FUZZY MULTI-ITEM PRODUCTION INVENTORY MODEL IN AN AGRICULTURAL FIELD WITH RANDOM DEMAND RATE UNDER SIZE CONSTRAINTS

[1] K.Dhanam, [2] W.Jesintha
[1]Associate Professor, [2]Research Scholar
[1&2]Department of Mathematics,
[1&2]Government Arts College for Women (Autonomous), Pudukkottai, India.

Abstract : ***In this paper, a fuzzy multi item production inventory model with size constraints has been considered with demand rate as random variable. The production rate depends directly on demand rate and time. This model has been proposed in fuzzy environment. The Lower-Upper (L-U) bud with rhombus fuzzy number is defined and its properties are given. The parameters involved in this model are represented by L-U bud with rhombus fuzzy number. The total cost is defuzzified by the technique of agreement index of L-U bud fuzzy number. This model is solved using matlab software. The Numerical example is given to illustrate the model.***

Keywords : multi item, probabilistic demand, level constraint, L-U bud with rhombus fuzzy number, agreement index technique.
AMS Subject Classification : 90B05.

I. INTRODUCTION

Inventory system is one of the main streams of the operations research which is essential in business enterprises and Industries. In business organization, inventory management is one of the major core competencies to compete in the global market place. Deterioration cannot be avoided in business scenarios. Deterioration is defined as change, damage, decay, spoilage, obsolescence and loss of utility or loss of original value in a commodity that results in the decreasing usefulness from the original one product. In real life and global market situations demand and various relevant costs are not exactly known. Often uncertainties may be associated with demand, supply and various relevant costs. In conventional inventory models uncertainties are treated as randomness and are handled by appealing to probability theory. However, in certain situations uncertainties are due to fuzziness and in these cases the fuzzy set theory is applicable. Fuzzy set theory is given in [1,6,10 & 12]. Zadeh introduced fuzzy concept in decision making in [13]. Fuzzy goals, costs and constraints are explained in [3].

Deterministic and probabilistic models in inventory control is given in [9]. A heuristic for replenishment for deteriorating items with linear trend in demand is explained in [5]. The inventory model for variable demand and production is given in [4]. Integrating maintenance planning and production scheduling are discussed in [8]. A production inventory model with shortages, fuzzy preparation time and variable production and demand is given in [7 &11].

In agricultural field, at the time of production and during supply, the rate of deterioration is constant. In general, the production rate changes according to the number of working machines, economy, seasonal promotion forecasting, etc., and demand rate changes due to weather, competition, customer status. Time also plays an important role in the inventory system. In our proposed inventory model, the production rate for plantain fibre extracting is constant and the production rate for plantain fibre products depends on the time and demand rate, in the period of making plantain fibre products with size constraints, the demand rate is taken as random variable which follows gamma distribution (For slow-moving products, the poisson distribution may be appropriate, in other cases, the gamma distribution provides a flexible means for modeling the randomness of the period demands).

II. Assumptions and Notations

The following assumptions and notations are used throughout this paper :

Assumptions :

1. The inventory system pertains multi - item in a time horizon.
2. The production rate depends on the demand rate and time

   ie., $d_i\left(\left(t_3 - t_2 / \left(t_2 - t_1\right)\right)\right)$.
3. The demand rate follows gamma distribution.
4. The deterioration occurred during the period of plantain fibre extracting and the plantain fibre products.
5. The deterioration rate is constant.

Notations:

The following are for i^{th} items (i = 1,2,3.......n)

n - number of items
N - number of replenishment cycles
t_1 - the time in which the production for plantain fibre bundles.
t_2 - the time in which the production and supply for plantain fibre products.
t_3 - the time in which the supply for plantain fibre products only.
I_f - order quantity for plantain trees (decision variable).
I_w - maximum inventory level for plantain fibre bundles (decision variable).
I_{mi} - maximum inventory level for plantain fibre products (decision variable).

