Hydro magnetic Flow with Constant Suction near An Oscillating Porous Flat Plate

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Abstract : In the present paper, the flow with constant suction of a viscous incompressible and electrically conducting fluid near an infinite porous flat plate oscillating in its own plane with constant suction has been discussed in the presence of uniform transverse magnetic field. It is assumed that the fluid extends on one side of the wall and is sucked away to the other side. Expressions for velocity profile and skin friction have been obtained. It is found that the velocity in the boundary layer decreases and the amplitude of the skin friction increases with the increase of Hartmann number M.

1. Fundamental Equations of Motion

We consider the flow of an incompressible electrically conducting viscous fluid over an infinite flat plate oscillating harmonically in its own plane in the presence of uniform magnetic field. Taking x' axis along the plate and y'-axis normal to it, the continuity and momentum equations are

$$\frac{\partial u'}{\partial t'}+u'\frac{\partial u'}{\partial x'}+v'\frac{\partial u'}{\partial y'}=\vartheta'\left(\frac{\partial^2 u'}{\partial x'^2}+\frac{\partial^2 u'}{\partial y'^2}\right)-\sigma B_0'^2 u'/\varphi' \quad \text{.......................} (1)$$

$$\frac{\partial v'}{\partial t'}+u'\frac{\partial v'}{\partial x'}+v'\frac{\partial v'}{\partial y'}=\frac{-1}{\vartheta'}\frac{\partial p'}{\partial y'}\vartheta'\left(\frac{\partial^2 v'}{\partial x'^2}+\frac{\partial^2 v'}{\partial y'^2}\right) \quad \text{.......................} (2)$$

$$\frac{\partial u'}{\partial x'}+\frac{\partial v'}{\partial y'}=0 \quad \text{.......................} (3)$$

Where B_0' is an external magnetic field and σ' is electrical conductivity of the fluid.

We look here for a solution when all the physical quantities are independent of x', the assumption that $\frac{\partial p'}{\partial x'}=0$ is based on the boundary layer analysis, within or outside the boundary $\frac{-1}{\vartheta'}\frac{\partial p'}{\partial x'}=\frac{\partial U'}{\partial t'}$ U'(t') is the velocity of the mean stream, since the fluid is at rest outside the boundary layer, $\frac{\partial p'}{\partial x'}=0$.

Further we assume that all physical properties of the fluid such as viscocity, electrical conductivity and magnetic permeability etc. are constant. We assume that magnetic prandtl number of the fluid is so small that the induced electric current does not effect the magnetic field, i.e. the applied magnetic field is essentially undisturbed by the flow field. In equation (1) all the electromagnetic quantities are measured in electromagnetic system of units.

Now equation (1), (2) and (3) reduce to

$$\frac{\partial v'}{\partial t'}+v'\frac{\partial u'}{\partial y'}+v'\frac{\partial^2 u'}{\partial y'^2}-\sigma'\frac{B_0'^2 u'}{\vartheta'} \quad \text{........................} (4)$$

$$\frac{\partial v'}{\partial t'}+v'\frac{\partial v'}{\partial y'}=-\frac{-1}{\vartheta'}.\frac{\partial p'}{\partial y'}+v'\frac{\partial^2 v'}{\partial y'^2} \quad \text{.......................} (5)$$

$$\frac{\partial v'}{\partial y'}=0 \quad \text{.......................} (6)$$

with the boundary conditions

$$y'=O, u'=U\delta(1+\varepsilon e^{in't'})$$

$$v'=-v_0' \quad \text{and} \quad y'-\infty u' \qquad \text{....................... (7)}$$

Here the study is confined to a region very close to the boundary. The pressure is taken to be constant along any normal and is given by its value outside the boundary layer.

Hence equation to be solved is

$$\frac{\partial u'}{\partial t'} - v_0' \frac{\partial u'}{\partial y'} v' \frac{\partial^2 v'}{\partial y'^2} - \sigma' B_0'^2 u'/\varphi^1 \qquad \text{...................... (8)}$$

With the condition

$$y'=0,\ u'=U_0'(1+\varepsilon e^{in't'})$$

$$y'-\infty,\ u'=0 \qquad \text{..................... (9)}$$

We now introduce the non-dimensional quantities

$$y=\frac{y'v_0'}{v},\quad t=\frac{v_0'^2 t'}{4v^1} \qquad \text{.................... (10)}$$

$$n=\frac{4n'v'}{v_0'^2},\ u=\frac{u'}{U'},\ M=\frac{4v'\sigma' B_0'^2}{v_0'^2 \varphi'}$$

Where U_0' is a reference velocity and n' is the frequency. Equation (8) now becomes

$$\frac{\partial^2 u'}{\partial t^2}+\frac{\partial u}{\partial y}-\frac{1}{4}\frac{\partial u}{\partial t}-\frac{Mu}{4}=0 \qquad \text{.................... (11)}$$

