A Study on Prime Labeling of Split Graph of Cycle C_n

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Abstract-
A graph G = (V, E) with n vertices is said to admit prime labeling if its vertices can be labeled with distinct positive integers not exceed n such that the label of each pair of adjacent vertices are relatively prime. A graph G which admits prime labeling is called a prime graph. In this paper, we investigate prime labeling for some classes of graph. In particular, we discussed on prime labeling of Split graph of Cycle C_n when n is odd or even. We have derived an algorithm which admits prime labeling to the Split graph of the cycle graph C_n.

Key words – Graph labeling, Prime labeling, Cycle graph and Split graph of a graph G.

I. INTRODUCTION

In labeling of graphs, we consider only simple, finite, undirected connected and non-trivial graph G = (V, E) with the vertex set V and edge set E. The number of elements of V, denoted as | V | is called the order of the graph while the number of elements of E denoted as | E | is called the size of the graph G. Spl (C_n) denotes the Split graph of the cycle graph C_n.

The notion of prime labeling originated with Entringer and was introduced in a paper by Tout, Dabbouchy and Howalla [2]. Entringer conjectured that all trees have a prime labeling. Haxell, Pikhuriko and Taraz [8] proved that all large trees are prime graph. Many researchers have studied prime graphs, for example in Fu.H.C and Huany K.C [4] has proved that the path P_n on n vertices is a prime graph. In [6] Ganesan. V et al proved that the Split graph of the path P_n admits prime labeling. In [7] S.Meena and Vaithelingam have proved that the prime labeling for some fan related graphs. For latest survey on graph labeling, we refer to
[5] (Gallian J. A. 2017). For various graph theoretic notations and terminology we follow Bondy. J. A and U. S. R Murthy [1]

We will give brief summary of definitions and other information which are useful for the present task.

II. PRELIMINARY DEFINITIONS

Definition 2.1
The graph labeling is an assignment of numbers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (edges) or both then the labeling is called a vertex labeling (edge labeling) or total labeling.

Definition 2.2
Let G = (V, E) be graph with p vertices. A bijection $f : V(G) \rightarrow \{1,2,\ldots|V|\}$ is called a prime labeling if for each edge e = uv, gcd (f(u), f(v)) = 1 . A graph which admits prime labeling is called a prime graph.

Definition 2.3
For $n \geq 3$, an n – cycle (or simply cycle graph) denoted by C_n, is a connected graph consisting of all vertices with degree two. A cycle graph C_n with n vertices has n edges.

Definition 2.4

For a graph G, the Split graph which is denoted by Spl(G) is obtained from G by adding to each vertex v, a new vertex v' such that v' is adjacent to every vertex that is adjacent to v in G.

III. MAIN RESULTS

3.1 Algorithm-

Prime Labeling of Split Graph of Cycle C_n - Step 1-

Let C_n be the cycle with n vertices $v_1, v_2, \ldots, v_n$ and let $v'_1, v'_2, \ldots, v'_n$ be the new vertices corresponding to each vertices $v_1, v_2, \ldots, v_n$ respectively.

Let G = Spl(C_n) be the Split graph of C_n

Step 2-

Obviously, |V(G)| = 2n and |E(G)| = 3n

Therefore, define a function $f : V(G) \rightarrow \{1,2,\ldots 2n\}$ as follows

$f(v_i) = 2i - 1$ for i = 1, 2, 3, …., n

$f(v'_i) = 2i$ for i = 1, 2, 3, …., n

Step 3-

Enumerate the different types of edges in G inwhich we have to check the relative prime of end points of each type of edges.

In G, there are three type of edges, namely v_iv_{i+1}, $v_iv'_{i+1}$, v'_iv_{i+1} for i = 1, 2, ..., n-1. Out of these three type of edges, we need to check only the relative prime of edges of type $v_iv'_{i+1}$ for i = 1, 2, ..., n-1.

