TRANSIENT RESPONSE OF NONLINEAR SYSTEMS

[1]Janaiah Bolleddu, [2]P Laxmi Narayana, [3]J Narsaiah
[1]Assistant Professor, [2]Professor, [3]Assistant Professor
[1]Department of Mechanical Engineering, AVNIET, Telangana, India. [2]Department of Mechanical Engineering, UCE Osmania University, Telangana, India. [3]Department of Mechanical Engineering, AVNIET, Telangana, India.

ABSTRACT- Vibrations on the strings of a guitar are desirable because it produces a beautiful sound. However, vibration in a vehicle is very undesirable it can cause discomfort to the vehicle occupants.
Vibrations can be subdivided into: free and forced, undamped and damped, and linear and nonlinear vibration. If the frequency of the external force coincides with one of the natural frequencies of the system, a condition known as resonance occurs, and the system undergoes dangerously large oscillations. Undamped vibration is when no energy is lost or dissipated in friction or other resistance during oscillations. Consider a typical case of nonlinearity found in the heat exchanger tubes commonly used in industry, where tube itself is linear, but the clearance between the baffle plate and the tube introduces nonlinearity in the boundary conditions.
For the full vehicle model, the frequency response diagrams are plotted by defining the amplitude as the maximum absolute value of the amplitude of the displacement the frequency response diagram of the heave motion for the sprung mass.

Index Terms: Vibrations, Damped, Undamped, Linear, Nonlinear, Oscillations Heat exchanger, Frequency.

I. INTRODUCTION

Vibration, which occurs in most machines, structures, and mechanical components, can be desirable and undesirable. Vibrations on the strings of a guitar are desirable because it produces a beautiful sound. However, vibration in a vehicle is very undesirable it can cause discomfort to the vehicle occupants. Vibration is undesirable, not only because of the unpleasant motion, the noise and the dynamic stress, which may lead to fatigue and failure of the structure, but also because of the energy losses and the reduction in performance which accompany the vibrations (Buchholz, 2000).

Vibrations can be subdivided into: free and forced, undammed and damped, and linear and nonlinear vibration. Free vibration is when a system after an initial disturbance, is left to vibrate on its own. No external force acts on the system. If a system is subjected to an external force, the resulting vibration is known as forced vibration. If the frequency of the external force coincides with one of the natural frequencies of the system, a condition known as resonance occurs, and the system undergoes dangerously large oscillations. Undamped vibration is when no energy is lost or dissipated in friction or other resistance during oscillations. On the other hand, when energy is lost in this way, it is known as damped vibration. Linear vibration is when all the variable forces are directly proportional to the displacement, or to the derivative of the displacement, with respect to time. On the other hand, if any of the variable forces are not proportional to the displacement or to its derivatives with respect to time nonlinear vibration occurs. Nonlinear element can reside either in material properties or geometric effects. Nonlinearities could also manifest from the manufacturing tolerances in the form of backlash or play and also by design features like clearance. For example consider a typical case of nonlinearity found in the heat exchanger tubes commonly used in industry, where tube itself is linear, but the clearance between the baffle plate and the tube introduces nonlinearity in the boundary conditions.

II. LITERATURE REVIEW

This chapter presents a review of literature relevant to study the response of a non-linear system subjected to pulse excitation is analyzed. Anderson's ultra-spherical polynomial approximation method is applied to Cubic Duffing oscillator subjected to various pulses. The pulses considered are cosine, sine, exponentially decaying, step function, combination of step function and sine or cosine, combination exponential and sine or cosine, and combination of sine and cosine pulses. The analytical results are compared with the digital solution obtained, by using Runge-Kutta fourth order method. The method of transforming the displacement variable and applying the ultra-spherical approximation give results which compare well with the digital solution. In some cases the analytical results and digital results are not matched with each other and also as the nonlinear parameter increases the variation in between analytical and digital solutions are increased.
A vehicle can be modeled as a complex multi-body dynamic system. The degree of complexity depends on the aim of modeling. Several models have been developed in the researches related to the dynamic behavior of vehicle and its vibration control. The simplest representation of a ground vehicle is a quarter car model (2 DOF system) with a spring and damper connected to the body to single vehicle, which in turn connected to the ground via the tire spring. A half car model (4 DOF system) as two wheel model is used for studying the heave and pitch motions. The four DOF model allows the study of both heave and pitch motions along with deflection of tires and suspensions. A more complex model is the full vehicle model with four wheel model with seven DOF for studying the heave, pitch and roll motions.