$I_1(t)$ - inventory level at any instant of time t, $0 \le t \le t_1$.
$I_{2i}(t)$ - inventory level at any instant of time t, $t_1 \le t \le t_2$.
$I_{3i}(t)$ - inventory level at any instant of time t, $t_2 \le t \le t_3$.
p - production rate (for plantain fibre extracting)
θ_1 - deterioration rate (for plantain fibre extracting)
θ_2 - deterioration rate (for plantain fibre products)
$\tilde{o}_{ci}$ - fuzzy ordering cost per order
$\tilde{p}_{ci}$ - fuzzy purchasing cost per unit per unit time
$\tilde{h}_{c1i}$ - fuzzy holding cost per unit per unit time for plantain fibre extracting.
$\tilde{h}_{c2i}$ - fuzzy holding cost per unit per unit time for plantain fibre products.
$\tilde{d}_{c1i}$ - fuzzy deteriorating cost per unit per unit time for plantain fibre extracting.
$\tilde{d}_{c2i}$ - fuzzy deteriorating cost per unit per unit time for plantain fibre products.

III. Mathematical Model in Crisp Environment

At the time t = 0, on-hand inventory level (plantain trees) is I_f (decision variable) . The production takes place in the period $[0,t_1]$. Here the production means of warping the plantain trees into fibre bundles, the fibre bundle level is I_w at the time of t_1. During the period $[t_1, t_2]$, the fibre products are produced and also meets out the demand. During the period $[t_2, t_3]$, only the demand takes place. The level of I_w (decision variable) can be written as $aI_f - b$.

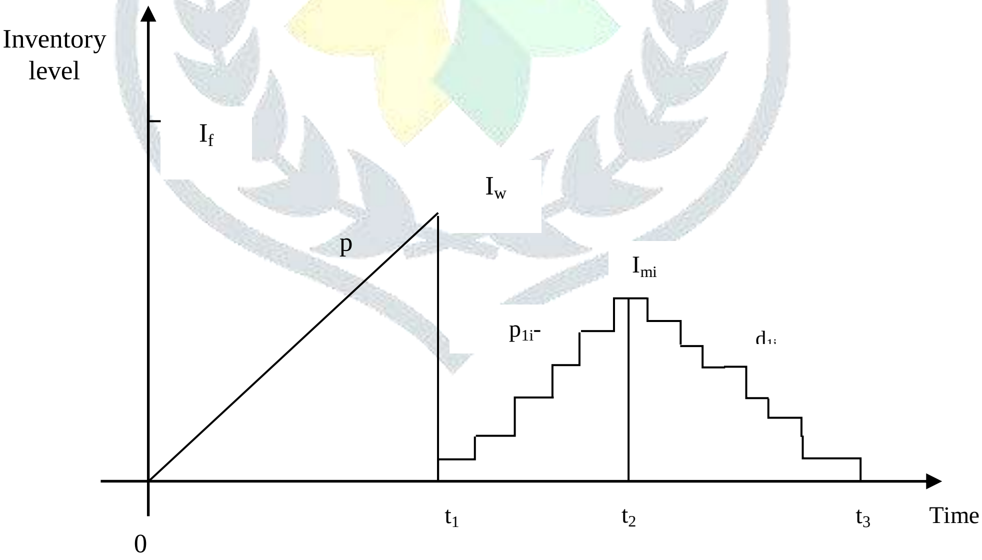
The rate of change of the inventory level for one cycle are governed by following differential equations for plantain fibre extracting and plantain fibre products. (according to Arindam Roy et.al.,)

$$\frac{dI_1(t)}{dt} + \theta_1 I_1(t) = p, \qquad 0 \le t \le t_1 \qquad ---(1)$$

$$\frac{dI_{2i}(t)}{dt} + \theta_{2i} I_{2i}(t) = p_{1i} - d_{1i}, \quad t_1 \le t \le t_2 \qquad ---(2)$$

$$\frac{dI_{3i}(t)}{dt} + \theta_{2i} I_{3i}(t) = -d_{1i}, \qquad t_2 \le t \le t_3 \qquad ---(3)$$

under the boundary conditions are $I_1(0)=0$, $I_1(t_1)=I_w$, $I_{2i}(t_1)=0$, $I_{2i}(t_2)=I_{mi}$, $I_{3i}(t_2)=I_{mi}$, $I_{3i}(t_3)=0$.