Step 4-

In this step, we check the relative prime pair of vertices (v_i, v'_{i+1}) for i = 1, 2, ..., n-1 of the labeled graph G obtained in step 2-

If gcd($f(v_i)$, $f(v'_{i+1})$)) = 1 for i = 1, 2, …, n-1 then the graph G admits prime labeling If gcd($f(v_i)$, $f(v'_{i+1})$)) ≠ 1 for i = 1, 2, …, n-1 then we have to do the following step 5.

Step 5-

Suppose gcd($f(v_i)$, $f(v'_{i+1})$)) ≠ 1 for some i then select all those pairs of vertices v_i and v'_{i+1} for which $f(v_i)$ and $f(v'_{i+1})$ are not relatively prime and encircle each pairs with in a circle. Now, interchange the labels of v'_i and v'_{i+1} (where v'_{i+1} is the encircled vertex and v'_i is not). The procedure is repeated until all encircled vertex v'_i are exhausted. Now, the newly labeled graph admits prime labeling.

3.2 Illustrations-

Illustrations 1- Let n = 7 (n is odd), Spl (C_7) is a prime graph

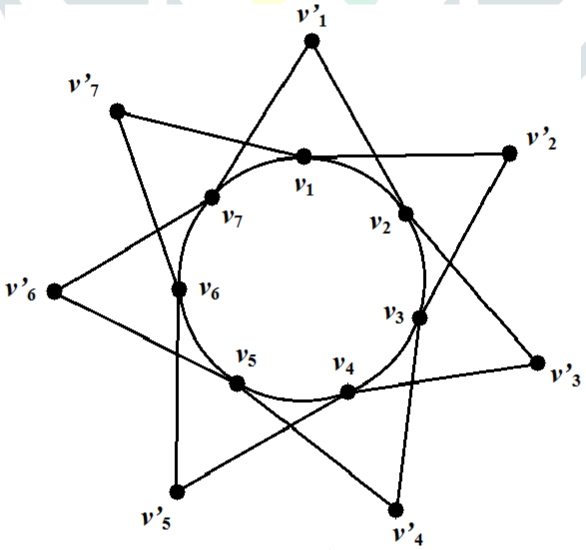

Figure 1. The Split graph of Cycle C_7, Spl (C_7)

Labeling the vertices of Spl (C_7) by using

$f(v_i) = 2i - 1$ for i = 1, 2,.......... 7

and $f(v_i') = 2i$ for i = 1, 2,.......... 7

we get the following labeled graph.

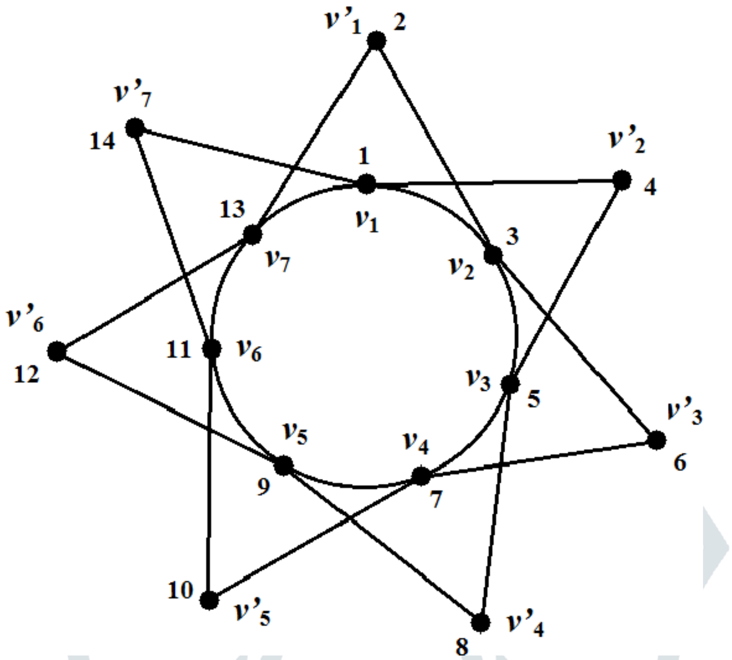

Figure 2. Labeling the vertices of Spl (C_7)

Checking the relative prime of each pair of vertices v_i and v'_{i+1} and mark the vertex v_i and v'_{i+1} within circles which are not relatively prime. We get the following graph.