State space modeling is just another way of presenting differential equations and describing a dynamic system. The time responses are obtained using the state space modeling. The responses obtained in state space are similar to those obtained in Laplace. And also we clearly noted that the response of sprung mass is exactly similar to those obtained with Laplace in case of 2 DOF modeling. By increasing the damping coefficient from 0 to 3000 N.s/m, we got different values for sprung mass. The time response curves are plotted by increasing the damping coefficient from 0 to 2000 N.s/m. In case of 4-DOF modeling, state variables which are analyzed are, front body displacement, rear body displacement, front wheel displacement, rear wheel displacement, front sprung mass velocity, rear unsprung mass displacement and rear unsprung mass velocity. The time response curves are plotted, by increasing the damping coefficient value from 0 to 1500 N.s/m for sprung mass and unsprung masses. The mathematical modeling and MATLAB coding is used to get the different plots for changing system parameters. The simulation tool MATLAB is used to obtain the displacement response, velocity and vertical acceleration of sprung mass at different values.

III. PULSE RESPONSE OF NON-LINEAR SYSTEMS

The response of a non-linear, non-conservative, single degree of freedom system subjected to a pulse excitation is analyzed. A transformation of the displacement variable is affected. The transformation function chosen is the solution of the linear problem subjected to same pulse (Srirangarajan,H. R., and Srinivasan, P., 1975). With this transformation the equation of motion is brought into a form where Anderson's ultra-spherical polynomial approximation is applicable for the solution of the problem. The method is applied to system subjected to various pulses. The pulses considered are cosine, exponentially decaying, step function, sine, combination of exponential and trigonometric etc.

The problem of single-degree-of-freedom mechanical vibratory system subjected to a transient disturbance is found in many fields of engineering. When the restoring elements are linear, the solution is straight forward but when they are non-linear it is often necessary to use experimental, graphical and approximate analytical methods. While the effects of steady-state disturbances on the non-linear case are fairly well known the transient state has not received much attention. The ground motions were of various single-pulse types, including the rectangular, the cosine and "skewed" cosine shapes. Under some circumstances the non-linearity is reduced maximum distortion, but in other cases it acts to increase it (Robert S. Ayre.,).

ANALYSIS

Consider a single degree of freedom system subjected to a pulse $f(t)$. The equation of motion is $m\ddot{x} + c\dot{x} + kx + ax^3 = F(t)$,

$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x + \frac{a}{m}x^3 = \frac{F(t)}{m}$, $\frac{c}{m} = 2\gamma$, $\frac{k}{m} = \Omega^2$, $\frac{a}{m} = \mathcal{E}$, $\emptyset(x,\dot{x}) = x^3$,

$$\ddot{x} + 2\gamma\dot{x} + \Omega^2 x + \mathcal{E}\emptyset(x,\dot{x}) = f(t)$$

Where $\Theta(t,y,\dot{y}) = \emptyset\left(y + F, \dot{y} + \dot{F}\right)$. And Anderson's (1) ultra spherical polynomial approximation method is now applied to equation subjected to the initial conditions. The solution of the equation is assumed as

$y(t) = A(t)\cos\Psi$,

Where $\Psi(t) = \omega t + \theta(t)$, $\omega = (\Omega^2 - \Upsilon^2)^{1/2}$. Now, following Anderson yields

IV. RESPONSE OF SYSTEM TO PARTICULAR PULSES

As an example, a damped Doffing oscillator subjected to various pulses is considered. The equation of motion of system is

$\ddot{x} + 2\gamma\dot{x} + \Omega^2 x + \mathcal{E}\emptyset(x,\dot{x}) = f(t)$, (14)

$\Theta(t, A\cos\Psi, -\gamma A\cos\Psi - \omega A\sin\Psi) = (A^3/4)\cos 3\Psi + (3A^2F/2)\cos 2\Psi + (3A/4)(A^2 + F^2)\cos\Psi + F(F^2 + 3A^2/2)$.

