MULTI-ITEM PRODUCTION INVENTORY MODEL WITH REMANUFACTURING OF DEFECTIVE ITEMS AND RETURN ITEMS USING HEXAGONAL FUZZY NUMBER

[1]K.Dhanam ,[2] T.Kalaiselvi
[1] Associate Professor (Retd), [2] Assistant Professor,
[1]Department of Mathematics,
Government Arts College for Women(Autonomous),Pudukkottai
[2]Department of Mathematics,
J.J. College of Arts and Science(Autonomous),Pudukkottai.

Abstract: This paper explores a Multi-Item production Inventory Model with Remanufacturing of defective items and return items using Hexagonal fuzzy number. Imperfect quality items are unavoidable in an inventory system due to production process, natural disasters, damages and many other reasons. Our proposed model is considered as the regular production during the production time some items are defected, wholesaler found that some items are damaged before the sales and after sales the customers identify the defects. The rate of demand considered as random variable and it follows Dagum distribution. The cost parameters represented by Hexagonal fuzzy number. Expected total cost is derived and the model is defuzzified by Mean Deviation Method. Finally, a numerical example is given to illustrate the model and sensitivity analysis also made.

Keywords: Inventory, Defected items, Remanufacturing, reworked items, Dagum Distribution, Hexagonal Fuzzy number, Mean Deviation Method.
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Introduction:

Inventory planning and control is concerned with the acquisition and storage of the materials required for supporting various business operations. In classical model such as EOQ and EPQ developed by Harris.F.W (1913) [6] and Taft .E.W (1918) [17], it is assumed that the rate of replenishment /production and unit price of an item are constant. But later theorists worked out more comprehensive, realistic market – sensitive models. Multi -item classical inventory models are presented in well known books by Hadley.G and Whitin.T.M (1958)[5], Silver.E.A and Peterson. R(1985) [16] and Taha.H.A (2005)[18]., etc. Cheng.T.C.H (1989) [4] was related to the EOQ model with demand dependent unit cost using geometric programming technique. Pureto.J et.al (1997)[11] considered inventory models in both constrained and unconstrained situation.

While modeling an inventory problem, it is assumed that demand and various relevant costs are defined with certainty. But, in real life, demand and various relevant costs are not exactly known. In this situation, uncertainties are treated as randomness and are handled through probability theory. In certain situations, uncertainties are due to fuzziness and in such cases the fuzzy set theory, originally introduced by Zadeh.L.A (1965) [19] may be applied. After Bellman.R.E and Zadeh.L.A (1970) [3]initiated fuzzy optimization through aggregation operations that combine fuzzy goals and fuzzy - decision space. Arnold Kaufmann and Madan M Gupta (1991) [2] provided an introduction of fuzzy arithmetic operations. Later, the fuzzy linear programming model was formulated and an approach for solving problem in linear programming model with fuzzy numbers has been presented by Zimmerman.H.J (1996) [20].

Generally the inventory model are formulated by considering that only the perfect items are produced. However, in reality production items may not always be perfect. So, a proportion of the produced items can be found to be defective. Samanta.G.P (2004) [14] worked a continuous production control inventory model for deteriorating items with shortages. Hejazi.S.R et.al(2008) [7] examined the EPQ model was investigated by considering production of various types of non-perfect products. OlhaYegorova (2014)[10] developed the economic order quantity model for deteriorating items with two level of trade credit in one replacement cycle. Ritha.W and Nivetha Martin (2013) [12] explained the inventory model with waste disposal method. The inventory model with imperfect quality items with shortages has been presented by Ritha.W and Rexlin Jayakumari.S (2013) [13]. Arindum Mukhopadhyay and Adrijit Goswami (2014) [1] investigates an economic production quantity(EPQ) model with imperfect quality items with varying set-up cost. Nirmal Kumar Mandal (2014) [9] formulated a multi-objective imperfect quality inventory model with defective items solved by modified geometric programming approach. Medhi.J (1994) [8] provided the concept of probability distribution.

In recent time, power scarcity has affected the large scale industries in manufacturing goods, to solve this problem, solar plants are being installed in many forms. It incurs a cost. The cost as alternative power supply cost, its operating and maintenance cost. In this paper, defects in products are discovered during manufacture, marketing and after sales. Some defects are noticed, immediately after the manufacturing process. During marketing wholesaler and retailer discover breakages and scrape. Finally the end user identify the defects in products and brings then up after sales services.

Miner defects are reworked and such returned products are rechecked and send to the market. The defective items are occurred at the time of manufacturing and products that are beyond repair items are remanufacturing. Remanufacturing goods are also send to the market and remaining unused products should be contaminate.

Assumptions and Notations

Our proposed model is constructed under the following assumptions and notations.