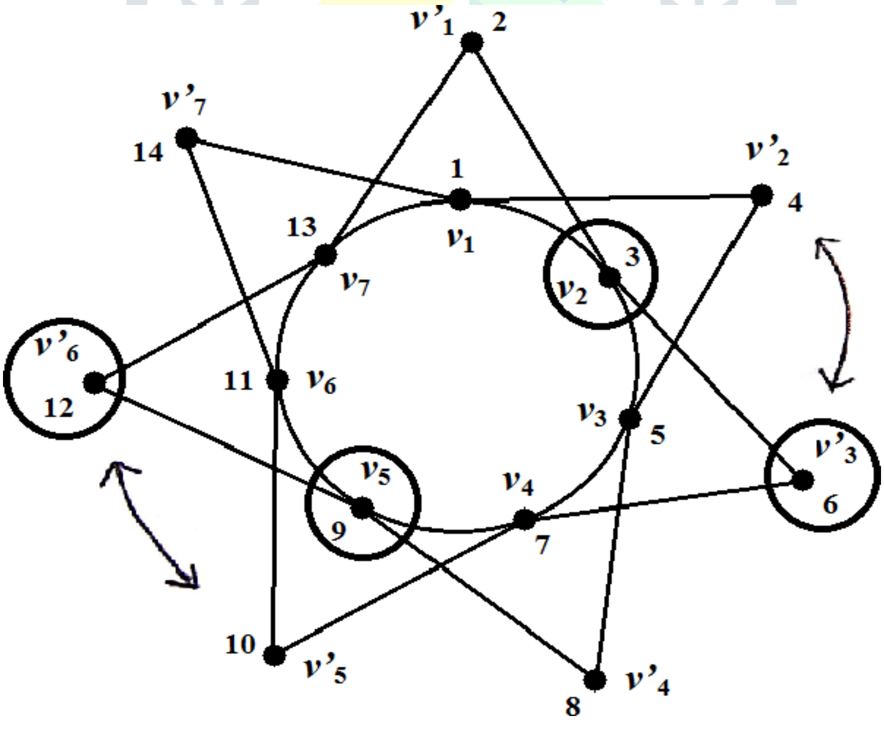

Figure 3. Checking and Encircling the non – prime pairs (v_i, v'_{i+1})

Interchange the labels of v'_{i+1} (which is encircled) and v'_i (which is not encircled). The procedure is continued for all encircled vertex v'_i we get the following resulting prime graph.

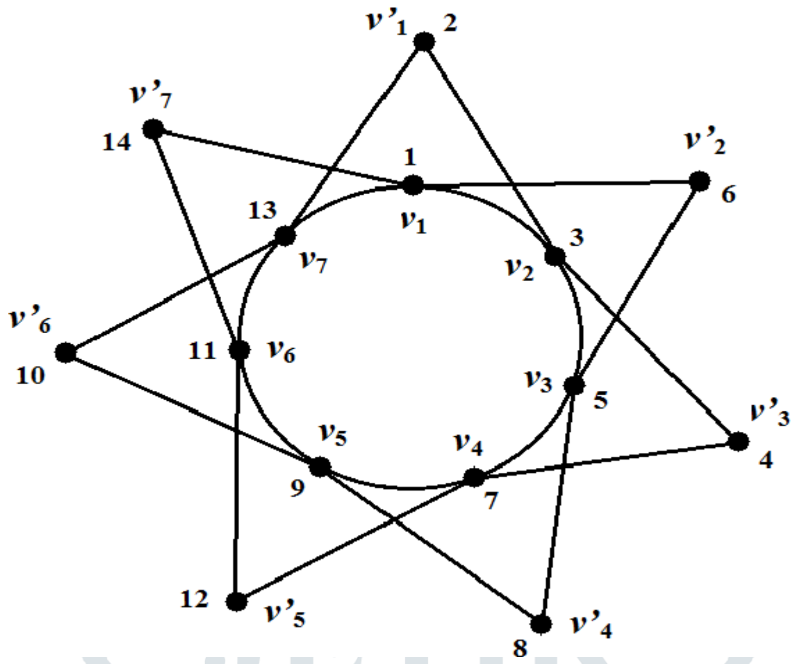

Figure 4. Prime labeling of $Spl(C_7)$

In the above graph,

gcd $(v_i, v_{i+1}) = 1$ for i = 1, 2,..........n-1
gcd $(v_i, v'_{i+1}) = 1$ for i = 1, 2,……..n-1
gcd $(v'_i, v_{i+1}) = 1$ for i = 1, 2,……..n-1

Therefore, Spl (C_7) admits prime labeling
Hence, Spl (C_7) is a prime graph.