COSINE PULSE:

Consider the system to be acted upon, from time $t = 0$, by an external force consisting of a cosine pulse of finite duration, T. Then the motion of the mass, in the interval in which the external force acts, is governed by the differential equation (14) subjected to initial conditions(2) , with $f(t) = \cos\alpha t$, where α is a constant. The displacement of the mass at time $t(<T)$ is obtained by applying the above analysis, as follows,

The equation (5) becomes

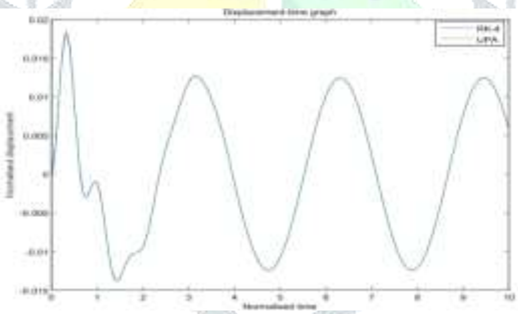


Figure: Plot of X vs t for Cosine Pulse

Equations (23), (19), (21), (9) and (3) determine the response of the system. In the numerical examples considered here, response curves are obtained for two values of the ultra-spherical polynomial index, $\lambda = 1/2$ (krylov and Bogoliubov) and $\lambda = 0$ and for the value non-linearity parameter $\varepsilon = 0.6$.

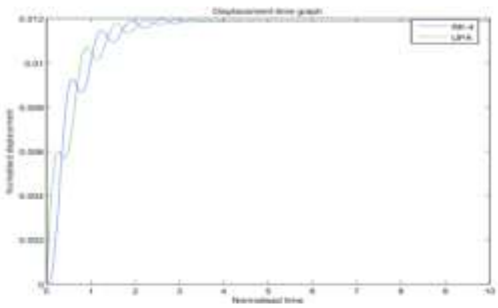


Figure: Plot of X vs t

V. VEHICLE DYNAMIC SYSTEMS

Since the vehicle dynamics is concerned with controllability and stability of automobile, it is important in design of a ground vehicle. Many mathematical models for vehicles were proposed and the analysis of the dynamic response of them has been examined in a larger number of previous investigations. In studying vehicle dynamics, three essential components are considered for typical ground vehicle, the main body of the vehicle, the suspension system and the wheels. The simplest representation of a ground vehicle is a quarter-car model with a spring and a damper connecting the body to a single wheel, which is in turn connected to the ground via the tire spring (Robson, 1979; Williams, 1997; Yang et al., 2001).

The mass representing the wheel, tire, brakes and part of the suspension linkage mass, is referred to as the unsprung mass. The quarter-car model is used only when the heave motion needs to be considered. A half car model is two wheel model (front and rear) for studying the heave and pitch motions (Moran and Nagai, 1994; Vetturi et al., 1996; Campos et al., 1999). This four degree-of-freedom model allows the study of the heave and pitch motions with the deflection of tires and suspensions. Comparing to the full 3-D vehicle model, the half-car model is relatively simple to analyze and yet can reasonably predict the response of the system (Oueslati and sankar, 1994). Therefore many researchers often use it. A more complex model is the full vehicle model which is a four wheel model with seven degree of-freedom done for studying the heave, pitch and roll motions (Ikenaga et al., 2000).

State Space Modeling: Quarter-Car

State space modeling is just another way of presenting differential equations and describing a dynamic system. It uses a set of 1st order differential equations. Physicists and control engineers have been using this modeling technique for years. The simplest representation of ground vehicle model is quarter car model. The model represents a single suspension from one of the corners of the vehicle; it is a 2-DOF often referred to as "quarter-car" model. The input to this model is a ground displacement input which is representative of a typical road profile. The input excites the 1-DOF through a spring element which represents tire stiffness. The unsprung mass is connected to the second degree of freedom, through the primary suspension spring and damper. This passive model has been used extensively for research without considering the damping of the tire, as it complicates the calculation and its effect is small and here it is considered as to make the model as close as possible to real model.

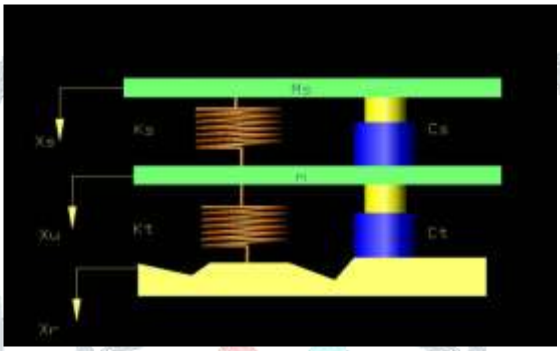


Figure: A Quarter Car Model (AUTOCAD)

State Space Modeling: Half Car Model

The model of a half-car suspension system is shown in figure below 4.2 (Campos et al., 1999); huang and lin., (2002, 2003); smith and wang, (2002)). The model is represented as linear four DOF system. It consists of car body connected to front and rear wheels at each corner. The sprung mass is free to heave and pitch motions. The sprung masses are free to bounce vertically with respect to the sprung mass. The suspensions between the sprung and unsprung masses are modeled as linear viscous dampers and spring elements, while the tire is modeled as simple linear spring without damping characteristic.

