A View on Intuitionistic Uniform Lattice Structure Spaces

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Abstract—Motivated by the concept of uniform space which is a generalization of a metric spaces introduced by A. Weil [10], in this paper, the concepts of intuitionistic uniform structure spaces, intuitionistic uniform lattice structure spaces, intuitionistic uniform lattice B-normal spaces and intuitionistic uniform lattice almost B-normal spaces are introduced and studied in this paper. In this connection characterizations are established.

Keywords—intuitionistic uniform lattice structure spaces, intuitionistic uniform lattice B-normal spaces and intuitionistic uniform lattice almost B-normal spaces.

I. INTRODUCTION

The concept of intuitionistic sets in topological spaces was introduced by Coker in [1]. He studied topology on intuitionistic sets in [2]. Motivated by the concept of uniform space which is a generalization of a metric spaces introduced by A. Weil [10], in this paper, the concepts of intuitionistic uniform structure spaces, intuitionistic uniform lattice structure spaces, intuitionistic uniform lattice B-normal spaces and intuitionistic uniform lattice almost B-normal spaces are introduced and studied in this paper. In this connection characterizations are established.

II. PRELIMINARES

In this section, the concepts of intuitionistic symmetric members, intuitionistic uniform structure spaces, intuitionistic t-open symmetric members and intuitionistic B-open symmetric members are introduced and studied.

Notation :2.1.

Given the non-empty set X, define the diagonal Δ as the following set in $X \times X$. $\Delta = \{(x; y) \in X \times X : x = y\}$.

Definition :2.1.

Let $X \times X$ be a non-empty set. An intuitionistic symmetric member A is an object having the form $A = \langle (x,y), A^1, A^2 \rangle$, where A^1 and A^2 are subsets of $X \times X$ satisfying $A^1 \cap A^2 = \Delta$. The set A^1 is called the set of members of A, while A^2 is called the set of nonmembers of A.

Notation 2.2.

Let $X \times X$ be a non-empty set and it is simply denoted by X.

Definition:2.2

Let X be a non-empty set. Let $\mathcal{L} = \langle (x,y), \mathcal{L}^1, \mathcal{L}^2 \rangle$ and $\mathcal{R} = \langle (x,y), \mathcal{R}^1, \mathcal{R}^2 \rangle$ be any two intuitionistic symmetric members in X. Then

(i) $\mathcal{L} \subseteq \mathcal{R}$ if and only if $\mathcal{L}^1 \subseteq \mathcal{R}^1, \mathcal{L}^2 \supseteq \mathcal{R}^2$

(ii) $\mathcal{L} = \mathcal{R}$ if and only if $\mathcal{L} \subseteq \mathcal{R}$ and $\mathcal{R} \subseteq \mathcal{L}$.

(iii) $\bar{\mathcal{L}} = \langle (x,y), \mathcal{L}^2, \mathcal{L}^1 \rangle$.

(iv) $\Delta_{\sim} = \langle (x,y), \Delta, X \rangle$.

(v) $X_{\sim} = \langle (x,y), X, \Delta \rangle$.

Definition:2.3

Let X be a non-empty set. Let $\mathcal{L}_i = \langle (x,y), {\mathcal{L}_i}^1, {\mathcal{L}_i}^2 \rangle : i \in J\}$ be intuitionistic symmetric members in X. Then

(i) $\cup \mathcal{L}_i = \langle (x,y), \cup {\mathcal{L}_i}^1, \cap {\mathcal{L}_i}^2 \rangle$.

(ii) $\cap \mathcal{L}_i = \langle (x,y), \cap {\mathcal{L}_i}^1, \cup {\mathcal{L}_i}^2 \rangle$.

Notation 2.3.

Let X be a non-empty set. Let $\mathcal{L} = \langle (x,y), \mathcal{L}^1, \mathcal{L}^2 \rangle$ be any intuitionistic symmetric member in X. Then

(i) $(x, y) \in \mathcal{L}$ denotes $(x, y) \in \mathcal{L}^1$ and $(x, y) \notin \mathcal{L}^2$.

