SOLUTION OF NONLINEAR HIGHER ORDER FUZZY FREDHOLM INTEGRO-DIFFERENTIAL EQUATIONS APPLYING MATHEMATICA IN MODIFIED ADOMIAN DECOMPOSITION METHOD

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ABSTRACT: The nonlinear Integro-Differential equations play a dominated role in the current applications of Mathematical Modelling and Engineering. We use Adomian polynomials to find the approximate solution in crisp case and extend it to fuzzy case. Using this method, we solve problems that the classical methods could not be applied for them in crisp caseand its variations are calculated using MATHEMATICA.

Keywords: Non-linear integro differential equation, adomian decomposition, modified adomian decomposition, Mathematica.

I. INTRODUCTION

We present the study for Fredhlom Integro-differential equations using Adomian decomposition method and existing Numerical solutions. Tables and figures are provided for comparison of the two methods. It is observed that values obtained using Adomian decomposition method agree very well with the existing numerical results.

The fuzzy integral equations and fuzzy differential equations have been rapidly growing in the recent years. The fuzzy mapping function was intriduced by chang and Zadeh (2002).later, Dubois and Prade (1998) presented an elementary fuzzy calculus based on the extension principle.also the concept of integration of fuzzy functions was first introduced by them.then the fuzzy integration is discussed by allahviranloo et (2003).

In the existence of the solution of fuzzy integral equation,the Ascoli's theorem or metric fixed point theorems are used.for the existence and uniqueness, the main tools is the banach fixed point principle.such discussions can be found in (2002,2003,2006). Babolian et al. and abbasbandy et al (2006) obtained a numerical solution of linear Fredholm fuzzy integral equations of the second kind.then Otadi and Mosleh(2007) considered fuzzy nonlinear integral equations of the second kind and obtanied an approximate solution to the fuzzy nonlinear integral equations.

Consider the nonlinear fuzzy Fredholm integral equations such as

$$F(s) = f(s) \oplus \int_a^b K(s,t,F(t))dt,$$

We generalize the nonlinear fuzzy integral equation to the nonlinear fuzzy integro- differential equations

$$F'(s) = f(s) \oplus \int_a^b K(s,t,F(t))dt, \; F(a) = F_0.$$

II. PRELIMINARIES

In this section the basic notations used in fuzzy operations are introduced.

Definition 2.1

A fuzzy number is a function u: R into I= [0, 1] having the properties (2008)

(i) *u is normal, that is* $\exists x_0 \in \mathbb{R}$ *such that* $u(x_0) = 1$;

(ii) *u is a fuzzy convex set;*

(iii) *u is upper semicontinuous on* $\mathbb{R}$;

(iv) *The support* $\overline{\{x \in \mathbb{R} \mid u(x) > 0\}}$ *is a compact set.*

The set of all the fuzzy numbers is denoted by E.

Definition 2.2

A fuzzy number u is a pair $(\underline{u}, \overline{u})$ *of functions* $\underline{u}(r)$ *and* $\overline{u}(r)$, $0 \le r \le 1$, *which satisfies the following requirements*

(i) $\underline{u}(r)$ *is a bounded monotonically non-decreasing, left continuous function on* $(0,1]$ *and right continuous at* 0;

(ii) $\overline{u}(r)$ *is a bounded monotonically non-increasing, left continuous function on* $(0,1]$ *and right continuous at* 0;

(iii) $\underline{u}(r) \le \overline{u}(r), 0 \le r \le 1$.