With the boundary conditions

$$y'=0,\ u=1+\varepsilon e^{int}$$

$$y=\infty,\ u=0 \qquad \text{.................... (12)}$$

We assume

$$u=f_0(y)+\varepsilon f_1(y)e^{int} \qquad \text{................... (13)}$$

Substituting (13) in (11) and comparing harmonic terms, we get following differential equations neglecting coefficient of ε^2

$$\frac{d^2 f_0}{dy^2}+\frac{df_0}{dy}-\frac{M}{4} f_0=0 \qquad \text{.................. (14)}$$

$$\frac{d^2 f_0}{dy^2}+\frac{df_1}{dy}-\frac{in}{4} f_1-\frac{M}{4} f_1=0 \qquad \text{.................. (15)}$$

With the conditions

$$f_0(O)=f_1(O),\ y=0$$

$$f_0(\infty)=f_1(\infty)=0y-\infty \qquad \text{.................. (16)}$$

The solution of (14) with the help of (16) is

$$f_0=\varepsilon^{-\alpha y} \qquad \text{................... (17)}$$

Where

$$\alpha=\frac{1}{2}\left(1+\sqrt{1+M}\right)$$

Similarly solution of (15) with the help of (16)

$$f_1=e^{-\beta y} \qquad \text{.................... (18)}$$

Where $\beta = \frac{1}{2}\left(1+\sqrt{1+in, M}\right)$

Hence the velocity field is

$$u = e^{-\alpha y} + \varepsilon\ e^{-\beta y}\ e^{int} \qquad \text{..................... (19)}$$

Non-dimensional skin friction $\Im_0$ is given by

$$-\Im_0 = \alpha + \varepsilon\ e^{int} \qquad \text{..................... (20)}$$

From equation (19) we have

$$u = e^{-\frac{1}{2}\left(1+\sqrt{1+M}\right)y} + \varepsilon\ e^{-\beta_r y} Cos\left(nt - \beta i^{y}\right) \qquad \text{.................... (21)}$$

Where $\beta_r = \frac{1}{2}\left[1+\frac{1+M}{2}+\frac{1}{2}v\left(1+M\right)^2+n^2\right]$

and $\beta_i = \frac{1}{2}\left[\frac{1}{2}v\left(1+M\right)^2+n^2-\frac{1+M}{2}\right]^{\frac{1}{2}}$

Also from equation (20) we have

$$-\Im_0 = \frac{1}{2}\left(1+\sqrt{1+M}\right)+\varepsilon\left|\beta\right| Cos\left(nt+\gamma\right) \qquad \text{..................... (22)}$$

Where $\beta = \beta_r + i\beta$ and $Tan\,\gamma = \beta_i / \beta_r$

The expression for the transient velocity profile is given by taking $nt = \frac{\pi}{2}$.

$$u\left(y,t\right) = \exp - \frac{1}{2}\left(1+\sqrt{1+M}\right)y + \varepsilon\ e^{-\beta_r y}\ Sin\,\beta_i y \qquad \text{..................... (23)}$$

Table - 1

y	0	1	2	3	4
u	1	.3687	.1361	.0503	.0186
M=1, nt = π/2					
y	0	1	2	3	4
u	1	.3023	.0915	.0270	.0081
M=2, nt = π/2					
y	0	1	2	3	4
u	1	.2568	.0653	.0166	.0043
M=4, nt = π/2					
y	0	1	2	3	4
u	1	.1989	.0396	.0078	.0017

From Table-1 we find that the velocity decreases by increasing the Hartmann number M. Also it can be seen that the amplitude $|\beta|$ of the skin friction increases with the increases of M.

For small values of frequency parameter n, the velocity is given by

$$u = e^{-cy}\left[1+\varepsilon\left(1+\frac{n^2 y}{16(1+M)}\right)\left\{Cos\left(nt-\frac{ny}{4\sqrt{1+M}}\right)+1\sin\left(nt-\frac{ny}{4\sqrt{1+M}}\right)\right\}\right] \qquad \text{.................. (24)}$$

For large value of frequency parameter n the velocity is given by

$$u = e^{-\alpha y} + \varepsilon \exp \frac{1}{2}\left(1+\sqrt{1+M+\sqrt{n}}\right) y \left(1-\frac{1}{2\sqrt{2}}\right) \frac{(1+M)^2 y}{\sqrt{n}\sqrt{1+M+\sqrt{n}}}$$

$$\left[Cos(nt - \left\{ \frac{v v_n - (1+M)}{2\sqrt{2}} - \frac{1}{8\sqrt{2}} \frac{(1+M)^2}{\sqrt{n}\sqrt{n-(1+M)}} \right\} y) \right.$$

$$\left. + i \sin\left(nt - \left\{ \frac{\sqrt{n-(1+M)}}{2\sqrt{2}} y - \frac{1}{8\sqrt{2}} \frac{(1+M)^2 y}{\sqrt{n}\sqrt{n-(1+M)}} \right\} \right) \right] \qquad \text{........................ (25)}$$

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