GAMES OVER AN IDEAL OF A TOPOLOGICALSPACE AND ITS FUZZIFICATION

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Abstract : *A game, over an ideal of a topological space is studied. The theory of fuzzy sets has also been applied to get the better results.*

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1. Introduction

An ideal over a topological space X determines both a topology of X, called I-topology as well as a local function I (X) analogous to the derived set. Associated with I it is possible to formulate a topological series as well as a localisation-series of set properties. These two series simplify the play of a game in the game theory. This paper is concerned with developing a theory of a particular game of pursuit and evasion over an ideal of a topological space and the theory of fuzzy sets has been applied on some results.

2. Games over an ideal

Let G (I, X) be an infinite positional game of pursuit and evason over I where X is a topological space and $I \subset P(X)$ s. t.

(i) I is closed with respect to union

(ii) I possesses heriditory property, Such collection I is called an ideal over X.

This game is played as follows : There are two players- P (Pursuer) and E (Evader). They choose alternately consecutive terms of a sequence $<E_n/n\in N$, where $N = \{0, 1, 2, ... n,......\} >$ of subsets of Xs.t. each player knows I, E_0, E_1 ,......, E_n when he is choosing E_{n+1}

A sequence $<E_n>$ of subset of X is said to be a play of the game if for all $n \in N$ the following holds :

(i) $E_0 = X$.

(ii) $E_1, E_3, E_5, ..., E_{2n+1}$ are the choice of P.

(iii) $E_1, E_3 E_5, ..., E_{2n+1} \in I$

(iv) $E_2, E_4, E_6, ..., E_{2n+2}$ are the choice of E.

(v) $E1, E2 \subset E0, E3, E4 \subset E2, ...; E2n+1, E2n+2 \subset E2n$.

(vi) $E_1 \cap E_2 = \phi$, $E_3 \cap E_4 = \phi$, ..., $E_{2n+1} \cap E_{2n+2} = \phi$.

If $\cap <E_{2n}> = \phi$ then player P wins the play, otherwise Evader wins the play.

A finite sequence $<E_m / m \leq n>$ is admissible for the game if the sequence $<E_0, E_1, ... E_n, \phi, \phi, ... , \phi>$ is a play of the game. For admissible sequence $<E_0, ... , E_n>$ and even n if $s : <E_0, ... , E_n> \rightarrow P(X)$ and $s\,(<E_0, ..., E_n>) = E_{n+1}$ then s is a strategy for player P.

In case of odd n, s is said to be strategy for evader E.

A strategy s is said to be winning for player P in the game $G\,(1, X)$ if P wins each play of the game with the help of this s.

Similarly s is said to be winning for E if E wins each play of the game with the help of s.

We denote by $P\,(I, X)$, the set of all winning strategies of P in the game $G\,(I, X)$ and by $E\,(I, X)$, the set of all winning strategies of E in the game $G\,(I, X)$.

A topologicl space X is said to be I-like if the set of all winning strategies of player P is not empty i.e. if $P\,(I, X) \neq \phi$.

Similarly, a space X is said to be anti I-like if the set of all winning strategies of player E is not empty. That is $E(I, X) \neq \phi$.

The game $G\,(I, X)$ is said to be determined, if $P\,(I, X) \neq \phi$ or $E\,(I, X) \neq \phi$ i.e. if X is I-like or X is anti 1-like.

The following properties follow from the definition of winning strategies :

If $I_1, I_2 \subset P(X)$ then

$$I_1 \subset I_2 \rightarrow P(I_1, X) \in P(I_2, X)$$
$$\rightarrow [P(I_1, X) \neq \phi \Rightarrow P(I_2, X) \neq \phi] ;$$
$$I_1 \subset I_2 \Rightarrow E(I_2, X) \subset E(I_1, X)$$
$$\Rightarrow [E(I_2, X) \neq \phi \Rightarrow E(I_1, X) \neq \phi].$$

THEOREM : 1

If the game $G\,(I,X)$ is determined in the favour of player P then the game G (I, E) is also determined in favour of P for all $E \in P\,(X)$.