Assumptions:

1. Production rate is finite.
2. All demands must be satisfied.
3. Some returned items are allowed and reworked.
4. Defected items are remanufactured.
5. Alternative power supply(solar plants, their operating and maintenance) costs are allowed.
6. Demand rate is random variable, which follows Dagum distribution.

$$E(d_i) = -\frac{B}{A} \times \frac{\Gamma\left(-\frac{1}{A}\right)\Gamma\left(\frac{1}{A}+P\right)}{\Gamma P}, P > 0, A > 0, B > 0.$$

Notations: The following notations are for the i^{th}item(i=1,2,3,….n)

Q_{si} - Inventory level at time t_1.

Q_{mi} - Maximum inventory level at time t_2.

d_i - demand rate(random variable).

R_{ri} - rate of remanufacturing items. $\left(i.e, R_{ri} = a\left(\theta_i + (1-b)(R_{1i} + R_{2i})\right)\right)$

R_{wi} - rate of reworked items. $\left(i.e, R_{wi} = b(R_{1i} + R_{2i})\right)$

θ_i - rate of defective items.

γ_i - rate of wastage items. $\left(i.e, \gamma_i = (1-a)\left(\theta_i + (1-b)(R_{1i} + R_{2i})\right)\right)$

R_{1i} - rate of return items during $[0, t_1]$.

R_{2i} - rate of return items during $[t_1, t_2]$.

R_{3i} - rate of return items during $[t_2, t_3]$.

$\tilde{k}_i$ - fuzzy Production rate.

$\tilde{H}_{mi}$ - fuzzy Holding cost of manufacturing product per unit per unit time.

$\tilde{H}_{si}$ - fuzzy Holding cost of reworked and remanufacturing product per unit per unit time.

$\tilde{H}_{Ri}$ - fuzzy Holding cost of returned product per unit per unit time.

$\tilde{s}_i$ - fuzzy setup cost per cycle.

$\tilde{R}_{ci}$ - fuzzy reworking cost.

$\tilde{w}_c$ - fuzzy wastage cost.

$\tilde{s}_{ei}$ - fuzzy operating and maintenance cost of solar plants per cycle.

$\tilde{t}_c$ - fuzzy transportation cost

W_t - working time of solar plants per cycle.

S_E - fixed solar plants cost for plan period.

a,b - arbitrary constant(0<a,b<1)

3. FORMULATION OF THE CRISP MODEL

To derive the inventory level function, divide the time interval $[0,t_3]$ into three parts: $[0,t_1]$, $[t_1,t_2]$ and $[t_2,t_3]$. The regular production started at time t=0 and stop at time $t=t_1$. During $[0,t_1]$, the inventory level gradually increases and some items defective and returned. During the period $[t_1,t_2]$, remanufacturing process and set right the returned items are done. So, stock builds up during the period $[0, t_2]$ and declines during the period$[t_2,t_3]$. The stock is reduced to zero at t_3.

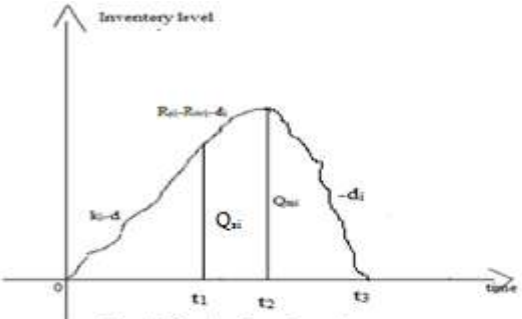

Fig:1 Production inventory model

Let $Q_i(t)$ be the inventory level during the period $[0, t_3]$. Then the differential equations governing the instantaneous state of any time t are given by,

$$\frac{dQ_i(t)}{dt}+(\theta_i+R_{1i})Q_i(t)=k_i-E(d_i) \qquad 0\le t\le t_1 \qquad \text{--------------(1)}$$

$$\frac{dQ_i(t)}{dt}+(\gamma_i+R_{2i})Q_i(t)=R_{ri}+R_{wi}-E(d_i) \qquad t_1\le t\le t_2 \qquad \text{--------------(2)}$$

$$\frac{dQ_i(t)}{dt}+R_{3i}Q_i(t)=-E(d_i) \qquad t_2\le t\le t_3 \qquad \text{--------------(3)}$$

with the boundary conditions are

$$Q_i(0)=0,\ Q_i(t_1)=Q_{si},\quad Q_i(t_2)=Q_{mi},\quad Q_i(t_3)=0. \qquad \text{--------------(4)}$$

The solution of the equations (1)-(3) are given by

$$Q_i(t)=\begin{cases}\dfrac{k_i-E(d_i)}{\theta_i+R_{1i}}+c_1e^{-(\theta_i+R_{1i})t} & 0\le t\le t_1\\[2ex] \dfrac{R_{ri}+R_{wi}-E(d_i)}{\gamma_i+R_{2i}}+c_2e^{-(\gamma_i+R_{2i})t} & t_1\le t\le t_2\\[2ex] -\dfrac{E(d_i)}{R_{3i}}+c_3e^{-R_{3i}t} & t_2\le t\le t_3\end{cases} \qquad \text{--------(5)}$$

