COUETTE FLOW OF GAS USING KAMAL TRANSFORM METHOD

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Abstract— In this paper we discuss the applicability of Kamal Transform technique [5] for finding the analytical solution of unsteady couette flow of gas.

Index Terms—Unsteady Couette flow, Partial differential equations, Kamal Transform.

I. Introduction

In the field of Fluid mechanics, most of them are familiar with problems of Couette fluid flow. The flow between two plates such that the bottom plate is fixed and the top plate is in rest initially and it is set into motion in its own plane with constant velocity is called Couette flow [1], [2]. This kind of flow has application in the hydro-static lubrication, viscosity pumps and turbine. Many authors have investigated such type of flows using different methods. But still much work has to be done in this area. Hall effects on Couette flow through a porous straight channel was studied by Bhaskara and Bathaiah [3].The analytic solutions of couette flows of a second grade fluid is obtained using Laplace Transform by Hayat et al.[4].

One of the newly developed transformations was introduced by Abdelilah Kama[5]l in 2016, to ease the process of solving ordinary and partial differential equations in the time domain. Kamal Transform is derived from the classical Fourier integral. Based on the mathematical simplicity of the Kamal transform and its fundamental properties. This transformation more related with the Laplace and Sumudu Transform (Abdelilah Kamal, [5]; Abdelilah Kamal, [6]).

Further, applications of these new integral transforms to the solution of partial differential equations are demonstrated in [8] and [9], respectively for interested readers.

Recently, Vennila [9] studied unsteady couette flow of a gas using Elzaki Transform. Motivated by above mentioned studies, an attempt has made to find the solution of Couette flow of a gas by applying new integral transform named as Kamal transform [5].

II. Mathematical analysis

Let us consider the unsteady laminar flow of a fluid between two infinite parallel plates. It is assumed that the fluid and the plates are at rest initially. At time t = 0^+, the lower plate suddenly starts to slide in its own plane with velocity proportional to U and the upper plate is fixed which is at a distance l from the lower plate. A coordinate system is introduced such that x-axis is chosen along the lower plate, y-axis is normal it.

The equation of motion can be written as $\frac{\partial}{\partial t}\psi+(\psi.\nabla)\psi=-\frac{1}{\eta}gardp+\varsigma\nabla^2\psi$ ____________(1)

$div(\psi)=0$; Now from (1), we have $\frac{\partial\psi}{\partial t}=\varsigma\frac{\partial^2\psi}{\partial y^2}$ ____________(2)

and the Boundary conditions are $\begin{cases}\psi=\mu,\ y=0\ \ for\ t>0\\ \psi=0,y=\gamma\ \ for\ \ t>0\\ \psi=0,in\ 0\le y\le\gamma\ \ for\, t\le 0\end{cases}$ ___________ (3)

Let $G(y,\tau)$ be the Kamal transform of $\psi(y,t)$. Taking kamal transform on both sides of (2), then we have

$$k\left\{\frac{\partial\psi}{\partial t}-\zeta\frac{\partial^2\psi}{\partial y^2}\right\}=k(0)\qquad\Rightarrow k\left\{\frac{\partial^2\psi}{\partial y^2}\right\}-\frac{1}{\zeta}k\left\{\frac{\partial\psi}{\partial t}\right\}=0$$

$$\Rightarrow\frac{d^2}{dy^2}G(y,\tau)-\frac{1}{\zeta}\left\{-\psi(y,0)+\frac{1}{\tau}G(y,\tau)\right\}=0$$

____________(4)

$$\Rightarrow\frac{d^2}{dy^2}G(y,\tau)-\frac{1}{\zeta}\frac{1}{\tau}G(y,\tau)=0$$

The general solution of (4) is given by $G(y,\tau)=c_1\exp\left(\frac{1}{\sqrt{\zeta\tau}}\right)(y)+c_2\exp\left(\frac{1}{\sqrt{\zeta\tau}}\right)(-y)$ ________ (5)

Applying kamal transform on (3); $G(0,\tau)=\mu\tau,\quad G(\gamma,\tau)=0,\quad G(y,0)=0$ ___________ (6)

Solving (5) and (6), we have $c_1 = \dfrac{\mu\tau}{\left(1-e^{\frac{2\gamma}{\sqrt{\zeta\tau}}}\right)}$ and $c_2 = -\dfrac{\mu\tau e^{\frac{2\gamma}{\sqrt{\zeta\tau}}}}{1-e^{\frac{2\gamma}{\sqrt{\zeta\tau}}}}$ ——— (7)

