width of the viscous fingers experimentally and found that, due to inertia, the finger width can increase with increasing speed.

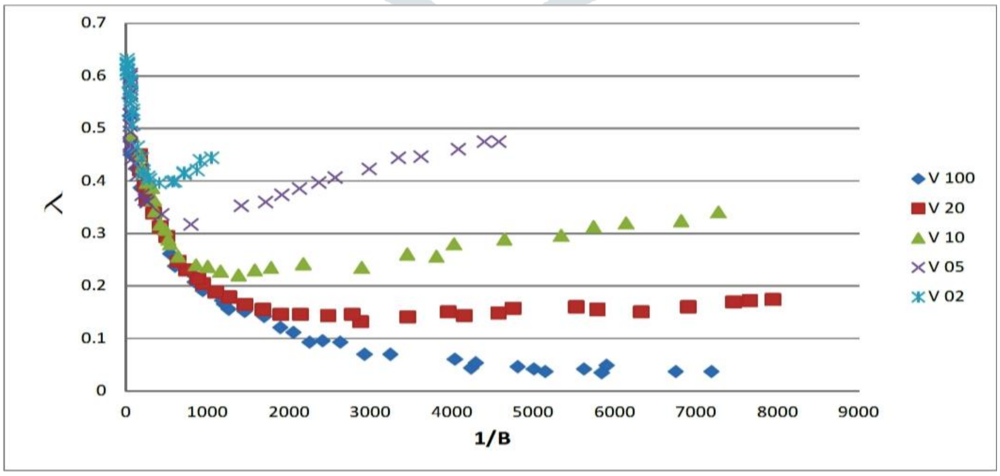

Fig 3: Results for the finger width λ as a function of the classical control parameter 1/B: for fixed geometry [8].

Authors did two type of experiment first they fixed the geometry and used different fluid and next they consider the different geometry for the same fluid. The different fluid used is silicon oils 47V02, 47V05, 47V10, 47V20 and 47V100. Figure 3 represent the relative finger width λ, (λ = w/W, W= width of the cell and w= width of finger) as a function of the classical control parameter 1/B .The classical control parameter define as

$$\frac{1}{B} = 12\left(\frac{w}{b}\right)^2 Ca \qquad (1)$$

Where w=finger width, b= spacing between plates and Ca is capillary number. Capillary is ratio of viscous to surface or interfacial tension

$$Ca = \frac{\mu V}{\gamma} \qquad (2)$$

Where μ is the dynamic viscosity of the liquid, V is a characteristic velocity and γ is the surface or interfacial tension.

First, varying the viscosity of the fluid for a given geometry b=0.75 mm, W =4 cm, is considered. This Figure 3 showed for low value of classical parameter 1/B the relative finger width decrease. As the value of 1/ B increase the relative finger width also increases. This surprising observation systematically appears at high Reynolds numbers, and it can be concluded that it must be related to inertial effects. Indeed, for a given geometry, only the fluid of highest viscosity gives results that agree with the classical Saffman–Taylor instability. Now the different geometry has been considered for the, silicon oil 47V05. The different geometry considered is as follows:

Table 1: Different Cell Geometries

	Spacing b	Width W	Aspect Ratio W/b
Geometry 1	0.25 mm	40 mm	160
Geometry 2	0.75 mm	80 mm	107
Geometry 3	0.75 mm	40 mm	53
Geometry 4	1.43 mm	40 mm	28

Figure 4 showed the increase of the finger width occurs for lower 1/B for a channel with larger plate spacing. All these observations agree with the suggestion that the increase in finger width with increasing velocity is due to inertial effects.

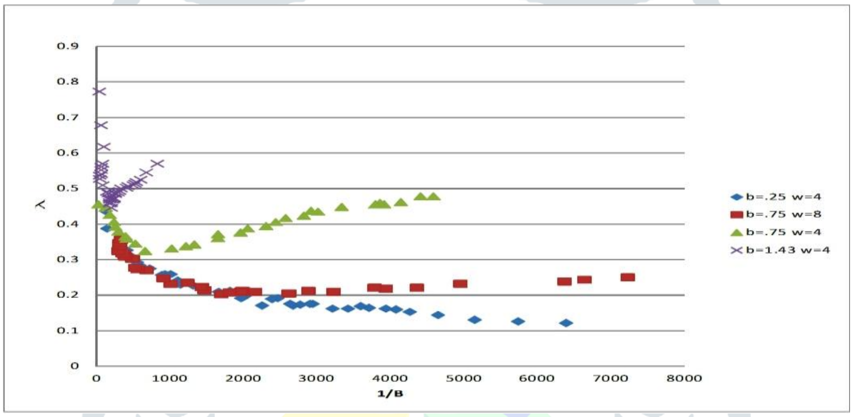

Fig 4: Results for the finger width λ as a function of the classical control parameter 1/B: for fluid silicon oil 47V05 [8]

Comparing the data for a fixed gap thickness (b=0.75 mm) and two different channel widths (W =4 and W =8 cm) in Figure 4, It was concluded that the cross over value of 1/B also depends on the channel width W. It was observed that for smaller channel width the relative finger width increased and for higher channel width relative finger width decreased. Hence, it was concluded that due to inertia experimental results deviate from the classical results. A regime of increasing finger width was observed. This increase occurs at lower1/B for lower viscosity, larger gap thickness or smaller gap width.