Using equations (4) and (5) , the solution of the equations are obtained.

$$Q_i(t)=\begin{cases}\dfrac{k_i-E(d_i)}{\theta_i+R_{1i}}+\left[1-e^{-(\theta_i+R_{1i})t}\right] & 0\le t\le t_1\\[2ex] \dfrac{R_{ri}+R_{wi}-E(d_i)}{\gamma_i+R_{2i}}+\left[\begin{array}{l}\left(\dfrac{k_i-E(d_i)}{\theta_i+R_{1i}}\right)\left[1-e^{-(\theta_i+R_{1i})t_1}\right]\\ -\left(\dfrac{R_{ri}+R_{wi}-E(d_i)}{\gamma_i+R_{2i}}\right)\end{array}\right]e^{(\gamma_i+R_{2i})(t_1-t)} & t_1\le t\le t_2\\[2ex] \dfrac{E(d_i)}{R_{3i}}+\left[e^{R_{3i}(t_3-t)}-1\right] & t_2\le t\le t_3\end{cases}$$

-----(6)

Holding cost of Production $=H_{mi}\int_0^{t_1}Q_i(t)dt$

$$=H_{mi}\left[\frac{k_i-E(d_i)}{\theta_i+R_{1i}}\right]\left[\left(t_1+\frac{e^{-(\theta_i+R_{1i})t_1}-1}{(\theta_i+R_{1i})}\right)\right]$$

Holding cost of Remanufacturing

and reworked items $=H_{si}\int_{t_1}^{t_2}Q_i(t)dt$

$$= H_{si}\left\{\left[\frac{R_{ri}+R_{wi}-E(d_i)}{\gamma_i+R_{2i}}\right](t_2-t_1)-\left[\begin{array}{l}\left(\frac{k_i-E(d_i)}{\theta_i+R_{1i}}\right)\left(1-e^{-(\theta_i+R_{1i})t_1}\right)\\-\left(\frac{R_{ri}+R_{wi}-E(d_i)}{\gamma_i+R_{2i}}\right)\end{array}\right]\left[\frac{e^{(\gamma_i+R_{2i})(t_1-t_2)}-1}{(\gamma_i+R_{2i})}\right]\right\}$$

Holding cost of Return items $= H_{Ri}\int_0^{t_3} Q_i(t)dt$

$$= H_{Ri}\left\{\begin{array}{l}\left[\left(\frac{k_i-E(d_i)}{\theta_i+R_{1i}}\right)\right]\left[t_1+\frac{e^{-(\theta_i+R_{1i})t_1}-1}{(\theta_i+R_{1i})}\right]\\+\left[\frac{R_{ri}+R_{wi}-E(d_i)}{\gamma_i+R_{2i}}\right](t_2-t_1)-\left[Q_{si}-\left(\frac{R_{ri}+R_{wi}-E(d_i)}{\gamma_i+R_{2i}}\right)\right]\left[\frac{e^{(\gamma_i+R_{2i})(t_1-t_2)}-1}{(\gamma_i+R_{2i})}\right]\\+\frac{E(d_i)}{R_{3i}}\left[\frac{e^{R_{3i}(t_3-t_2)}-1}{R_{3i}}+(t_2-t_3)\right]\end{array}\right\}$$

Setup cost $=s_i$

Wastage cost $= w_c\gamma_i(t_2-t_1)$

Cost of reworked items $=R_{ci}\, b(R_{1i}+R_{2i})(t_2-t_1)$

Transportation cost $=t_cE(d_i)$

Solar plants operating and maintenance cost $= s_{ei}W_t$

Solar Plant cost $=S_E$

Expected total cost=[Regular Production Holding cost + Remanufacturing and reworked items Holding cost + Return items Holding cost+ Setup cost + wastage cost+cost of reworked items + Transportation cost + Solar plants operating and maintenance cost + Solar Plant cost] Expected Total cost is given by