(ii) $(x, y) \notin \mathcal{L}$ denotes either $(x, y) \notin \mathcal{L}^1$ and $(x, y) \in \mathcal{L}^2$ or $(x, y) \notin \mathcal{L}^1$ and $(x, y) \notin \mathcal{L}^2$.

Notation 2.4. Let X be a non-empty set. Let ny two intuitionistic symmetric members in X.

(i) $\mathcal{L}^t = \{(y, x) \in X : (x, y) \in \mathcal{L}\}$.

(ii) $\mathcal{L} \circ \mathcal{R} = \{(x, z) : (x, y) \in \mathcal{R}$ and $(y, z) \in \mathcal{L}$ for some $y \in X\}$.

Definition:2.4.

An intuitionistic uniform structure on a non-empty set X is a collection $\mathcal{E}$ of intuitionistic symmetric members in X satisfying the following axioms:

(i) $\Delta_{\sim}, X_{\sim} \in \mathcal{E}$.

(ii) $\mathcal{L}_1 \cap \mathcal{L}_2 \in \mathcal{E}$ for any $\mathcal{L}_1 , \mathcal{L}_2 \in \mathcal{E}$.

(iii) $\cup \mathcal{L}_i \in \mathcal{E}$ for any arbitrary family $\{\mathcal{L}_i : i \in J\} \subseteq \mathcal{E}$.

(iv) If $\mathcal{L}_1 \subset \mathcal{L}_2$ and $\mathcal{L}_1 \in \mathcal{E}$ then $\mathcal{L}_2 \in \mathcal{E}$.

(v) If $\mathcal{L}_1 \in \mathcal{E}$ then $\mathcal{L}_1^t = \{(x, y) : (y, x) \in \mathcal{L}_1\} \in \mathcal{E}$.

(vi) For any $\mathcal{L}_1 \in \mathcal{E}$ there is some $\mathcal{L}_2 \in \mathcal{E}$ such that $\mathcal{L}_2^2 \subset \mathcal{L}_1$.

In this case the pair (X, $\mathcal{E}$) is called an intuitionistic uniform structure space and any intuitionistic symmetric member in $\mathcal{E}$ is called an intuitionistic open symmetric member. The complement of an intuitionistic open symmetric member is called an intuitionistic closed symmetric member.

III INTUITIONISTIC UNIFORM LATTICE SPACES

In this section, the concepts of intuitionistic lattice symmetric members, intuitionistic uniform lattice spaces and intuitionistic lattice chain are introduced and some of their properties are established.

Definition:3.1.

Let (X, $\mathcal{E}$) be an intuitionistic uniform structure space. Let A = $\langle (x, y), A^1 \; A^2 \rangle$ be an intuitionistic symmetric member in X.

(i) If a family $\{\mathcal{L}_i = \langle (x,y), \mathcal{L}_i{}^1, \mathcal{L}_i{}^2 \rangle : i \in J\}$ of intuitionistic open symmetric members in X satisfies the condition $A \subseteq \cup\{\mathcal{L}_i : i \in J\}$, then it is called an intuitionistic uniform open cover of A.

(ii) A finite subfamily of an intuitionistic uniform open cover $\{\mathcal{L}_i : i \in J\}$ of A, which is also an intuitionistic uniform open cover of A, is called an intuitionistic uniform finite subcover of $\{\mathcal{L}_i : i \in J\}$.

(iii) An intuitionistic symmetric member A in an intuitionistic uniform structure space is called an intuitionistic uniform compact iff every intuitionistic uniform open cover of A has an intuitionistic uniform finite subcover.

Notation 3.1.

Let (X, $\mathcal{E}$) be an intuitionistic uniform structure space. Let $\mathcal{L} = \langle (x,y), \mathcal{L}^1, \mathcal{L}^2 \rangle$ and $\mathcal{R} = \langle (x,y), \mathcal{R}^1, \mathcal{R}^2 \rangle$ be any two intuitionistic symmetric members. Then $\mathcal{L} \sim D = (\mathcal{L} \cap \overline{\mathcal{R}}) \cup (\mathcal{R} \cap \bar{\mathcal{L}})$.