A crisp number r is simply represented by $\underline{u}(\alpha) = \overline{u}(\alpha) = r, 0 \le \alpha \le 1$

For arbitrary

$u = (\underline{u}(r), \overline{u}(r)), v = (\underline{v}(r), \overline{v}(r))$

And ***k in R*** we define addition and multiplication by k

$$\underline{(u+v)}(r) = (\underline{u}(r) + \underline{v}(r)),$$

$$\overline{(u+v)}(r) = (\overline{u}(r) + \overline{v}(r)),$$

$$\underline{ku}(r) = k\underline{u}(r), \overline{ku}(r) = k\overline{u}(r) \text{ if } k \ge 0,$$

$$\underline{ku}(r) = k\overline{u}(r), \overline{ku}(r) = k\underline{u}(r) \text{ if } k < 0.$$

Definition 2.3

For arbitrary fuzzy numbers *u, v, w* we use the Hausdorff distance D

$E \times E \to \mathbb{R}_+ \cup \{0\}$

$$D(u,v) = \sup_{0 \le r \le 1} \max\{|\overline{u}(r) - \overline{v}(r)|, |\underline{u}(r) - \underline{v}(r)|\}.$$

We denote $\|\cdot\|_E = D(\cdot, \widetilde{0})$, where $\widetilde{0} \in E, \widetilde{0} = \chi_{\{0\}}$.

(i) *The pair* $(E, \oplus)$ *is a commutative semigroup with* $\widetilde{0} = \chi_{\{0\}}$ *zero element;*

(ii) *For fuzzy numbers which are not crisp, there is no opposite element (that is,* $(E, \oplus)$ *cannot be a group);*

(iii) *For any* $a, b \in \mathbb{R}$ *with* $a, b \ge 0$ *or* $a, b \le 0$, *and for any* $u \in E$, *we have* $(a+b) \odot u = a \odot u \oplus b \odot u$;

(iv) *For any* $\lambda \in \mathbb{R}$ *and* $u, v \in E$, *we have* $\lambda \odot (u \oplus v) = \lambda \odot u \oplus \lambda \odot v$;

(v) *For any* $\lambda, \mu \in \mathbb{R}$ *and* $u \in E$, *we have* $\lambda \odot (\mu \odot u) = (\lambda.\mu) \odot u$;

(vi) *The function* $\|\cdot\|_E : E \to \mathbb{R}_+ \cup \{0\}$ *has the usual properties of the norm, that is,* $\|u\|_E = 0$ *if and only if* $u = \widetilde{0}$, $\|\lambda \odot u\|_E = |\lambda| \, \|u\|_E$ *and* $\|u \oplus v\|_E = \|u\|_E \oplus \|v\|_E$ *for any* $u, v \in E$;

(vii) $|\,\|u\|_E - \|v\|_E\,| \le D(u,v)$ *and* $D(u,v) \le \|u\|_E + \|v\|_E$ *for any* $u, v \in E$.

Theorem 2.1 (S.G.Gal (2001))

Theorem 2.2(C.X.Wu (2001))

(i) (E, D) *is complete metric space;*

(ii) $D(u \oplus v, v \oplus w) = D(u, w)$ *for all* $u, v, w \in E$;

(iii) $D(k \odot u, k \odot v) = |k| D(u, v)$ *for all* $u, v \in E$ *and* $k \in \mathbb{R}$;

(iv) $D(u \oplus v, w \oplus e) \le D(u, w) + D(v, e)$ *for all* $u, v, w, e \in E$.

Definition 2.4

Let $f : [a,b] \longrightarrow E^1$, *for each partition* $P = \{t_0, t_1, \cdots, t_n\}$ *of* $[a,b]$

$$R_p = \sum_{i=1}^{n} f(\xi_i)(t_i - t_{i-1}),$$

$$\int_a^b f(x)dx = \lim_{\Delta \to 0} R_p,$$

where

$$\Delta := \max\{|t_i - t_{i-1}|, i = 1, 2, \cdots, n\}$$

provided that this limit exists in the metric D.