PROOF :

Let $s \subset P\,(I, X)$.

We set $E_0 = X, F_0 = E, E_1 = s <E_0>, F_1 = F_0 \cap E_1$.

We form $t \in P\,(I, E\,)$ such that $t <F_0> = F_1$ then

$F_1 \in P(E) \cap I.$

Let $F_2 \in P(E)$ with $F_1 \cap F_2 = \phi$.

We set $E_2 = F_2, E_3 = s <E_0, E_1, E_2> F_3 = E_3$, and $t <F_0 , F_1, F_2> = F_3$.

Continuing in this manner, the plays $<E_n>$ of $G\,(I, X)$ and $<F_n>$ of $G\,(I, E\,)$ are obtained.

Now $E_{2n} = F_{2n}$ for all $n \geq 1$. $\cap <E_{2n}> = \cap <F_{2n}>$.

But $s \in P\,(I, X) \Rightarrow \cap <E_{2n}> = \phi$.

It follows that $\cap < E_{2n}> = \phi \Rightarrow t \in P\,(I, E)$.

THEOREM : 2

If $E \in P(X)$ and $E(I, E) \neq \phi$ then $E\,(I, X) \neq \phi$.

PROOF :

Let $s \in E\ (I, E)$.

To prove the theorem, it will be sufficient to define $t \in E\ (I, X)$, For, set $E_0 = X$, $F_0 = E$, and let $E_1 \in P(X) \cap I$.

Also set $F_1 = F_0 \cap E_1$, $F_2 = s <F_0, F_1>$, $E_2 = F_2$, and $t<E_0, E_1> = E_2$.

Assume $E_3 \in P(X) \cap$ I with $E_0 \subset E_2$.

Again set $F_3 = E_0$, $F_4 = s<F_1, F_2, F_3>$, $E_4 = F_4$ and $t <E_0, E_1, E_2, E_3> = E_4$.

Continuing in this manner, the plays $<E_n>$ of $G\ (I, X)$ and $<F_n>$ of $G\ (I, E)$ are obtained such that

$E_{2n} = F_{2n}$ for all $n \geq I \Rightarrow \cap <E_{2n}> = \cap <F_{2n}>$.

But $s \in E\ (I, E) \Rightarrow \cap <F_{2n}> \neq \phi$.

Hence $\cap <E_{2n}> \neq \phi \Rightarrow t \in E\ (I, X) \Rightarrow E\ (I, X) \neq \phi$.

DEFINITION : 3

A subset E of a topological space X is said to be locally I at a point x if there exists a neighbourhood (nhd) $U_{(x)}$ such that $E \cap U_{(x)} \in I$.

Space X is said to be locally I if it is locally I at each of its points. It is now possible to introduce the notion of I-derivative corresponding to the ideal I by setting : $I(E) = \{ X \in X/E$ is not locally I at x };

$$= \{x \in X/E \cap U(x) \notin \text{I, for all nhd } U_{(x)} \text{ of } x\} ,$$

which has the following properties ;

P1. $I(E \cup F) = I\ (E) \cap I\ (F)$, for all $E, F \subset X$.

P2. $I\ (E) = \phi$, if $E \in I$.

P3. $I\ I\ (E) \subset I\ (E)$

P4. $I(E)$ is a closed set and contained in E where $E = \phi\ (E) = I$ - derivative of null ideal.

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P5. $E \cup I\ (E)$ is a closure function (denoted by E^{-1}) satisfying :

Kuratowski's Postulates :

C1. $(E \cup F)^{-1} = E^{-1} \cup F^{-1}$

C2. $E \subset E^{-1}$,

C3. $E^{-1} = E$

C4. $\phi^{-1} = \phi$

Proof is obvious. .

3 . I-SCATTERED SPACE

The topology of X is defined by the closure function $E^{-1} = E \cup I\ (E)$ may be termed as I-closure topology.

DEFINITION : 4

A topological space X is said to be scattered if every nonviod subspace E has an isolated point related to E, i.e. if x is not a limit point of the subspace E.

Now 1-scatteredness of a space X with reference to the ideal I can be formulated by following definition.