$$E(TC)=\sum_{i=1}^{n}\left[\begin{array}{l}H_{mi}\left(\left[\left(\frac{k_i-E(d_i)}{\theta_i+R_{1i}}\right)\right]\left[t_1+\frac{e^{-(\theta_i+R_{1i})t_1}-1}{(\theta_i+R_{1i})}\right]\right)\\+H_{si}\left(\left[\frac{R_{ri}+R_{wi}-E(d_i)}{\gamma_i+R_{2i}}\right](t_2-t_1)-\left[\begin{array}{l}\left[\left(\frac{k_i-E(d_i)}{\theta_i+R_{1i}}\right)\right]\left[1-e^{-(\theta_i+R_{1i})t_1}\right]\\-\left(\frac{R_{ri}+R_{wi}-E(d_i)}{\gamma_i+R_{2i}}\right)\end{array}\right]\left[\frac{e^{(\gamma_i+R_{2i})(t_1-t_2)}-1}{(\gamma_i+R_{2i})}\right]\right)\\+H_{Ri}\left\{\begin{array}{l}\left[\left(\frac{k_i-E(d_i)}{\theta_i+R_{1i}}\right)\right]\left[t_1+\frac{e^{-(\theta_i+R_{1i})t_1}-1}{(\theta_i+R_{1i})}\right]+\left[\frac{R_{ri}+R_{wi}-E(d_i)}{\gamma_i+R_{2i}}\right](t_2-t_1)\\-\left[\begin{array}{l}\left[\left(\frac{k_i-E(d_i)}{\theta_i+R_{1i}}\right)\right]\left[1-e^{-(\theta_i+R_{1i})t_1}\right]\\-\left(\frac{R_{ri}+R_{wi}-E(d_i)}{\gamma_i+R_{2i}}\right)\end{array}\right]\left[\frac{e^{(\gamma_i+R_{2i})(t_1-t_2)}-1}{(\gamma_i+R_{2i})}\right]\\+\frac{E(d_i)}{R_{3i}}\left[\frac{e^{R_{3i}(t_3-t_2)}-1}{R_{3i}}+(t_2-t_3)\right]\end{array}\right\}\\+s_i+w_c\gamma_i(t_2-t_1)+R_{ci}b(R_{1i}+R_{2i})(t_2-t_1)+t_cE(d_i)+s_{ei}W_t+S_E\end{array}\right]$$

---(7)

Using the values of t_1, t_2 and t_3 then the values of Q_{si} and Q_{mi} are

$$Q_{si}=\left[\frac{k_i-E(d_i)}{\theta_i+R_{1i}}\right]\left[1-e^{-(\theta_i+R_{1i})t_1}\right]$$

and

$$Q_{mi}=\left[\frac{E(d_i)}{R_{3i}}\right]\left[e^{R_{3i}(t_3-t_2)}-1\right]$$

4.Hexagonal fuzzy number and its properties

A Hexagonal fuzzy number $\tilde{A}$ is described as a fuzzy subset on the real line R whose membership function $\mu_{\tilde{A}}(x)$ is defined as follows

$$\mu_{\tilde{A}}(x)=\begin{cases} 0 & for \quad x\le a \\ W_A\left(\dfrac{x-a}{b-a}\right) & for \quad a\le x\le b \\ W_A+(1-W_A)\left(\dfrac{x-b}{c-b}\right) & for \quad b\le x\le c \\ 1 & for \quad c\le x\le d \\ W_A+(1-W_A)\left(\dfrac{c-x}{c-d}\right) & for \quad d\le x\le e \\ W_A\left(\dfrac{f-x}{f-e}\right) & for \quad e\le x\le f \\ 0 & for \quad x\ge f \end{cases}$$

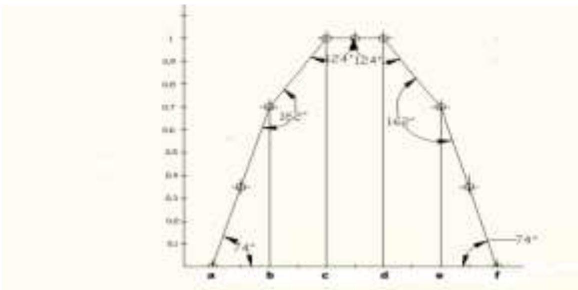

Figure 2: Graphical Representation of Hexagonal Fuzzy Number

Where $0.6\le W_A<1$, a, b, c, d, e and f are real numbers.

This type of fuzzy number is denoted by $\tilde{A}=(a,b,c,d,e,f;W_A)_{HFN}$

$\mu_{\tilde{A}}$ satisfies the following conditions:

1. $\mu_{\tilde{A}}$ is a continuous mapping from R to the closed interval [0,1] .
2. $\mu_{\tilde{A}}$ is a convex function.
3. $\mu_{\tilde{A}}(x)=0, -\infty<x\le a$.
4. $\mu_{\tilde{A}}(x)=\delta_l(x)$ is strictly increasing on (a , c) .
5. $\mu_{\tilde{A}}(x)=1, x\in[c,d]$.
6. $\mu_{\tilde{A}}(x)=\delta_r(x)$ is strictly decreasing on (d ,f) .
7. $\mu_{\tilde{A}}(x)=0, f<x\le\infty$.

Remark:

If $0<W_A<0\cdot6$, then $\tilde{A}$ becomes a trapezoidal fuzzy number.

5.Mean Deviation of the fuzzy number:

Let us consider $\tilde{A}$ is a Hexagonal fuzzy number such that $\tilde{A}=(a,b,c,d,e,f)_{,HFN}$, then the parameters of Mean deviation of Hexagonal fuzzy number is explained as follows.

