Application of Fuzzy Linear Programming In Mathematical Programming

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Abstract : We know that a linear programming problem is define as - $Max\ Z = C^T X$ Subject to the constraints $AX \leq B, X \geq 0$ Where $C, X \in R^n$ & $B \in R^m$. X & B are column vector and C is a row vector. $A = (a_{ij})$ is a vector of the order $m \times n, m \leq n$.
In this paper, we survey on methods and technique for solving fuzzy linear programming (FLP). We classified the problems in three type of categories based on fuzziness in constraints, coefficients and objective functions and provide its solution. These three ways are-

a) There is fuzziness in the accomplishment of the constraints.
b) Fuzziness in the coefficients a_{ij} and b_i.
c) There is a situation where there is fuzziness in accomplishment of the constraints as well as fuzziness in "a_{ij}" and "b_i".

Keywords: Fuzzy Linear Programming, Fuzzy Objective Function, Fuzziness in constraints and optimal value.

I. INTRODUCTION

Linear programming problem (LP) is one of the main branches of operation research technique. A linear programming model deals the real world problems. It composes the optimization model whose objective function and constraints are linear on the decision variable. Due to their simplicity it has been extensively used in various branches of science, Engineering, Medicine etc. Its application in mathematical programming was started by Belman and Zadeh in 1970 [12], since then, models have been suggested which allow flexibility in constraints and fuzziness in the objective function in linear & Non-linear programming. After that many researchers adopted this concept for solving fuzzy linear programming problem like. H. Tanaka in 1974[11] , H. J. Zimmerman in 1978[10], H.R. Meleki in 2000[5], K.Ganesan in 2006[4], A. Ebrahimjad in 2010[3], C. Veeramani in 2012 [2], D. Stephen in 2017[1]. Here we will consider some fuzzy models in linear programming and provided the solution of related example.

II. SUBJECT MATTER

Generally, we know the fuzzy linear programming is a crisp LP problem consists in maximizing/minimizing a function subject to the constraints over the variables.

$$\left.\begin{array}{l} MaxZ = C^T X \\ s.t.\ Ax \leq b, X \geq 0 \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where $A \in M_{m\times n}(R)$ is a matrix of the real number $C \in R^n$ is a cost vector and $b \in R^m$ is a vector.
In the above formulation all coefficients are assumed to be perfectly known. Now fuzziness in (1) can be considered in different way. For example, elements of either A, b or C can be fuzzy number rather than crisp numbers, the constraints can be represented by fuzzy sets rather than by crisp inequalities or the objective functions can either be represented by a fuzzy set or by a fuzzy function etc. In our discussion, we will consider only those fuzzy linear programming problems (in short FLP problems) in which there is no fuzziness in the objective function or its cost coefficients.
Now we consider three type of FLP
(a) First we consider the case in which the decision maker permits violations in the accomplishment of the constraints in the sense that he allows the constraints to be satisfied "as far as possible" , Now this "as far as possible" is a fuzzy expression can be explained by a membership function.

$$\mu_i(X) = \begin{cases} 1 & \text{if } a_i X \leq b_i \\ f_i(a_i, X) & \text{if } b_i \leq a_i X \leq b_i + \bar{b}_1 \\ 0 & \text{if } a_i X \geq b_i + \bar{b}_1 \end{cases}$$
$$i = 1,2 \ldots\ldots\ldots..m.$$

Where $a_i X \leq b_i$ is the ith constraint in "Eq.1" and f_i are assumed to be continuous and non-increasing. The fuzzy constraints in this case will be represented Symbolically as

$$a_i X \leq b_i\ , i = 1,2,3 \ldots\ldots\ldots\ldots..m$$

In above, b_i is a value up to which the decision maker tolerates violation in each constraint. The associated FLP in this case, can be represented as follows -

$$\left.\begin{array}{l} Max\ C^T X \\ \text{Subject to } AX \leq b, X \geq O \end{array}\right\} \quad \ldots\ldots\ldots\ldots..(2)$$

This problem was described and solved in [11] and [10] in different ways. Later on in [8] a general method was found for solution of (2) , which involved the above solution as particular cases. This solution was obtained from the solution of the following LP problem.

$$\begin{array}{l} \text{Max } C^T X \\ \text{Subject to} \\ \quad AX \leq g(\propto) \\ \quad X > O, \propto\in (0.1] \end{array}$$

Where $g(\propto)$ is a column vector defined by the inverse function of the $f_i, i = 1,2, \ldots\ldots\ldots..m$.