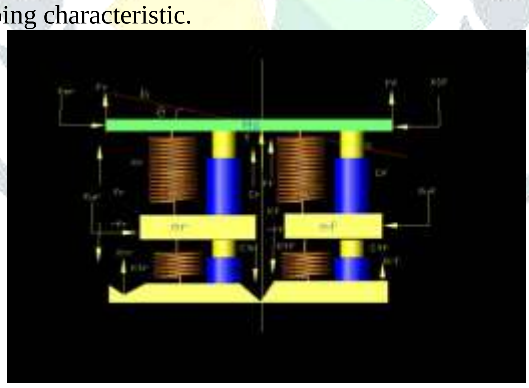


Figure: Half Car Model (AUTOCAD)

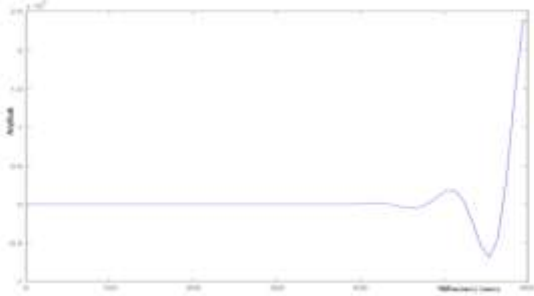
2- DOF Model

2-DOF model is solved by using Laplace transformation and state space variables. So the time response analysis for the displacement of sprung and unsprung is done here using MATLAB software. In order to get optimized step response, the damping coefficient "C_s" has been varied using true vehicle data. The road disturbance or the bump height is taken as 10 cm for simulating the response and vehicle is assumed to be moving with constant speed.

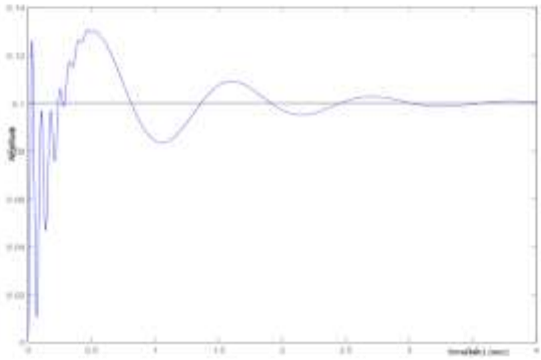
2-DOF MODEL:

Referring to chapter 3, 2-DOF model is solved by using Laplace transformation and state space variables. So the time response analysis for the displacement of sprung and unsprung masses software. In order to get optimized step response, the damping coefficient "C_s"has been varied using true vehicle data. The road disturbance or the bump height is taken as 10 cm for simulating the response and vehicle is assumed to be moving with constant speed.Laplace Transformation:

The damping coefficient has been varied from value zero up to optimum value to suggest the range for optimized response. Observations from figure (4.3) to figure (4.22) show the displacement characteristic of sprung mass with time with varying "C_S".Displacement characteristic of sprung mass and unsprung mass with time with varying "C_s".



Amplitude Response of Sprung Mass at C_S= 0 N.S/m

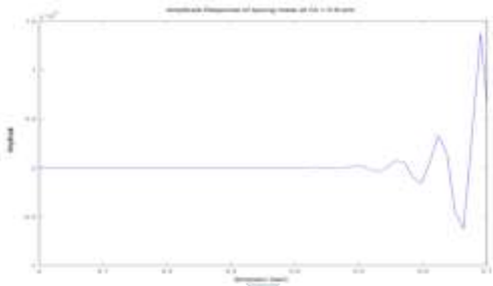


Amplitude Response of Unsprung Mass at C_S= 1000 N.S/m

On increasing the damping coefficient valve above 3000N.s/m it is noted that the amplitude of displacement is still decreasing but one can observe that the maximum overshoot (Mp) is high compared to the peak time (tp), which is very low. This condition is acceptable for the optimum comfort for the ride and as result of that vehicle will continue to oscillate with time resulting in discomfort to passenger and an undesirable condition for the vehicle stability. Hence it can be said that the damping value for this particular model should be between 2000N.s/m to 3000N.s/m.