The membership function is divided into two parts. One at the left of $x_{m1}, x_l(\alpha)$ and the other at right of $x_{m2}, x_r(\alpha)$ .The quantity $\delta_l(\tilde{A})=\int_{\alpha=0}^{1}[x_m-x_l(\alpha)]d\alpha$ is called the left mean deviation,the quantity $\delta_r(\tilde{A})=\int_{\alpha=0}^{1}[x_r(\alpha)-x_m]d\alpha$ is called the right mean deviation and $\delta(\tilde{A})=\delta_l(\tilde{A})+\delta_r(\tilde{A})$ is called the mean deviation of the fuzzy number.

By construction, a fuzzy number have $x_l(\alpha)\le x_{m1}\le x_m\le x_{m2}\le x_r(\alpha)$, $\delta_l(\tilde{A})\ge 0, \delta_r(\tilde{A})\ge 0, \delta(\tilde{A})\ge 0$ .Also $\delta_l(\tilde{A})$ represents the area to the left of x_m and $\delta_r(\tilde{A})$ represents the area of the right. Where $\delta(\tilde{A})$ is the sum of the two areas.

Therefore the mean deviation of the Hexagonal fuzzy number is

$$\delta(\tilde{A})=\begin{bmatrix} \left(\dfrac{W_A}{2(a-b)}\right)\{(x_m-b)^2-(x_m-a)^2\}+\left(\dfrac{1-W_A}{2(b-c)}\right)\{(x_m-c)^2-(x_m-b)^2\} \\ +\left(\dfrac{W_A}{2(e-f)}\right)\{(e-x_m)^2-(f-x_m)^2\}+\left(\dfrac{1-W_A}{2(d-e)}\right)\{(d-x_m)^2-(e-x_m)^2\} \end{bmatrix}$$

7. The proposed inventory model in fuzzy Environment

If the cost parameters are fuzzy number, then the problem (13) is transformed to

$$E(\tilde{T}C)=\sum_{i=1}^{n}\begin{bmatrix} \tilde{H}_{mi}\left(\left[\left(\dfrac{\tilde{k}_i-E(d_i)}{\theta_i+R_{1i}}\right)\right]\left[t_1+\dfrac{e^{-(\theta_i+R_{1i})t_1}-1}{(\theta_i+R_{1i})}\right]\right) \\ +\tilde{H}_{si}\left(\left[\dfrac{R_{ri}+R_{wi}-E(d_i)}{\gamma_i+R_{2i}}\right](t_2-t_1)-\begin{bmatrix}\left[\left(\dfrac{\tilde{k}_i-E(d_i)}{\theta_i+R_{1i}}\right)\left[1-e^{-(\theta_i+R_{1i})t_1}\right]\right] \\ -\left(\dfrac{R_{ri}+R_{wi}-E(d_i)}{\gamma_i+R_{2i}}\right)\end{bmatrix}\left[\dfrac{e^{(\gamma_i+R_{2i})(t_1-t_2)}-1}{(\gamma_i+R_{2i})}\right]\right) \\ +\tilde{H}_{Ri}\left\{\begin{matrix}\left[\left(\dfrac{\tilde{k}_i-E(d_i)}{\theta_i+R_{1i}}\right)\right]\left[t_1+\dfrac{e^{-(\theta_i+R_{1i})t_1}-1}{(\theta_i+R_{1i})}\right]+\left[\dfrac{R_{ri}+R_{wi}-E(d_i)}{\gamma_i+R_{2i}}\right](t_2-t_1) \\ -\left[\left[\left(\dfrac{\tilde{k}_i-E(d_i)}{\theta_i+R_{1i}}\right)\left[1-e^{-(\theta_i+R_{1i})t_1}\right]\right]-\left(\dfrac{R_{ri}+R_{wi}-E(d_i)}{\gamma_i+R_{2i}}\right)\right]\left[\dfrac{e^{(\gamma_i+R_{2i})(t_1-t_2)}-1}{(\gamma_i+R_{2i})}\right] \\ +\dfrac{E(d_i)}{R_{3i}}\left[\dfrac{e^{R_{3i}(t_3-t_2)}-1}{R_{3i}}+(t_2-t_3)\right]\end{matrix}\right\} \\ +\tilde{s}_i+\tilde{w}_c\gamma_i(t_2-t_1)+\tilde{R}_{ci}b(R_{1i}+R_{2i})(t_2-t_1)+\tilde{t}_cE(d_i)+\tilde{s}_{ei}W_t+S_E \end{bmatrix} \quad (13)$$

Where ~ represents the fuzzification of the parameters.

In the proposed model, the parameters k_i,H_{mi},H_{si},H_{Ri},s_i,w_{ci},S_{ri},t_{ci},s_{ei} are considered as Hexagonal fuzzy number.