(b) Next we consider FLP problem with fuzzy coefficients. Suppose in (1) the decision maker does not know exactly the value of the coefficients $a_{ij}'s$ and $b_i's$. In order establish some linear constraint.

$$a_{i1}x_1 + a_{i2}x_2 + - - - - + a_{in}x_n \leq b_i$$

He has some information about the constraint as follow coefficients have values approximately equal to $a_{ij}, j = 1,2 \dots \dots \dots . . n$ and never greater than $(a_{ij} + \bar{a}_{ij})$ nor less than$(a_{ij} - \bar{a}_{ij})$. Further, the value on the right hand side is about b_i with limit $(b_i + \bar{b}_1)$ and $(b_i - \bar{b}_1)$. Then the corresponding FLP problem can be explained as follow-

$$\text{Max } C^T X$$
$$\text{Subject to}$$
$$\sum_{J=1}^{n} a_{iJ} X_J \leq b_i \qquad \dots\dots\dots\dots\dots(3)$$
$$X \geq 0 \quad \& \quad i = 1,2, \dots \dots \dots \dots m$$
$$J = 1,2, \dots \dots \dots \dots n$$

Where a_{iJ}, b_i are LR type fuzzy numbers in the sensor of [9].

This problem was proposed and solved by Tanaka el al [7].

(c) In [6] general model for FLP problems has been discussed which includes (a) & (b) as particular cases. Here they have dealt with the situation in which a_{iJ} and $b_i, i = 1,2, \dots \dots \dots \dots m, J = 1,2 \dots \dots \dots \dots n$ are LR type Fuzzy numbers and there is fuzziness in the accomplishment of the constraints also in this case the corresponding FLP can be modeled as-

$$\text{Max } C^T X$$
$$\text{Subject to}$$
$$\sum_{J=1}^{n} a_{iJ} X_J \leq b_i \qquad \dots\dots\dots\dots\dots(4)$$
$$X_J \geq 0$$
$$i = 1,2, \dots \dots \dots \dots m$$
$$J = 1,2, \dots \dots \dots \dots n$$

The author has proposed an auxiliary problem to solve (4) which has been given in the following lines. Let t_i be a fuzzy number fixed by the decision maker giving his allowed maximum violation in the accomplishment of the constraint. Further let us assume that there exists a relation in the set N(R) of fuzzy number ranking them which preserves a ranking when the fuzzy number are multiplied by positive scalars. Then the i[th] constraints in (4) can be replaced by

$$\sum_{i=1}^{n} a_{iJ} X_J (<) b_i + t_i(1-\propto),$$
$$i = 1,2 \dots \dots \dots \dots \dots \dots . . m, \propto \in (0,1]$$

Which expresses that for $\propto = 1$, the constraint is completely satisfied as the decision maker wishes. The smaller the value of $\propto$, the smaller the accomplishment degree for the decision maker will be. Then the authors propose the following auxiliary problem to solve (4).

$$\text{Max } C^T X$$
$$\text{Subject to}$$
$$\sum_{j=1}^{n} a_{ij} X_J (<) b_i + t_i(1-\propto),$$
$$X_J \geq 0, \quad \propto \in (0,1]$$
$$i = 1,2 \dots \dots \dots \dots \dots \dots . . m$$

Here different solutions will be obtained according as different relations are considered.