If the fuzzy function $f(t)$ *is continuous in the metric D, its definite integral exists*

$$(\underline{\int_a^b f(t;r)dt}) = \int_a^b \underline{f}(t;r)dt,$$

$$(\overline{\int_a^b f(t;r)dt}) = \int_a^b \overline{f}(t;r)dt.$$

Theorem 2.3

The following properties hold

(i) $D(f(x), f(y)) \leq \omega_{[a,b]}(f, |x-y|)$ *for any* $x, y \in [a,b]$;

(ii) $\omega_{[a,b]}(f,\delta)$ *is increasing function of* δ;

(iii) $\omega_{[a,b]}(f, 0) = 0$;

(iv) $\omega_{[a,b]}(f, \delta_1 + \delta_2) \leq \omega_{[a,b]}(f,\delta_1) + \omega_{[a,b]}(f,\delta_2)$ *for any* $\delta_1, \delta_2 \geq 0$;

(v) $\omega_{[a,b]}(f, n\delta) \leq n\omega_{[a,b]}(f,\delta)$ *for any* $\delta \geq 0$ *and* $n \in \mathbb{N}$;

(vi) $\omega_{[a,b]}(f, \lambda\delta) \leq (\lambda + 1)\omega_{[a,b]}(f,\delta)$ *for any* $\delta, \lambda \geq 0$;

(vii) *If* $[c,d] \subseteq [a,b]$, *then* $\omega_{[a,b]}(f,\delta) \leq \omega_{[a,b]}(f,\delta)$.

III . Fuzzy Integro-differential Equations

We consider the nonlinear Fredholm integro-differential equations of the second kind

$$F'(s) = f(s) \oplus \int_a^b K(s,t,F(t))dt, \quad F(a) = F_0,$$

Where

$$f : [a,b] \to E \text{ and } K : [a,b] \times [a,b] \times E \to E$$

Are continuous. Moreover, K is uniformly continuous with respect to s

Theorem 3.1 *Let* $f : [a,b] \to E$ *and* $K : [a,b] \times [a,b] \times E \to E$ *are continuous. Consider the nonlinear fuzzy Fredholm integro-differential* (1). *A mapping* $F : [a,b] \to E$ *is a solution to* (1) *if and only if F is continuous and satisfies the integral equation*

$$F(s) = F_0 \oplus \int_a^s f(z)dz \oplus \int_a^s \int_a^b K(z,t,F(t))dtdz, \quad s \in [a,b].$$

Proof

Since f and K are continuous by O.Kaleva (1987) it must be integrable. So for

$$F'(s) = f(s) \oplus \int_a^b K(s,t,F(t))dt, \ s \in [a,b].$$

We have equivalently [37]

$$F(s) = F(a) \oplus \int_a^s f(z) \oplus \int_a^b K(z,t,F(t))dtdz;$$

$$F(s) = F(a) \oplus \int_a^s f(z)dz \oplus \int_a^s \int_a^b K(z,t,F(t))dtdz.$$

Since $F(a)=F_0$ we have

$$F(s) = F_0 \oplus \int_a^s f(z)dz \oplus \int_a^s \int_a^b K(z,t,F(t))dtdz.$$

Consider the space of functions

$$X = \{f : [a,b] \to E \mid f \text{ continuous}\}$$

With the metric $D^*(f,g)=\sup D(f(s),g(s))$. Recall the fact that (X,D^*) is complete metric space.
Define the operation A by

$$A(F)(s) = F_0 \oplus \int_a^s f(z)dz \oplus \int_a^s \int_a^b K(z,t,F(t))dtdz, \ s \in [a,b], \ \forall f \in X.$$

Theorem 3.2 *Suppose that the functions f and K are continuous. In addition, K is uniformly continuous with respect to s and there exist $L > 0, M_1 > 0, M_2 > 0$ such that*

$$\|K(z,t,u)\|_E \le M_2, \quad \forall z,t \in [a,b], \ \forall u \in E,$$

$$\|f(z)\|_E \le M_1, \quad \forall z \in [a,b]$$

And

$$D(k(z,t,u),k(z,t,v)) \le LD(u,v), \quad \forall z,t \in [a,b], \ \forall u,v \in E.$$