$\tilde{k}_i = [k_1, k_2, k_3, k_4, k_5, k_6]$ $\tilde{H}_{mi} = [H_{m1}, H_{m2}, H_{m3}, H_{m4}, H_{m5}, H_{m6}]$

$\tilde{H}_{si} = [H_{s1}, H_{s2}, H_{s3}, H_{s4}, H_{s5}, H_{s6}]$ $\tilde{H}_{Ri} = [H_{R1}, H_{R2}, H_{R3}, H_{R4}, H_{R5}, H_{R6}]$

$\tilde{s}_i = [s_1, s_2, s_3, s_4, s_5, s_6]$ $\tilde{w}_c = [w_{c1}, w_{c2}, w_{c3}, w_{c4}, w_{c5}, w_{c6}]$

$\tilde{t}_c = [t_{c1}, t_{c2}, t_{c3}, t_{c4}, t_{c5}, t_{c6}]$ $\tilde{R}_{ci} = [R_{c1}, R_{c2}, R_{c3}, R_{c4}, R_{c5}, R_{c6}]$ $\tilde{s}_{ei} = [s_{e1}, s_{e2}, s_{e3}, s_{e4}, s_{e5}, s_{e6}]$

The corresponding fuzzy Problem (13) is

$$E(\tilde{T}C) = \sum_{i=1}^{n} \left[\begin{array}{l} \delta_{\tilde{H}_{mi}} \left(\left[\left(\frac{\delta_{\tilde{k}i} - E(d_i)}{\theta_i + R_{1i}} \right) \right] \left[t_1 + \frac{e^{-(\theta_i + R_{1i})t_1} - 1}{(\theta_i + R_{1i})} \right] \right) \\ + \delta_{\tilde{H}_{si}} \left(\begin{array}{l} \left[\frac{R_{ri} + R_{wi} - E(d_i)}{\gamma_i + R_{2i}} \right] (t_2 - t_1) \\ - \left[\begin{array}{l} \left[\left(\frac{\delta_{\tilde{k}_i} - E(d_i)}{\theta_i + R_{1i}} \right) \right] \left[1 - e^{-(\theta_i + R_{1i})t_1} \right] \\ - \left(\frac{R_{ri} + R_{wi} - E(d_i)}{\gamma_i + R_{2i}} \right) \end{array} \right] \left[\frac{e^{(\gamma_i + R_{2i})(t_1 - t_2)} - 1}{(\gamma_i + R_{2i})} \right] \end{array} \right) \\ + \delta_{\tilde{H}_{Ri}} \left\{ \begin{array}{l} \left[\left(\frac{\delta_{\tilde{k}_i} - E(d_i)}{\theta_i + R_{1i}} \right) \right] \left[t_1 + \frac{e^{-(\theta_i + R_{1i})t_1} - 1}{(\theta_i + R_{1i})} \right] \\ + \left[\frac{R_{ri} + R_{wi} - E(d_i)}{\gamma_i + R_{2i}} \right] (t_2 - t_1) \\ - \left[\begin{array}{l} \left[\left(\frac{\delta_{\tilde{k}_i} - E(d_i)}{\theta_i + R_{1i}} \right) \right] \left[1 - e^{-(\theta_i + R_{1i})t_1} \right] \\ - \left(\frac{R_{ri} + R_{wi} - E(d_i)}{\gamma_i + R_{2i}} \right) \end{array} \right] \left[\frac{e^{(\gamma_i + R_{2i})(t_1 - t_2)} - 1}{(\gamma_i + R_{2i})} \right] \\ + \frac{E(d_i)}{R_{3i}} \left[\frac{e^{R_{3i}(t_3 - t_2)} - 1}{R_{3i}} + (t_2 - t_3) \right] \end{array} \right\} \\ + \delta_{\tilde{s}_i} + \delta_{\tilde{w}_c} \gamma_i (t_2 - t_1) + \delta_{\tilde{R}_{ci}} b(R_{1i} + R_{2i})(t_2 - t_1) + \delta_{\tilde{t}_c} E(d_i) + \delta_{\tilde{s}_{ei}} W_t + S_E \end{array} \right]$$

Where

$$\delta_{\tilde{H}m_i} = \left[\begin{array}{l} \left(\frac{W_A}{2(H_{m1} - H_{m2})} \right) \left\{ (x_{H_m} - H_{m2})^2 - (x_{H_m} - H_{m1})^2 \right\} \\ + \left(\frac{1 - W_A}{2(H_{m2} - H_{m3})} \right) \left\{ (x_{H_m} - H_{m3})^2 - (x_{H_m} - H_{m2})^2 \right\} \\ + \left(\frac{W_A}{2(H_{m5} - H_{m6})} \right) \left\{ (H_{m5} - x_{H_m})^2 - (H_{m6} - x_{H_m})^2 \right\} \\ + \left(\frac{1 - W_A}{2(H_{m4} - H_{m5})} \right) \left\{ (H_{m4} - x_{H_m})^2 - (H_{m5} - x_{H_m})^2 \right\} \end{array} \right]$$

Similarly for

$\delta_{\tilde{k}_i}, \delta_{\tilde{H}_{Ri}}, \delta_{\tilde{H}_{si}}, \delta_{\tilde{s}_i}, \delta_{\tilde{w}_c}, \delta_{\tilde{t}_c}, \delta_{\tilde{R}_{ci}}$ *and* $\delta_{\tilde{s}_{ei}}$.