From equation (1),

$$I_1(t)e^{\int \theta_i dt} = \int p e^{\int \theta_i dt} dt + c_1$$

$$I_1(t) = \frac{p}{\theta_1}\left(1 - e^{-\theta_1 t}\right); \qquad 0 \le t \le t_1$$

using the boundary condition $I_1(t_1)=I_w$,

$$I_w = \frac{p}{\theta_1}\left(1 - e^{-\theta_1 t_1}\right)$$

From equation (2),

$$I_{2i}(t)=\left(\frac{p_{1i}-d_{1i}}{\theta_{2i}}\right)+c_2e^{-\theta_{2i}t}$$

using the boundary condition $I_2(t_1)=0$,

$$I_{2i}(t)=\left(\frac{p_{1i}-d_{1i}}{\theta_{2i}}\right)\left(1-e^{\theta_{2i}(t_1-t)}\right)$$

using the boundary condition $I_{2i}(t_2)=I_{mi}$,

$$I_{mi}=\left(\frac{p_{1i}-d_{1i}}{\theta_{2i}}\right)\left(1-e^{-\theta_{2i}(t_2-t_1)}\right)$$

From equation (3),

$$I_{3i}(t)=\left(\frac{-d_{1i}}{\theta_{2i}}\right)+c_3e^{-\theta_2 t}$$

using the boundary condition $I_{3i}(t_{3i})=0$,

$$I_{3i}(t)=\left(\frac{d_{1i}}{\theta_{2i}}\right)\left(e^{-\theta_{2i}t}-1\right)$$

$$\text{Purchasing cost}=\mathrm{p_{ci}}\left[\left(\frac{1}{a}\right)\left(\left(\frac{p}{\theta_1}\right)\left(1-e^{-\theta_1 t_1}\right)+b\right)\right]$$

$$\text{Warping cost}=\mathrm{w_{ci}}\left[\left(\frac{1}{a}\right)\left(\left(\frac{p}{\theta_1}\right)\left(1-e^{-\theta_1 t_1}\right)+b\right)\right]$$

$$\text{Holding cost for fibre extracting}=\mathrm{h_{c1i}}\left[\int_0^{t_1}I_1(t)\,dt\right]$$

$$=\mathrm{h_{c1i}}\left(\frac{p}{\theta_1}\left(t_1+\frac{e^{-\theta t_1}}{\theta_1}-\frac{1}{\theta_1}\right)\right)$$

$$\text{Holding cost for plantain fibre products}=\mathrm{h_{c2i}}\left[\int_{t_1}^{t_2}I_{2i}(t)\,dt+\int_{t_2}^{t_3}I_{3i}(t)\,dt\right]$$

$$=\mathrm{h_{c2i}}\left(\left(\left(\frac{p_{1i}-d_{1i}}{\theta_{2i}}\right)\left((t_2-t_1)+\frac{e^{\theta_{2i}(t_1-t_2)}}{\theta_{2i}}-\frac{1}{\theta_{2i}}\right)\right)-\left(\left(\frac{d_{1i}}{\theta_{2i}}\left((t_3-t_2)+\frac{e^{-\theta_{2i}t_3}}{\theta_{2i}}-\frac{e^{-\theta_{2i}t_2}}{\theta_{2i}}\right)\right)\right)\right)$$

Holding cost per cycle

$$=\mathrm{h_{c1i}}\left(\frac{p}{\theta_1}\left(t_1+\frac{e^{-\theta t_1}}{\theta_1}-\frac{1}{\theta_1}\right)\right)+\mathrm{h_{c2i}}\left(\begin{array}{l}\left(\left(\frac{p_{1i}-d_{1i}}{\theta_{2i}}\right)\left((t_2-t_1)+\frac{e^{\theta_{2i}(t_1-t_2)}}{\theta_{2i}}-\frac{1}{\theta_{2i}}\right)\right)-\\ \left(\left(\frac{d_{1i}}{\theta_{2i}}\left((t_3-t_2)+\frac{e^{-\theta_{2i}t_3}}{\theta_{2i}}-\frac{e^{-\theta_{2i}t_2}}{\theta_{2i}}\right)\right)\right)\end{array}\right)$$

$$\text{Deteriorating cost for fibre extracting}=\mathrm{d_{c1i}}\left[pt_1-I_w\right]$$

$$=\mathrm{d_{c1i}}\left(pt_1-\left(\frac{p}{\theta_1}\left(1-e^{-\theta_1 t_1}\right)\right)\right)$$

$$\text{Deteriorating cost for plantain fibre products}=\mathrm{d_{c2i}}\left[(p_{1i}-d_{1i})(t_2-t_1)-d_{1i}(t_3-t_2)\right]$$