Definition:3.2.

Let (X, $\mathcal{E}$) be an intuitionistic uniform structure space. Let $\mathcal{L} = \langle (x,y), \mathcal{L}^1, \mathcal{L}^2 \rangle$ be an intuitionistic closed symmetric member in (X, $\mathcal{E}$). Then the intuitionistic uniform lattice of $\mathcal{L}$ is denoted and defined as $[\mathcal{L}] = \{ \mathcal{R} = \langle (x,y), \mathcal{R}^1, \mathcal{R}^2 \rangle : \mathcal{R}$ is an intuitionistic closed symmetric member and IUcl(A ~ D) is intuitionistic uniform compact }.

Notation :3.2.

Let (X, $\mathcal{E}$) be an intuitionistic uniform structure space. Let $\mathcal{L}_1 = \langle (x,y), \mathcal{L}_1{}^1, \mathcal{L}_1{}^2 \rangle$ and $\mathcal{L}_2 = \langle (x,y), \mathcal{L}_2{}^1, \mathcal{L}_2{}^2 \rangle$ be any two intuitionistic uniform lattices. Then

(i) $[\mathcal{L}_1] \wedge [\mathcal{L}_2] = [\mathcal{L}_1 \cap \mathcal{L}_2]$.

(ii) $[\mathcal{L}_1] \vee [\mathcal{L}_2] = [\mathcal{L}_1 \cup \mathcal{L}_2]$.

(iii) $[\mathcal{L}_1]$ and $[\mathcal{L}_2]$ are disjoint denotes there is no common intuitionistic uniform lattices.

(iv) [$\mathcal{E}$] denote the collection of all intuitionistic uniform lattices.

Definition:3.3.

Let (X, $\mathcal{E}$) be an intuitionistic uniform structure space. An intuitionistic uniform lattice structure, $\mathcal{L}_\mathcal{E}$ is a non-empty collection of intuitionistic uniform lattices in (X, $\mathcal{E}$) satisfying the following axioms:

(i) $[L_1] \wedge [L_2] \in \mathcal{L}_\mathcal{E}$ for any $[L_1], [L_2] \in \mathcal{L}_\mathcal{E}$.

(ii) $[L1] \vee [L2] \in \mathcal{L}_\mathcal{E}$ for any $[L1], [L2] \in \mathcal{L}_\mathcal{E}$.

In this case $\mathcal{L}_\mathcal{E}$ is said to be an intuitionistic uniform lattice structure space and every member of $\mathcal{L}_\mathcal{E}$ is said to be an intuitionistic lattice open symmetric member.

Definition:3.4.

Let $\mathcal{L}_\mathcal{E}$be an intuitionistic uniform lattice structure space and [A] be an intuitionistic lattice open symmetric member. Then $[\bar{A}] = [\mathcal{E}] \setminus [A]$ is said to be an intuitionistic lattice closed symmetric member.

Definition:3.5

Let $\mathcal{L}_\mathcal{E}$ be an intuitionistic uniform lattice structure space. Let [A] be an intuitionistic uniform lattice. Then the intuitionistic uniform lattice closure and intuitionistic uniform lattice interior of [A] are denoted and defined as

IU$\mathcal{L}$cl([A]) = $\cap$\{[K] : [K] is an intuitionistic lattice closed symmetric member and [A] $\subseteq$ [K]\}.

IU$\mathcal{L}$int([A]) = $\cup$\{[K] : [K] is an intuitionistic lattice open symmetric member and [K] $\subseteq$ [A]\}.

Definition:3.6

Let $\mathcal{L}_\mathcal{E}$ be an intuitionistic uniform lattice structure space. An intuitionistic uniform lattice [A] is said to be an

(i) intuitionistic lattice t-open symmetric if IU$\mathcal{L}$int([A]) = IU$\mathcal{L}$int(IU$\mathcal{L}$cl([A])).