Rodolfo Brandao et. al. (2013) [9] have investigated stretch flow of a non-Newtonian fluid Confined in Hele-Shaw cell for which the upper plate is lifted. Large number of investigations is done on the of the lifting Hele-Shaw problem, performed mainly through experiments. The authors added to this body of work with weakly nonlinear analytical solutions. Authors try to gain analytical insight into the dynamic process of finger shape and competition by exploiting the onset of nonlinear effects. This is achieved by using a modified Darcy model. It describes a lifting Hele-Shaw interfacial flow where the displaced fluid exhibits a shear-dependent viscosity. In this direction, the dominant direction of fingering was determined, and the finger width behavior is analyzed depending on the shear nature (shear thinning or shear thickening) of the stretched fluid.

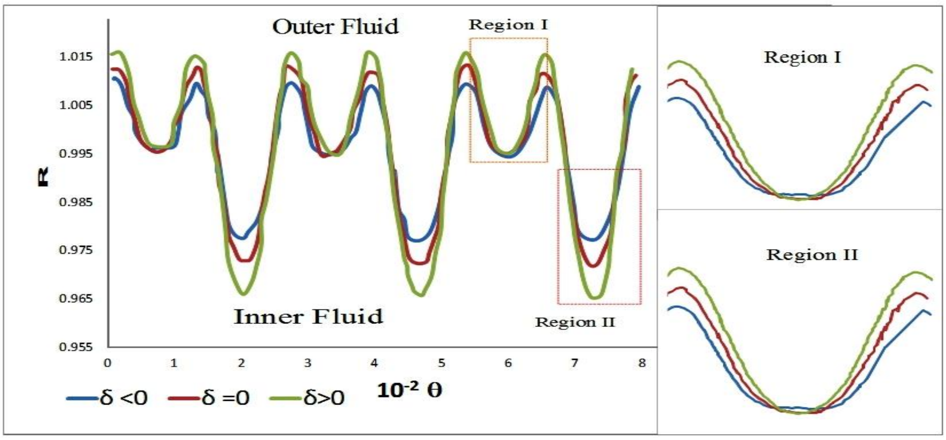

Fig 5: Fluid-fluid interface position R as a function of the polar angle θ. Three different values of δ are used as follows: -5.0×10^5, 0 and 5.0×10^5[9].

Authors have deeply understanding of influence of the non-Newtonian effects on the morphology and finger competition dynamics of the evolving interface. Figure 5 is a plot of polar angle θ and R interface position. The three cases shear thinning, shear thickening and Newtonian case was considered. δ is a small parameter (|δ| <<1) that expresses the non- Newtonian nature of the displaced fluid: δ = 0 corresponds to the Newtonian case, while δ >0 describes the shear-thinning case, and δ <0 refers to the shear-thickening situation. If the displaced fluid is shear thinning the finger competition among inward-pointing fingers is more intense. On the other hand, finger competition is inhibited if the bulk fluid is shear thickening. Shear thickening fluid being even less intense than the finger competition in the purely Newtonian case, which is clearly shown in Figure 5.

A close-up view of regions I and region II of Figure 5 (areas shown by square box toward positive value of x axis) is shown in inset. In inset one can see that the inward-pointing fingers are clearly wider, presenting flatter tips, if the stretched fluid is shear thickening. In contrast, the equivalent fingers produced in the shear-thinning case are not as wide, being actually narrower than the inward-pointing fingers obtained for the Newtonian lifted fluid situation.

Joao V. Fontana et. al. [10] concentrated on controlling and minimizing protocols considering the situation, in which the displaced fluid is a non-Newtonian, power-law fluid. In this work authors considered the displacement of a viscous non-Newtonian (power-law) fluid by an inviscid fluid in a radial Hele-Shaw cell. Initially authors fixed the total amount of injected fluid. They employed a variational approach for obtaining the optimal injection process and it was found that the optimization process is substantially dependent on the power-law index. Then, considered the total amount of injected fluid is not fixed, and focused on process that intended to control the total number of resulting fingers arising at the interface. It was found that regardless the nature of the displaced fluid (Newtonian, shear thinning, or shear thickening) the desired injection rate (with proportionality constant) depends on the power-law index α. By simulations efficiency of these two different controlling strategies has been verified.

III. CONCLUSION

The viscous fingering instability leading to micro fractals has been extensively studied for more than five decades. Research papers related to viscous fingering of Non-Newtonian Fluid, modeling and simulation of fingering formation are critically reviewed. It is seen from the literature survey that:

1. Many of the attributes like, time dependent viscous fluid, varying velocity of upper plate of Hele- Shaw cell, inclination of confined area and different cavity patterns on flat surfaces which affect the formation of different viscous fingering pattern is not explored by the researchers.
2. Furthermore various governing models were proposed in the literature on mechanics of formation of micro fractals under different conditions. However, generalized governing modeling have not been presented and established.
3. Maximum research is done on the Hele- Shaw cell using Newtonian fluid. Some researchers [1-4] [6-7] [9-11] used non-Newtonian fluid in the Hele-Shaw cell. However, the factors concerning to controlling and minimizing fractal geometry were not studied exhaustively.

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