Deteriorating cost per cycle

$$=\mathrm{d_{c1i}}\left(pt_1-\left(\frac{p}{\theta_1}\left(1-e^{-\theta_1 t_1}\right)\right)\right)+\mathrm{d_{c2i}}\left[(p_{1i}-d_{1i})(t_2-t_1)-d_{1i}(t_3-t_2)\right]$$

Our proposed model is to minimize the expected total cost,

$$
\mathrm{E}\left(\mathrm{TC}\left(t_1,t_2,t_3\right)\right)
$$

$$
=\sum_{i=1}^{n}\left[\begin{array}{l}
o_{ci}+p_{ci}\left(\dfrac{p}{a\theta_1}\left(1-e^{-\theta_1 t_1}\right)+\dfrac{b}{a}\right)+w_{ci}\left(\dfrac{p}{a\theta_1}\left(1-e^{-\theta_1 t_1}\right)+\dfrac{b}{a}\right)+\\
h_{c1i}\left(\dfrac{p}{\theta_1}\left(t_1+\dfrac{e^{-\theta_1 t_1}}{\theta_1}-\dfrac{1}{\theta_1}\right)\right)+\mathrm{d}_{c1i}\left(pt_1-\left(\dfrac{p}{\theta_1}\left(1-e^{-\theta_1 t_1}\right)\right)\right)\\
+\mathrm{h}_{c2i}\left(\begin{array}{l}\left(\left(\left(\dfrac{\left(\left(t_3-t_2\right)/\left(t_2-t_1\right)\right)\left(\alpha_i/\beta_i\right)-\left(\alpha_i/\beta_i\right)}{\theta_{2i}}\right)\left(\left(t_2-t_1\right)+\dfrac{e^{\theta_{2i}\left(t_i-t_2\right)}}{\theta_{2i}}-\dfrac{1}{\theta_{2i}}\right)\right)-\right.\\
\left.\left(\left(\dfrac{\left(\alpha_i/\beta_i\right)}{\theta_{2i}}\right)\left(\left(t_3-t_2\right)+\dfrac{e^{-\theta_{2i}t_3}}{\theta_{2i}}-\dfrac{e^{-\theta_{2i}t_2}}{\theta_{2i}}\right)\right)\right)\end{array}\right)+\\
\mathrm{d}_{c2i}\left[\left(\left(t_3-t_2\right)/\left(t_2-t_1\right)\right)\left(\alpha_i/\beta_i\right)_1-\left(\alpha_i/\beta_i\right)\left(t_2-t_1\right)-\left(\alpha_i/\beta_i\right)\left(t_3-t_2\right)\right]
\end{array}\right]Nt_3
$$

Subject to

Case (i) : $I_{mi} > I_W$;

Case (ii) : $I_{mi} \le I_W$;

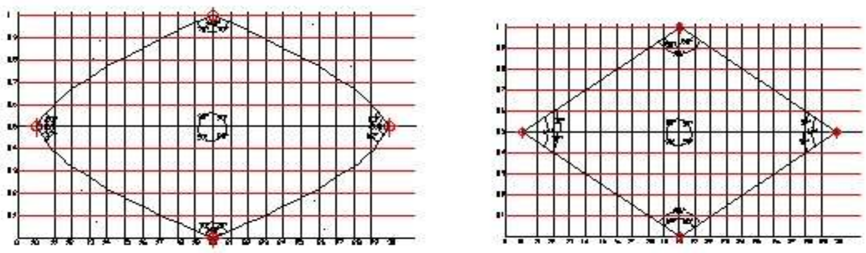
The optimal value of t_1, t_2 and t_3 have been obtained from the expected total cost E(TC(t_1,t_2,t_3)) by using MATLAB Software.