(ii) intuitionistic lattice t-closed symmetric if IU$\mathcal{L}$cl([A]) = IU$\mathcal{L}$cl(IU$\mathcal{L}$int([A])).

(iii) intuitionistic lattice B-open symmetric member if [A] = [K]$\cap$[L] where $[K] \in \mathcal{L}_\mathcal{E}$and [L] is an intuitionistic lattice t-open symmetric member.

(iv) intuitionistic lattice B-closed symmetric member if [A] = [K]$\cup$[[L] where [K] is an intuitionistic lattice closed symmetric member and [L] is an intuitionistic lattice t-closed symmetric member.

Definition:3.7.

Let $\mathcal{L}_\mathcal{E}$ be an intuitionistic uniform lattice structure space. Let [A] be an intuitionistic uniform lattice. Then the intuitionistic uniform lattice B-interior and intuitionistic uniform lattice B-closure of A are denoted and defined as

IU$\mathcal{L}$Bint([A]) =$\cup$ { [K] : [K] is an intuitionistic lattice B-open symmetric member and[K] $\subseteq$ [A]}.

IU$\mathcal{L}$Bcl([A]) = $\cap${[K] : [K] is an intuitionistic lattice B-closed symmetric member and[A] $\subseteq$[K]}.

Definition:3.8

Let $\mathcal{L}_\mathcal{E}$ be an intuitionistic uniform lattice structure space. Then $\mathcal{L}_\mathcal{E}$is said to be an intuitionistic uniform lattice B-normal space if for any pair of disjoint intuitionistic lattice B-closed symmetric members [X], [Y], there exist intuitionistic lattice B-open symmetric members [U] and [V] such that [X] $\subseteq$ [U] and [Y] $\subseteq$ [V].

Proposition 3.1.

Let $\mathcal{L}_\mathcal{E}$ be an intuitionistic uniform lattice structure space. Then the following statements are equivalent:

(i) $\mathcal{L}_\mathcal{E}$ is an intuitionistic uniform lattice B-normal space.

(ii) For each intuitionistic lattice B-closed symmetric member [$\mathcal{F}$] and for each intuitionistic lattice B-open symmetric member [U] containing [$\mathcal{F}$], there exists an intuitionistic lattice B-open symmetric member [V] containing [$\mathcal{F}$] such that IU$\mathcal{L}$Bcl([V]) ⊆ [U].

(iii) For each pair of disjoint intuitionistic lattice B-closed symmetric members [A] and [B] in, there exists an intuitionistic lattice B-open symmetric member [U] containing [A] such that IU$\mathcal{L}$Bcl([U]) ∩ [B] = ∅.

(iv) For each pair of intuitionistic lattice B-closed symmetric members [A] and [B] in $\mathcal{L}_\mathcal{E}$, there exist intuitionistic lattice B-open symmetric members [U] and [V] such that [A] ⊆[U] and [B] ⊆ [V] such that IU$\mathcal{L}$Bcl([U]) ∩ IU$\mathcal{L}$Bcl([V]) = ∅.

Proposition 3.2.

Let $\mathcal{L}_\mathcal{E}$ be an intuitionistic uniform lattice structure space. Then the following statements are equivalent:

(i) $\mathcal{L}_\mathcal{E}$ is an intuitionistic uniform lattice B-normal space.

(ii) For every pair of intuitionistic lattice B-open symmetric members [U] and [V] whose union is [$\mathcal{E}$], there exist intuitionistic lattice B-closed symmetric members [A] and [B] such that [A] ⊆ [U], [B] ⊆ [V] and [A]∪[B] = [$\mathcal{E}$].

(iii) For every intuitionistic lattice B-closed symmetric member [$\mathcal{F}$] and every intuitionistic lattice B-open symmetric member [$\mathcal{g}$] containing [$\mathcal{F}$], there exists an intuitionistic lattice B-open symmetric member [U] such that [$\mathcal{F}$] ⊆ [U] ⊆ IU$\mathcal{L}$Bcl([U]) ⊆ [$\mathcal{g}$].

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