Now substituting (7) in (5), we get

$$G(y,\tau)=\frac{\mu\tau}{1-e^{\frac{2\gamma}{\sqrt{\zeta\tau}}}}\exp\left(\frac{1}{\sqrt{\zeta\tau}}\right)(y)-\frac{\mu\tau e^{\frac{2\gamma}{\sqrt{\zeta\tau}}}}{1-e^{\frac{2\gamma}{\sqrt{\zeta\tau}}}}\exp\left(\frac{1}{\sqrt{\zeta\tau}}\right)(-y)=\frac{\mu\tau}{1-e^{\frac{2\gamma}{\sqrt{\zeta\tau}}}}\left\{e^{\frac{y}{\sqrt{\zeta\tau}}}-e^{\frac{2\gamma}{\sqrt{\zeta\tau}}}e^{\frac{-y}{\sqrt{\zeta\tau}}}\right\}$$

$$G(y,\tau)=\frac{\mu\tau}{1-e^{\frac{2\gamma}{\sqrt{\zeta\tau}}}}\left\{e^{\frac{\gamma}{\sqrt{\zeta\tau}}}e^{\frac{-\gamma}{\sqrt{\zeta\tau}}}e^{\frac{y}{\sqrt{\zeta\tau}}}-e^{\frac{\gamma}{\sqrt{\zeta\tau}}}e^{\frac{\gamma}{\sqrt{\zeta\tau}}}e^{\frac{-y}{\sqrt{\zeta\tau}}}\right\}=\mu\tau\left\{\sinh\left(\frac{\gamma-y}{\sqrt{\zeta\tau}}\right)\Big/\sinh\left(\frac{\gamma}{\sqrt{\zeta\tau}}\right)\right\} \qquad \text{———(8)}$$

Now taking inverse of kamal transform we get

$$K^{-1}\left[G(y,\tau)\right]=\psi(y,t)=K^{-1}\left\{\frac{\mu\tau\sinh\left(\frac{\gamma-y}{\sqrt{\zeta\tau}}\right)}{\sinh\left(\frac{\gamma}{\sqrt{\zeta\tau}}\right)}\right\} \qquad \text{——— (9)}$$

This can be obtained by expressing into Laplace transform

$$K^{-1}\left(G(y,\tau)\right)=Sum\,of\ residues\,of\left\{e^{st}G\left(y,\frac{1}{s}\right)\right\} \qquad \text{——— (10)}$$

$G\left(y,\frac{1}{s}\right)$ has a pole at s=0 and also at $s=\dfrac{-n^2\pi^2\zeta}{\gamma^2}$ where $n=0,\pm1,\pm2,\ldots\ldots$

Now Residue of $e^{st}G\left(y,\frac{1}{s}\right)$ at s=0 is $\underset{s\to 0}{lt}\dfrac{d}{ds}\left\{\dfrac{s^2e^{st}\mu\sinh\left((\gamma-y)\sqrt{\frac{s}{\zeta}}\right)}{s\sinh\left(\gamma\sqrt{\frac{s}{\zeta}}\right)}\right\}=\mu\left(\dfrac{\gamma-y}{\gamma}\right)$

Residue of $e^{st}G\left(y,\frac{1}{s}\right)$ at $s=s_n$

$$\underset{s\to s_n}{lt}\frac{e^{st}\mu\sinh\left((\gamma-y)\sqrt{\frac{s}{\zeta}}\right)}{s\frac{d}{ds}\left\{\sinh\left(\gamma\sqrt{\frac{s}{\zeta}}\right)\right\}}=-\frac{2\mu\exp\left(-\left(\frac{n^2\pi^2\zeta}{\gamma^2}\right)t\right)\sin\left(\frac{n\pi y}{\gamma}\right)}{n\pi}$$

Substituting these values in (10), $\psi(y,t)=\mu\left(\dfrac{\gamma-y}{\gamma}\right)+\dfrac{\mu}{\pi}\displaystyle\sum_{n=1}^{\infty}\dfrac{(-2)}{n}\exp\left[-\left(\dfrac{n^2\pi^2\zeta}{\gamma^2}\right)t\right]\sin\left(\dfrac{n\pi y}{\gamma}\right)$

III. Conclusion

We have successfully demonstrated the analytical solution for the equation of motion in favor of unsteady couette flow between two analogous inestimable plates in which one plate is motionless and second plate is moved with constant velocity by kamal transform.

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