IV. LU-bud with rhombus Fuzzy Number and its agreement index method

Definition :(L-U bud with rhombus fuzzy number)

A L-U bud with rhombus fuzzy number $\tilde{A}$ described as a normalized convex fuzzy subset on the real line R whose membership function $\mu_{\tilde{A}}(x)$ is defined as follows

$$
\mu_{\tilde{A}}(x)=\begin{cases}
0.5 & at \quad x=a,c\\
\dfrac{1}{2}\left\{1-\left(\dfrac{x-a}{b-a}\right)^{\frac{n}{n+1}}\right\} & at \quad a\le x\le b\\
\dfrac{1}{2}\left\{1-\left(\dfrac{c-x}{c-b}\right)^{\frac{n}{n+1}}\right\} & at \quad b\le x\le c\\
\dfrac{1}{2}\left\{1+\left(\dfrac{x-a}{b-a}\right)^{\frac{n}{n+1}}\right\} & at \quad a\le x\le b\\
\dfrac{1}{2}\left\{1+\left(\dfrac{c-x}{c-b}\right)^{\frac{n}{n+1}}\right\} & at \quad b\le x\le c\\
0 \ and\ 1 & at \quad x=b
\end{cases}
\qquad \text{for all } n\ge 0.
$$



This type of fuzzy number is denoted as $\tilde{A}=\left[a,b,c;w_A\right]$ where $w_A=0.5$, whose membership function $\mu_{\tilde{A}}(x)$ satisfies the following conditions:

1. $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval [0,1].
2. $\mu_{\tilde{A}}(x)$ is a convex function, $\mu_{\tilde{A}}(x)=0,1 \ at \ x=b, \ \mu_{\tilde{A}}(x)=0.5 \ at \ x=a,c.$
3. $\mu_{\tilde{A}}(x)$ is strictly decreasing as well as increasing on [a, b] and [b, c].
4. When n=0, the membership function gives the four points (a,0.5), (b,0), (b,1), (c,0.5).
5. The two line joining of the points (a,0.5), (c,0.5) and (b,0), (b,1) are intersect orthogonally.
6. By joining the four points (a,0.5), (b,0), (c,0.5), (b,1), the rhombus is formed.
7. When n > 0, the L-U bud shape is formed by joining the membership values.
8. The opposite angle of the L-U bud curve is decreased in horizontal wise and increased in vertical wise, when the value of n > 0.

Properties:

1. Opposite angles are equal.
2. The horizontal and vertical diagonal bisect each other and meet at 90^0.
3. In the horizontal and vertical diagonal, the base of the adjacent angles are equal.
4. The length of the horizontal diagonal is twice of the length of the vertical diagonal.

V. Agreement index method

A function $h(x)$; $x \in R$ such that

$$h(x)=\begin{cases} 1 & at \quad x \le x_1 \\ \dfrac{x_2-x}{x_2-x_1} & at \quad x_1 \le x \le x_2 \\ 0 & at \quad x \ge x_2 \end{cases}$$

$h(x)$ represent by a fuzzy subset $H \subset R.$ consider a fuzzy number $A \subset R,$ which we call the agreement index of A with regard to H, the ratio being defined as

$$i(A,H)=\frac{area \ of \ \ A \cap H}{area \ of \ \ A}$$

When **A is nonfuzzy**, that is,

$$\mu(A)=\begin{cases} 1, & x=n \\ 0, & x \ne n \end{cases} \quad \text{then,} \quad i(A,H)=h(n).$$

The agreement index of a L-U bud fuzzy number is of the form

When **H is nonfuzzy**, that is,

$$h(x)=\begin{cases} 1, & x \le x_1 \\ 0, & x > x_1 \end{cases}$$ In this case compute the area of A to the left of x_1.

$$i_G(A,H)=\frac{1}{I_7}(I_1+I_2+I_3+I_4+I_5+I_6)$$

where,

$$I_1=\left(\frac{(c-x_2)(x_2-x_a)}{x_2-x_1}\right), I_2=\left(\frac{(x_2-a)(x_a-x_b)}{x_2-x_1}\right),$$

$$I_3=\left(\frac{x_a^{\ 2}-x_b^{\ 2}+2x_2(x_a-x_b)}{2(x_2-x_1)}\right),$$

$$I_4=\left(\frac{n(c-b)}{2(2n+1)}\right)\left(1-2\left(\frac{x_2-x_a}{x_2-x_1}\right)\right)^{\frac{2n+1}{n}}, I_5=\left(\frac{n(a-b)}{2(2n+1)}\right)\left(2\left(\frac{x_2-x_b}{x_2-x_1}\right)-1\right)^{\frac{2n+1}{n}},$$

$$I_6=\left(\frac{n(a-c)}{2(2n+1)}\right), I_7=\left(\frac{(c-a)(n+1)}{2n+1}\right)$$