DEFINITION : 5

A topological space X is I-scattered iff every nonviod subset E of X is locally I at every point $x$of X.

THEOREM : 6

If there exists an $F \in I$, so that $P(I, E) \neq \phi$ for each $E \in P(X)$ with $E \cap F = \phi$ then $P(I, X) \neq \phi$

PROOF :

Let $F \in I$ such that $P(I, E) \neq \phi$ for each $E \in P(X)$ with $E \cap F = \phi$.

Now $s_E \in P(I, E)$ can be picked up when $E \in P(X)$ such that $E \cap F = \phi$.

set $E_0 = X$, $s(E_0) = E_1 = F$.

Let $E_2 \in P(X)$ such that $E_1 \cap E_2 = \phi$

Then set $F_0 = E_2$, $F_1 = {}^{S}{}_{E}(F_0)$, $E_3 = F_1$ and $E_3 = s<E_0, E_1, E_2>$.

Assume $E_4 \in P(X)$ s.t. $E_4 \subset E_2$ and $E_3 \cap E_4 = \phi$.

Again set $F_2 = E_4$, $F_3 = s_E<F_0, F_1, F_2>$, $E_5 = F_3$ and $E_5 = s< E_0, ..., E_4>$ and so on.

Continuing in this manner, the plays $<E_n / n \in N>$ of $G(I, X)$ and $<F_n>$ of $G(I, F_0)$ where $F_n = E_{n+2}$ for all $n \in N$ can be obtained.

Now, $s_E \in P(I, F_0) \Rightarrow \cap <F_{2n}> = \phi \Rightarrow P(I, X) = \phi \Rightarrow S \in P(I, X)$.

THEOREM : 7

If X has a cover $\{ X_n / n \in N \}$ where $X_n \in P(X)$ and $G(I, X_n)$ is determined for each $n \in N$ then $G(I, X)$ is determined.

PROOF :

$G(I, X_n)$ is determined for each n.

$\Rightarrow P(I, X_n) \neq \phi$, for all n.

$\Rightarrow P(I, X) \neq \phi$.

Let $P(I, X_m) = \phi$ for some $m \in N$ and $G(I, X_n)$ is determined for each $n \in N$. Then $E(I, X_m) \neq \phi$, for some $m \in N$.

Hence by theorem 2, we have $E(I, X) \neq \phi$

$\Rightarrow G(I, X)$ is determined.

4 . Fuzzy Games

A game is determined by information, decisions and goals. But human notions (ideas) and decisions are fuzzy. For, a man with immense entropy functions may err, set right and understanding a little may increases his understanding in the pursuit of some knowledge. Therefore, in a game, perfect information, decisions & goals may not be feasible. We are therefore, led to the introduction of fuzzy games.

Let $G = (N,v)$ be a nonfuzzy game of the set $N = \{1, 2, 3,...., n\}$ of n players in which $v : S \rightarrow R$ is a real valued function (characteristic function) from a family of coalition $S \subset N$ to the set of real numbers R. Hence $v(A)$ means the gain which a coalition A can aquire only through the action of A. The coalition A can be specified by the characteristic function τ^A as follows :

$$\tau^A(i) = 0 \text{ if } i \notin A \text{ and } 1 \text{ if } i \in A.$$

A rate of participation τ^A (i) of a player i is defined by

τ^A (i) = 1, if a player i participates in A and

τ^A (i) = 0, if a player i does not participate in A.

Consequently, a coalition A is represented by

$\tau^A = (\tau^A(l), \tau^A(2),.... \tau^A(n))$

A fuzzy coalition τ is defined as a coalition in which a player i can participate with a rate of participation $\tau \in [0, 1]$ instead of $\{0, 1\}$. The characteristic function or coalitional worth function of a fuzzy game is a real valued function $f : [0, 1]^n \rightarrow R$ which specifies a real number $f(\tau)$ for any fuzzy coalition τ .

This fuzzy game is denoted by $FG = (N, f)$.

Conclusion :

In this paper, a new solution concept in fuzzy game, which is very useful in the field of game theory & fuzzy games.

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