6. Numerical Example:

The two wheeler manufacturing company produces two items. The relevant data for the two items are given below.

W_t=1200; W=0.7; S_E= 1000000 ; a=0.6; b=0.6

$\tilde{s}_{ei}$ = [1 3 5 7 9 11] $\tilde{t}_c$ = [1 1.5 2 2.5 3 3.5] $\tilde{w}_c$ = [0.2 0.3 0.4 0.5 0.6 0.7]

Item1:

R_{11}=5; R_{21}=2; R_{31}=3; A=82.6; B=2.1; P=14.5; $\theta_1 = 1$

$\tilde{k}_1 = [2000 \;\; 3000 \;\; 4000 \;\; 5000 \;\; 6000 \;\; 7000]$ $\quad \tilde{H}_{m_1} = [5 \;\; 10 \;\; 15 \;\; 20 \;\; 25 \;\; 30]$

$\tilde{H}_{s_1} = [3 \;\; 4 \;\; 5 \;\; 6 \;\; 7 \;\; 8]$ $\quad \tilde{H}_{R_1} = [7 \;\; 10 \;\; 13 \;\; 16 \;\; 19 \;\; 22]$

$\tilde{R}_{w_1} = [0.2 \;\; 0.3 \;\; 0.4 \;\; 0.5 \;\; 0.6 \;\; 0.7]$

$\tilde{s}_1 = [100 \;\; 150 \;\; 200 \;\; 250 \;\; 300 \;\; 350]$

Item2:

$R_{12}=4$; $R_{22}=2$; $R_{32}=2$; $A=15.5$; $B=7.1$; $P=13.2$;

$\theta_2 = 2$

$\tilde{k}_2 = [1000 \;\; 1500 \;\; 2000 \;\; 2500 \;\; 3000 \;\; 3500]$ $\quad \tilde{H}_{m_2} = [5 \;\; 7 \;\; 9 \;\; 11 \;\; 13 \;\; 15]$

$\tilde{H}_{s_2} = [2 \;\; 4 \;\; 6 \;\; 8 \;\; 10 \;\; 12]$ $\quad \tilde{H}_{R_2} = [3 \;\; 4 \;\; 5 \;\; 6 \;\; 7 \;\; 8]$

$\tilde{R}_{w_2} = [0.2 \;\; 0.4 \;\; 0.6 \;\; 0.8 \;\; 1.0 \;\; 1.2]$ $\quad \tilde{s}_2 = [80 \;\; 160 \;\; 240 \;\; 320 \;\; 400 \;\; 480]$

Using MATLAB software, the expected maximum inventory level, the expected inventory level at t_1 and expected minimum total cost are obtained in both crisp and fuzzy environment and are given in the following table.

Table 1 : Comparison of crisp and fuzzy result

Model	Item	k_i	H_{mi}	H_{si}	H_{Ri}	R_w	s	t_c	w_c	s_{ei}	t_1	t_2	t_3	$E(Q_{si}^*)$	$E(Q_m^*)$	E (TC)
crisp	1	2000	5	3	7	0.2	100	1	0.2	1	22.59	26.24	28.50	333	632	1178000
	2	1000	5	2	3	0.2	80							165	393	
	1	3000	10	4	10	0.3	150	1.5	0.3	3	22.59	26.24	28.50	500	642	1380500
	2	1500	7	4	4	0.4	160							249	397	
	1	4000	15	5	13	0.4	200	2	0.4	5	22.59	26.24	28.55	666	751	1661400
	2	2000	9	6	5	0.6	240							331	441	
	1	5000	20	6	16	0.5	250	2.5	0.5	7	22.59	26.22	28.60	833	903	2017800
	2	2500	11	8	6	0.8	320							415	499	
	1	6000	25	7	19	0.6	300	3	0.6	9	23.19	26.13	28.54	1000	1007	2378300
	2	3000	13	10	7	1.0	400							499	537	
	1	7000	30	8	22	0.7	350	3.5	0.7	11	23.14	26.13	28.60	1166	1187	2888300
	2	3500	15	12	8	1.2	480							582	600	
fuzz	1	3400	17	3.4	10.2	0.34	170	1.7	0.34	6.8	22.61	26.22	28.60	**566**	**909**	**1523100**
	2	1700	6.8	6.8	3.4	0.27	272							**282**	**501**	

Table 2 : Effect of changes in regular manufacturing holding cost of the fuzzy inventory model

%change	Item	H_{mi}	t_1	t_2	t_3	$E(Q_{si}^*)$	$E(Q_{mi}^*)$	E(TC)
-50	1	8.5	22.60	26.23	28.60	566	883	2581100
	2	3.4				282	491	
-25	1	12.75	22.60	26.23	28.61	566	906	2778100
	2	5.1				282	500	
+25	1	21.25	22.59	26.24	28.62	566	928	3172000
	2	8.5				282	508	
+50	1	25.5	22.59	26.24	28.63	566	953	3368900
	2	10.2				282	517	

Conclusion

From table 1, observed that the optimal values are given for the fuzzy models along with the crisp model. The expected inventory level in the fuzzy environment is high compared to the crisp value. The expected minimum total cost in the crisp environment is high compared to the fuzzy value. Finally, conclude that the fuzzy model can be executable in the real world.