VI. Inventory Model in Fuzzy Environment

The proposed inventory model in fuzzy environment is

$$E\left(T\tilde{C}(t_1,t_2,t_3)\right) = \sum_{i=1}^{n}\left[\begin{array}{l} \tilde{o}_{ci} + \tilde{p}_{ci}\left(\frac{p}{a\theta_1}\left(1-e^{-\theta_1 t_1}\right)-\frac{b}{a}\right)+\tilde{w}_{ci}\left(\frac{p}{a\theta_1}\left(1-e^{-\theta_1 t_1}\right)-\frac{b}{a}\right)+ \\ \tilde{h}_{c1i}\left(\frac{p}{\theta_1}\left(t_1+\frac{e^{-\theta_1 t_1}}{\theta_1}-\frac{1}{\theta_1}\right)\right)+\tilde{d}_{c1i}\left(pt_1-\left(\frac{p}{\theta_1}\left(1-e^{-\theta_1 t_1}\right)\right)\right)+ \\ \tilde{h}_{c2i}\left(\begin{array}{l}\left(\left(\frac{((t_3-t_2)/(t_2-t_1))(\alpha_i/\beta_i)_1-(\alpha_i/\beta_i)}{\theta_{2i}}\right)\left(\begin{array}{l}(t_2-t_1)+ \\ \frac{e^{\theta_{2i}(t_1-t_2)}}{\theta_{2i}}-\frac{1}{\theta_{2i}}\end{array}\right)\right)- \\ \left(\left(\frac{(\alpha_i/\beta_i)}{\theta_{2i}}\left((t_3-t_2)+\frac{e^{-\theta_{2i}t_3}}{\theta_{2i}}-\frac{e^{-\theta_{2i}t_2}}{\theta_{2i}}\right)\right)\right)\end{array}\right)+ \\ \tilde{d}_{c2i}\left[((t_3-t_2)/(t_2-t_1))(\alpha_i/\beta_i)_1-(\alpha_i/\beta_i)(t_2-t_1)-(\alpha_i/\beta_i)(t_3-t_2)\right]\end{array}\right]Nt_3$$

subject to

$$I_{mi} > I_w;$$
$$I_{mi} \le I_w;$$

where ~ represents the fuzzification of the parameters.

$$E\left(i_G\left(TC(t_1,t_2,t_3),H\right)\right) = \sum_{i-1}^{n}\left[\begin{array}{l} i_G(o_{ci},H)+i_G(p_{ci},H)\left(\frac{p}{a\theta_1}\left(1-e^{-\theta_1 t_1}\right)-\frac{b}{a}\right)+i_G(w_{ci},H)\left(\frac{p}{a\theta_1}\left(1-e^{-\theta_1 t_1}\right)-\frac{b}{a}\right)+ \\ i_G(h_{c1i},H)\left(\frac{p}{\theta_1}\left(t_1+\frac{e^{-\theta_1 t_1}}{\theta_1}-\frac{1}{\theta_1}\right)\right)+i_G(d_{c1i},H)\left(pt_1-\left(\frac{p}{\theta_1}\left(1-e^{-\theta_1 t_1}\right)\right)\right)- \\ i_G(h_{c2i},H)\left(\begin{array}{l}\left(\left(\frac{((t_3-t_2)/(t_2-t_1))(\alpha_i/\beta_i)-(\alpha_i/\beta_i)}{\theta_{2i}}\right)\left(\begin{array}{l}(t_{2i}-t_{1i})+ \\ \frac{e^{\theta_{2i}(t_i-t_2)}}{\theta_{2i}}-\frac{1}{\theta_{2i}}\end{array}\right)\right)- \\ \left(\left(\frac{(\alpha_i/\beta_i)}{\theta_{2i}}\left((t_3-t_2)+\frac{e^{-\theta_{2i}t_i}}{\theta_{2i}}-\frac{e^{-\theta_{2i}t_2}}{\theta_{2i}}\right)\right)\right)\end{array}\right)+ \\ i_G(d_{c2i},H)\left[((t_3-t_2)/(t_2-t_1))(\alpha_i/\beta_i)-(\alpha_i/\beta_i)(t_2-t_1)-(\alpha_i/\beta_i)(t_3-t_2)\right]\end{array}\right]Nt_3$$

Subject to

Case (i) : $I_{mi} \ge I_w;$

Case (ii) : $I_{mi} \le I_w;$

The optimal value of t_1, t_2, t_3 have been obtained from the expected total cost $i_G(E(TC(t_1,t_2,t_3)),H)$ by using MATLAB software.