III. EXAMPLE

Let us consider an example related to LP-

$$Max\ Z = u_1 + u_2$$
$$\text{Subject to}$$
$$u_1 \leq 1$$
$$u_2 \leq 1$$
$$u_1, u_2 \in R$$

Solution: - Given, Max Z = $u_1 + u_2$

Subject to

$$u_1 + 0 \times u_2 \leq 1$$
$$0 \times u_1 + u_2 \leq 1$$
$$u_1, u_2 \in R$$

With a unique solution

$$u_1^* = u_2^* = 1$$
$$\text{And} \quad u_1^* = u_2^* = 2$$

Let us assume that the decision maker sets the desired value of the objective Z to 3. Then we get the following linear programming

$$(1,\gamma)u_1 + (1,\gamma)u_2 \geq (3, b_0)$$
$$(1,\gamma)u_1 + (0,\gamma)u_2 \leq (1, b_1)$$
$$(0,\gamma)u_1 + (1,\gamma)u_2 \leq (1, b_2)$$
$$u_1, u_2 \in R \text{ , where } \gamma = 0.5 \ \& \ b_0 = b_1 = b_2 = 1$$

Which satisfies the in equations as much as possible.

Now we get the following statement: -

Find $Z = u_1 + u_2$ from the possible inequalities system.

$$(u_1, 0.5|u_1|) + (u_2, 0.5|u_2|) \geq (3,1)$$

$$(u_1, 0.5|u_1|) + (0, 0.5|u_2|) \leq (1,1)$$
$$(0, 0.5|u_1|) + (u_2, 0.5|u_2|) \leq (1,1)$$
$$u_1, u_2 \in R$$

Which satisfies the inequalities as much as possible.

Let us consider the situation

$$u_1 = u_2 = 1$$
$$(2,1) \geq (3,1)$$
$$(1,1) \leq (1,1)$$

Since we know that

$$\text{Pos}(A \leq B) = \begin{cases} 1 & \text{if } a \leq b \\ 1 - \frac{a-b}{\propto+\beta} & \text{otherwise} \\ 0 & \text{if } a \geq b + (\propto+\beta) \end{cases} \qquad \ldots\ldots\ldots\ldots.(\text{I})$$

i.e Pos $[A = (a, \propto) \leq \beta = (b, \beta) = 1 \Leftrightarrow a \leq b]$

and

$$\text{Pos }(A \geq B) = \begin{cases} 1 \text{ if } a \geq b \\ 1 - \frac{b-a}{\propto+\beta} \quad \text{otherwise} \\ 0 \text{ if } a \leq b - (\propto+\beta) \end{cases} \qquad \ldots\ldots\ldots\ldots\ldots.(\text{II})$$

i.e Pos $[A = B] = {}^{Sup}_{*}Min\,\{A(u), B(u)\}]$

$u \in R$ & A is a fuzzy number.

Using "Eq.I" & 'Eq.II" we get the degree of satisfactions of inequalities.

$$\mu_0(u_1, u_2) = \begin{cases} 1 & \text{if } u_1 + u_2 \geq 3 \\ 1 - \frac{3-(u_1+u_2)}{1+0.5(|u_1|+|u_2|)} & \text{otherwise} \\ 0 & \text{if } u_1 + u_2 \leq 3 - (1 + 0.5)(|u_1| + |u_2|) \end{cases}$$

$$\mu_1(u_1, u_2) = \begin{cases} 1 & \text{if } u_1 \leq 1 \\ 1 - \frac{u_1-1}{1+0.5(|u_1|+|u_2|)} & \text{otherwise} \\ 0 & \text{if } u_1 > 1 + [1 + 0.5(|u_1| + |u_2|)\,] \end{cases}$$

$$\mu_2(u_1, u_2) = \begin{cases} 1 & \text{if } u_2 \leq 1 \\ 1 - \frac{u_2-1}{1+0.5(|u_1|+|u_2|)} & \text{otherwise} \\ 0 & \text{if } u_1 > 1 + [1 + 0.5(|u_1| + |u_2|)\,] \end{cases}$$

The fuzzy solution is defined by

$$\mu(u_1, u_2) = Min\{\mu_0(u_1, u_2), \mu_1(u_1, u_2), \mu_2(u_1, u_2)\}$$

And the maximizing solution is obtained from-

$$\lambda^* = {}^{Max,Min}_{u_1,u_2}\{\mu_0(u_1, u_2), \mu_1(u_1, u_2), \mu_2(u_1, u_2)\}$$