Illustrations 2- Let n = 8 (n is even), Spl (C_8) is a prime graph

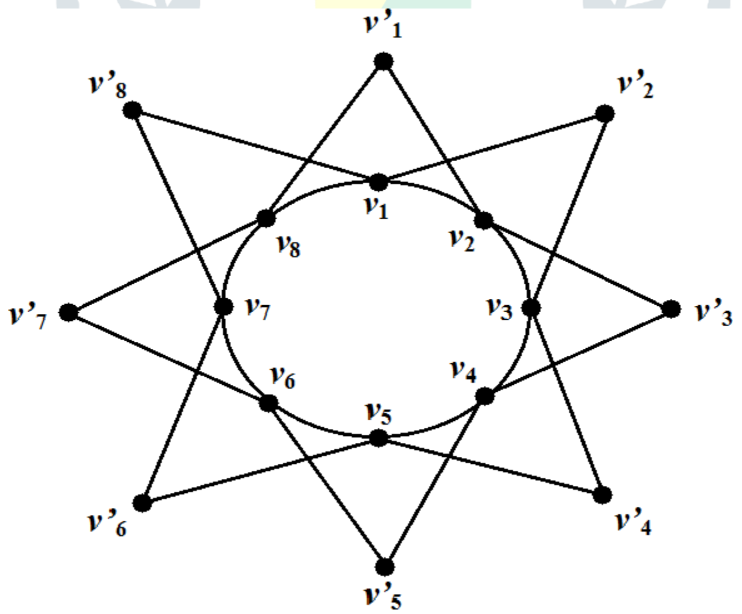

Figure 5. Split graph of C_8, $Spl(C_8)$

Labeling the vertices of $Spl(C_8)$ by using

$f(v_i) = 2i - 1$ for i = 1, 2, …….. n-1
and $f(v'_i) = 2i$ for i = 1, 2, …….. n-1

We get the following labeled graph.

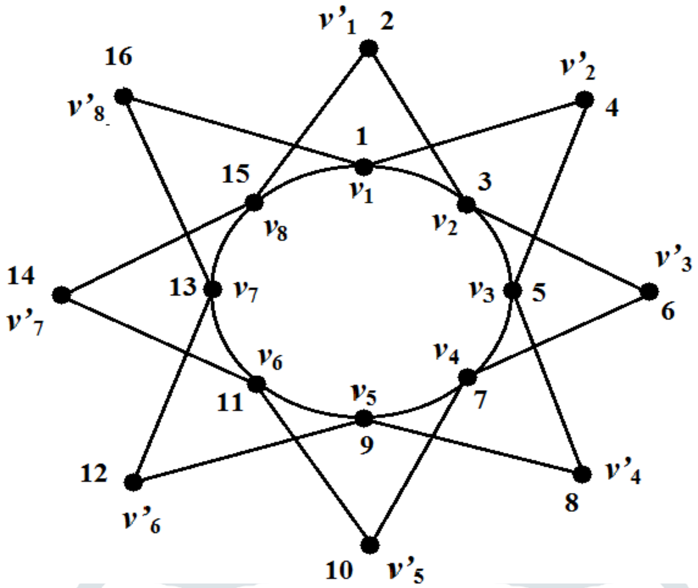

Figure 6. Labeling the vertices of Spl(C_8)

Check the relative prime of each pair of vertices v_i and v'_{i+1} within circle and mark the vertex with $f(v_i)$ and $f(v'_{i+1})$ within circles which are not relatively prime. We get the following graph.