STATE SPACE ANALYSIS

The time responses are obtained using state space modeling. The responses obtained in state space are similar to those obtained in Laplace. Here the sprung mass displacement and velocity with suspension travel (Y1=Xs-Xu) have been analyzed. It is clearly noted that response for sprung mass is exactly similar to those obtained with Laplace. With no damping, the amplitude of vibration goes on increasing and continues to vibrate for long a time. The setting time and maximum peak of amplitude decreases considerably as damping is increased, making ride comfortable.



Amplitude Response of Sprung Mass atC_S = 0 $N.S/m$

FOUR-DOF MODELING

Dynamic analysis of half car model was done using state space method. By using those equations in the time response of state space variables is obtained using MATLAB. The state variables analyzed are front body displacement, rear body displacement, front wheel displacement, rear wheel displacement, front sprung mass velocity, rear unsprung mass displacement and rear unsprung mass velocity.

Simulation Input Data for Half Car Model

M_s	575kg
m_f	60kg
m_r	60kg
j	769 kg/m^2
a	1.38m
b	1.36m
K_f	16812 N/m
K_r	16812 N/m
K_{tr}	190000 N/m
K_{tf}	190000N/m

Figure 4.30 to figure 4.33 show the amplitude response of front sprung mass at 0 N.s/m, 500 N.s/m, 1000 N.s/m and 1500 N.s/m damping value. It can be seen that when no damping is provided the amplitude is increasing with time which means passengers will be subjected to lot

of vibrations and damage to the vehicle. After increasing the damping, amplitude of vibration goes on decreasing resulting in decrease in settling time.

VI. 7-DOF NONLINEAR FULL VEHICLE MODEL

Introduction

The vehicle body is represented by a 3D model with seven degree-of-freedom rigid cuboids with mass $'m_s'$. The heave, pitch and roll motions of the sprung mass are considered. The four unsprung masses (front-left, front-right, rear-left, rear right) are connected to each corner of the rigid cuboids. It is assumed that the four unsprung masses are free to bounce vertically. The suspensions between the sprung mass and unsprung masses are modeled as nonlinear spring and nonlinear damper elements, while the tires are modeled as nonlinear springs with viscous damping (Mitsuaki Ishitobi., oct2005).

VII. Results and Discussion

For the full vehicle model, the frequency response diagrams are plotted by defining the amplitude as the maximum absolute value of the amplitude of the displacement as in (Belato et al., 2001), and the frequency as frequency of the sinusoid road disturbance. Figure.3 represents the resonance curves of the heave, roll and pitch motion of sprung mass and heave motion of unsprung mass in front-left corner as the forcing frequency f is slowly increased and then slowly decreased. The diagrams were calculated by using an increment $\Delta f = 0.001$ Hz as the variation of control parameter was changed 0.001 Hz with the time interval 50 s, so that the response diagrams can be presumed to be continuous.

Figure 2(a) and (b) show the frequency response diagram of the heave motion for the sprung mass. When the forcing frequency is increasing, there are three jumps at $f = 3.51$Hz, f =3.91Hz and f =4.82Hz, while between $f = 5.07$Hz and $f = 5.39$Hz the oscillations change into beats. As shown if figure 2(b), there is a small upward jump at f =4.76Hz and several small upward jumps around $f = 2.92$Hz as the forcing frequency decreases. The beat begins at $f = 5.39$ Hz and at f =5.07Hz. The new unstable region which does not exist as f is increasing appears in the forcing frequency 2.92 Hz $< f <$ 3.34 Hz. Figure 2(a) and (b) show that the number of jumps and unstable regions when forcing frequency is increasing are different from ones as forcing frequency is decreasing. This phenomenon was not observed in study of dynamic responses of four degree-of-freedom nonlinear half-car model.

The frequency responses for roll motion of sprung mass are illustrated in figure 2(c) and 2(d). There is one unstable region in 5.09$< f < 5.39$Hz when the frequency is increased. However, there are two unstable regions for decreasing frequency. Although the amplitude of the response is small, existence of unstable region and jumps indicate that chaos in roll motion may exist.

CONCLUSION

It is concluded that the method of transforming the displacement variable and applying the ultra-spherical polynomial approximation gives results which compare well with the digital solution. It can be seen that though the approximation gives erroneous results at the turning points of the response curves, error is not large. In other cases error between analytical and numerical is significant. Also as the non-linearity parameter 'ε' increases, the results of the approximation depart from those of the digital solution.

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