In table 2, the sensitivity analysis are given for the fuzzy model, from the same the following are observed.

- In the regular manufacturing holding cost H_{mi}

1. If the regular manufacturing holding cost is taken as either 25% or 50% decrease then the expected inventory level Q_{si}, the expected maximum inventory level Q_{mi} and expected total cost will be decreases.

2. If the regular manufacturing holding cost is taken as either 25% or 50% increase then the expected inventory level Q_{si}, the expected maximum inventory level Q_{mi} and expected total cost will be increases.

- In the same manner, all cost parameters can be analyzed.

Reference:

[1] ArindumMukhopadhyayand AdrijitGoswami (2014), "*Economic production quantity (EPQ) model for three type imperfect items with rework and learning in setup*" , An International Journal of Optimization and Control: Theories & Applications, Vol.4(1), 57-65.

[2] Arnold Kaufmann, Madan M Gupta (1991), "*Introduction to Fuzzy Arithmetic:Theory and Applications* ", Van Nostrand Reinhold, New York.

[3] R.E. Bellman and L.A. Zadeh(1970), *Decision-making in a fuzzy environment*. Management Science, 17,141-164.

[4] T.C.E.Cheng (1989), *An economic order quantity model with demand-dependent unit cost*. European Journal of Operational Research ,40, 252-256.

[5] G.Hadley& T.M. Whitin(1958), Analysis of inventory systems. Englewood clifs, NJ:Printice Hall.

[6] F.W.Harries(1913), *How Many Parts to make at once. Factory*, The Magazine of Management,10(2), 135-136,152.[Reprinted in operations Research,38(6)(1990), 947-950] .

[7] S.R. Hejazi, J.C. Tsou& M. RastiBarzoki (2008), "*Optimal lot size of EPQ model considering imperfect and defective products*", Journal of Industrial Engineering, Vol.4(7), 59-68.

[8] J. Medhi (1994), " *Stochastic Processes* " 2nd Ed. Wiley Eastern Limited.

[9] Nirmal Kumar Mandal (2014), "*An Economic Production Inventory Problem with Reworking of Imperfect Items in Fuzzy Environment*", Matematika, © UTM Centre for Industrial and Applied Mathematics, Vol.30(1), 17-35.

[10] OlhaYegorova(2014) , " *Fuzzy Expected value model with inspection Errors and two level of trade credit in one Replenishment Cycle* " , International Journal "Information Models and Analysis"Vol.3(1), 37-52.

[11] J.Puerto and F.R.Fernandez(1998), *Pareto - optimality in classical inventory problems*. Naval Research Logistics ,45,83-98.

[12] W.Ritha and Nivetha Martin (2013)," *Environmental Oriented Inventory Model And Benefits Of Incineration As Waste Disposal Method*" Aryabhatta Journal of Mathematics & Informatics, Vol. 6(1), 159-164.

[13] W. Ritha and S. RexlinJeyakumari (2013)," *Fuzzy Inventory Model for Imperfect Quality Items with Shortages*", Annals of Pure and Applied Mathematics, Vol.4(2), 127-137.

[14] G.P. Samanta (2004), " *A Production inventory Model with Deteriorating items and Shortages*", Yugoslav Journal of Operations Research, Vol.14(2), 219-230.

[15] Samar Hazari, KalipadaMaity, Jayanta Kumar Dey and SamarjitKar (2014), "*Optimal Dynamic Production and price for reliability-dependent imperfect production with inventory-level-dependent demand in uncertain environment*", Journal of Uncertainty Analysis and Applications, 2:9, 1-17.

[16] E.A.Silvar and R.Peterson(1985), *Decision systems for inventory management and production planning*,John Wiley, New York.

[17] E.W.Taft(1918), *The most economical production lot*, Iron Age,101,1410-1412.

[18] H.A.Taha(2005), *Operations Research: An Introduction*, Prentice-Hall of India, Delhi.

[19] L.A. Zadeh (1965), "*Fuzzy Sets*", Information and Control, Vol.8(3),338-353

[20] H.J. Zimmerman(1996), "*Fuzzy Set Theory and its Application*", 3rd Ed.Dordrecht: Kluwer, Acad Publishers.