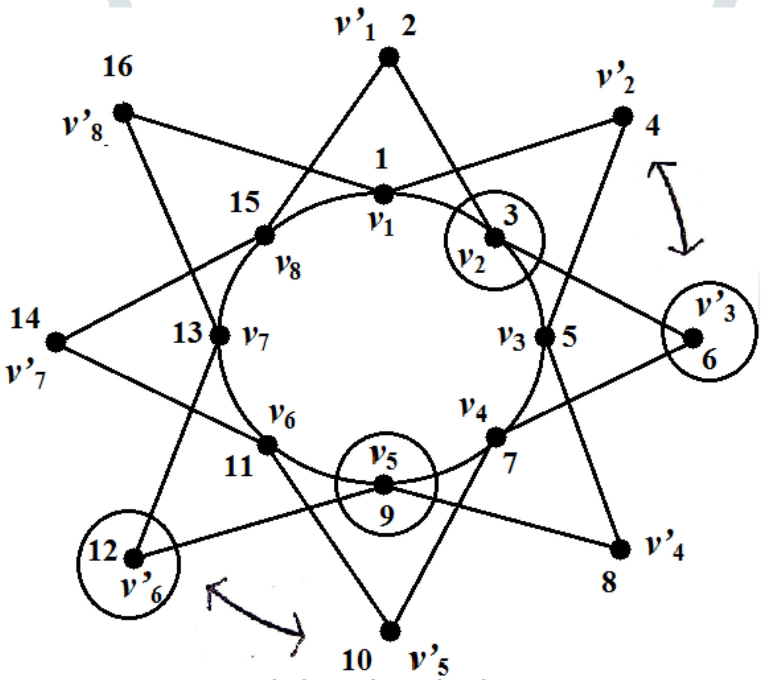

Figure 7. Checking and Encircling the non – prime pairs

Now, Interchange the labels of v'_{i+1} (which is encircled) and v'_i (which is not encircled). This procedure is continued for all encircled vertex v'_i we get the following prime graph.

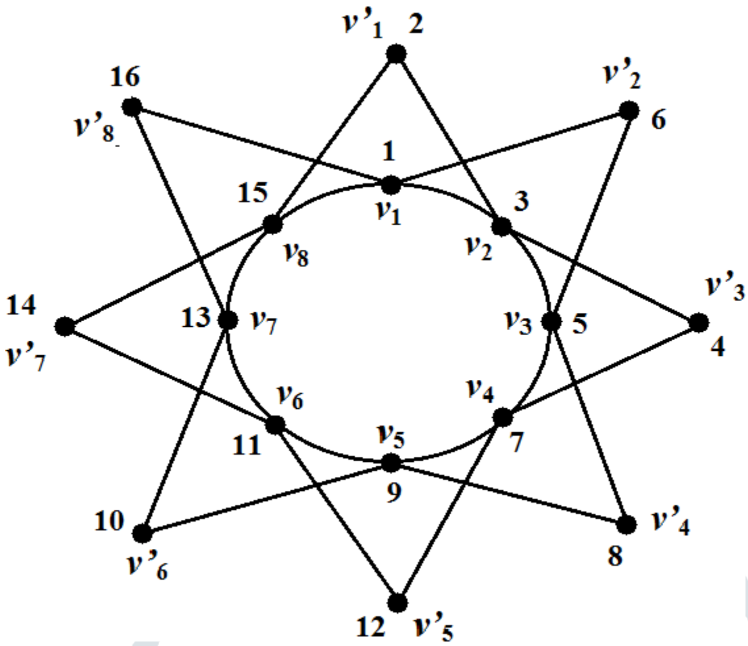

Figure 8. Prime labeling of Spl(C_8)

Therefore, Spl (C_8) admits prime labeling Hence, Spl (C_8) is a prime graph.

IV. CONCLUSION

We have presented an algorithm for prime labeling to some classes of graph such as Splitting graph of cycle C_n and illustrate with two examples for the cases n is odd and is even separately. Really, this will motivate the researcher to investigate the prime labeling of Split graph of other families like star, tree, etc., are prime graphs.

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