That is

$$\text{Max } \lambda$$
$$\mu_0(u) \geq \lambda$$
$$\mu_1(u) \geq \lambda$$
$$\mu_2(u) \geq \lambda$$
$$0 \leq \lambda \leq 1, u \in R^2$$

To find the maximizing solution to FLP problem we have to solve the following non-linear programming problem.

$$\text{Max } \lambda$$
$$1 - \frac{3(u_1+u_2)}{1+0.5(|u_1|+|u_2|)} \geq \lambda$$
$$1 - \frac{u_1-1}{1+0.5(|u_1|+|u_2|)} \geq \lambda$$
$$1 - \frac{u_2 - 1}{1 + 0.5(|u_1| + |u_2|)} \geq \lambda$$
$$0 \leq \lambda \leq 1\ , u_1, u_2 \in R$$

The unique solution is $u^* = (4/3,\ 4/3)$and the degree of consistency$\lambda^* = 6/7$. Thus the optional value of fuzzy objective function is

$$Z = u_1 + u_2 = 4/3\ + 4/3 = 8/3 \Rightarrow Z = 8/3.$$

$$[u_1 + u_2, 0.5(|u_1| + |u_2|)] \geq (3,1)$$
$$[u_1, 0.5(|u_1| + |u_2|)] \leq (1,1)$$
$$[u_2, 0.5(|u_1| + |u_2|)] \leq (1,1)$$
$$\Longrightarrow [4/3\ + 4/3, 0.5(4/3\ + 4/3\,)] \geq (3,1)$$
$$[4/3, 0.5(4/3\ + 4/3\,)\,] \leq (1,1)$$

IV. CONCLUSION: -

Fuzzy mathematical programming (FLP) has an area of high research potential. Several research papers have been published in this area, a list of those can be found in the following references and in the volumes of "Fuzzy sets and systems" which can provide an interested readers , direction for further research in this area.

REFERENCES:-

[1] D.Stephen & S. Kamalnathan, , solving Fuzzy linear programming problem.
Int Jou of App of Fuzzy set and Artificial intelligence, vol 7 (2017) PP 281-292.

[2] C- Veeramani, solving Fuzzy linear programming problem using symmetric fuzzy number approximation.
Int Jou OR vol 15 no 3 (2012)

[3] A. Ebrahimnejad, F.H.Lotif , A primal-dual method for LPP with fuzzy variables.
Eur.J.Ind.Eng^{n} ,4(2010)PP189-209

[4] K.Ganesan, P.Veeramani, FLP with trapezoidal fuzzy numbers
Ann. OR 143 (2006) PP 305-315.

[5] H.R . Meleki, M.Tata, M.Mashinchi, linear programming with fuzzy variable.
Fuzzy set system 109(2000) PP 21-33.

[6] M.Delgado, J.K verdegay, M.A.Vita, A general model for fuzzy linear programming .
Fuzzy sets and systems 29 (1989), 21-29.

[7] H.Tanaka, H.Lchihashi and K.Asal, A formation of FLP problem based on comparision of fuzzy numbers.
Control and cybernet 13 (1984) 185-194.

[8] J.L.Verdegay , Fuzzy Mathematical programming in M.M gupta & E. sanchex Eds.
Fuzzy information and decision process, North Holi and Amsterdum 1982, PP 231-237.

[9] D.Dubois and H.prade, Fuzzy set and system, Theory & Application .
Academic press, New York 1980.

[10] H.J.Zimmerman, fuzzy programming and linear programming with several objective functions.
Fuzzy set system 1 (1978) PP 45-55.

[11] H.Tanaka, T.Okuda, K.Asal, on fuzzy mathematical programming
J .Cybernet 3 (1974) PP 37-46.

[12] R.Bellman & L.A.Zadeh, Decision making a fuzzy environment management sci
B 17 (1970) PP 141-164.