VII. Numerical Example

The following values of the parameter in proper unit were considered as input for the numerical analysis of the above problem

$o_{c1}=150;\ p_{c1}=58;\ w_{c1}=36;\ h_{c11}=14;\ h_{c21}=19;\ d_{c11}=9;\ d_{c21}=10;$

$a=0.6;\ b=57;\ p=100;\ \theta_{11}=1.1;\ \theta_{21}=0.5;\ \alpha_1=0.8;\ \beta_1=0.1;\ I_w=122;$

$\tilde{o}_{c1}=[150\ 151\ 152]=150.2531;\ \tilde{p}_{c1}=[58\ \ 59\ \ 60]=58.7514;\ \tilde{w}_{c1}=[36\ 37\ 38]=36.6157;$

$\tilde{h}_{c11}=[14\ 15\ 16]=14.2564;\ \tilde{h}_{c21}=[19\ 20\ 21]=19.5432;\ \tilde{d}_{c11}=[9\ 10\ 11]=9.1908;$

$\tilde{d}_{c22}=[8\ 9\ 10]=8.2304;\ a=0.6;\ b=7;\ P=100;\ \theta_{11}=1.1;\ \theta_{21}=0.5;\ \alpha_1=0.8;\ \beta_1=0.1;\ N=1.$

$o_{c2}=151;\ p_{c2}=59;\ w_{c2}=37;\ h_{c11}=15;\ h_{c21}=20;\ d_{c11}=10;\ d_{c21}=11;$

$a=0.6;\ b=7;\ P=100;\ \theta_{11}=1.1;\ \theta_{21}=0.5;\ \alpha_1=0.8;\ \beta_1=0.1;\ I_w=135;$

$\tilde{o}_{c2}=[151\ 152\ 153]=151.1532;\ \tilde{p}_{c2}=[59\ 60\ 61]=59.4580;\ \tilde{w}_{c2}=[37\ 38\ 39]=37.1452;$

$\tilde{h}_{c12}=[15\ 16\ 17]=15.5698;\ \tilde{h}_{c22}=[20\ 21\ 22]=20.7216;\ \tilde{d}_{c12}=[10\ 11\ 12]=10.4536;$

$\tilde{d}_{c22}=[11\ 12\ 13]=11.2386;\ a=0.6;\ b=7;\ P=100;\ \theta_{11}=1.1;\ \theta_{21}=0.5;\ \alpha_1=0.8;\ \beta_1=0.1;\ N=1.$

Comparison table of crisp and fuzzy results :

Model	**Crisp**		**Fuzzy**	
Case 1	**Item 1**	**Item 2**	**Item 1**	**Item 2**
t_1^*	1.9107	1.7036	1.8546	1.6521
t_2^*	3.2566	2.7747	3.0214	2.3012
t_3^*	4.4257	3.8360	4.4100	3.9562
I_f	18.1454	16.8313	18.8750	15.9863
I_m	65.5233	36.0507	67.0012	32.2130
$E(TC(t_1,t_2,t_3))^*$	27,102	28,184	25,985	26,500
Case 2	**Item 1**	**Item 2**	**Item 1**	**Item 2**
t_1^*	1.9592	2.1740	1.5597	2.0689
t_2^*	2.3694	2.5296	2.9304	2.5123
t_3^*	2.7847	2.8892	3.8299	2.8120
I_f	19.3310	28.0973	22.0125	31.0256
I_m	36.3153	40.6781	37.2147	43.0010
$E(TC(t_1,t_2,t_3))^*$	22,115	22,548	21,023	21,956

Observation:

From the above table, it should be noted that compared to crisp model the fuzzy model is very effective method in the sense that the maximum inventory levels are obtained in fuzzy model is higher than the crisp model and the expected total cost is obtained in fuzzy model is lower than the crisp model. Also, the results obtained in two cases, the fuzzy value is obtained in the first case is best one.

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