Moreover for every there exists ϵ such that δ and $|s_2 - s_1| \le \delta$, the following inequalities are satisfied

$$D(\tilde{0}, \int_{s_1}^{s_2} f(z)dz) < \epsilon,$$

$$D(\tilde{0}, \int_{s_1}^{s_2} \int_a^b K(z,t,F(t))dtdz) < \epsilon,$$

$$L(b-a)(S-a) < 1,$$

It has a unique solution F^* in X, which can be obtained through the method of successive approximations starting by any element of X. Moreover in the approximation of solution by terms of sequence of successive approximations

$$(F_m)_{m\in\mathbb{N}}, F_1(s) = F_0,$$

$$F_{m+1}(s) = F_0 \oplus \int_a^s f(z)dz \oplus \int_a^s \int_a^b K(z,t,F_m(t))dtdz, \ s \in [a,b], m = 1,2,\cdots,$$

The prior error estimate is

$$D(F^*(s),F_{m+1}(s)) \le \frac{[L(b-a)(S-a)]^m}{1-L(b-a)(S-a)}[(S-a)M_1 + (S-a)(b-a)M_2],$$

$$s \in [a,b], \ m = 1,2,\cdots.$$

Proof Firstly, we prove that $A(X) \subset X$. In this aim, we see that for all

$$\begin{aligned}
D(A(F)(s_1), A(F)(s_2)) = D(F_0 \oplus \int_a^{s_1} f(z)dz \oplus \int_a^{s_1}\int_a^b K(z,t,F(t))dtdz,\\
F_0 + \int_a^{s_2} f(z)dz \oplus \int_a^{s_2}\int_a^b K(z,t,F(t))dtdz)\\
\le D(\int_a^{s_1} f(z)dz, \int_a^{s_2} f(z)dz) +\\
D(\int_a^{s_1}\int_a^b K(z,t,F(t))dtdz, \int_a^{s_2}\int_a^b K(z,t,F(t))dtdz)\\
\le D(\int_a^{s_1} f(z)dz \oplus \tilde{0}, \int_a^{s_1} f(z)dz \oplus \int_{s_1}^{s_2} f(z)dz) +\\
D(\int_a^{s_1}\int_a^b K(z,t,F(t))dtdz \oplus \tilde{0},\\
\int_a^{s_1}\int_a^b K(z,t,F(t))dtdz \oplus \int_{s_1}^{s_2}\int_a^b K(z,t,F(t))dtdz)\\
= D(\tilde{0}, \int_{s_1}^{s_2} f(z)dz) + D(\tilde{0}, \int_{s_1}^{s_2}\int_a^b K(z,t,F(t))dtdz)\\
< \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon.
\end{aligned}$$

So, $A(F)$ is uniformly continuous for any $F \in X$, and consequently continuous on $[a,b]$. Then, $A(X) \subset X$.

For $F, G \in X$ and $s \in [a,b]$ follows:

$$\begin{aligned}
D(A(F)(s), G(F)(s)) = D(F_0 \oplus \int_a^{s} f(z)dz \oplus \int_a^{s}\int_a^b K(z,t,F(t))dtdz,\\
F_0 + \int_a^{s} f(z)dz \oplus \int_a^{s}\int_a^b K(z,t,G(t))dtdz)\\
\le D(\int_a^{s}\int_a^b K(z,t,F(t))dtdz, \int_a^{s}\int_a^b K(z,t,G(t))dtdz)\\
\le D(\int_a^{s}\int_a^b L.D(F(t),G(t))dtdz\\
\le LD^*(F,G)(b-a)(s-a)\\
= LD^*(F,G)(b-a)\varphi(s).
\end{aligned}$$

Let $\varphi(S) = \sup_{s\in[a,b]}\{\varphi(s)\} = (S-a)$. Consequently,

$$D(A(F)(s), G(F)(s)) \le LD^*(F,G)(b-a)\varphi(S),\ \forall F, G \in X.$$

Since, $L(b-a)\varphi(S) < 1$, the operator A is a contraction. Using the Banach's fixed point principle we infer that (1) has a unique solution F^* in X and the following inequality holds:

$$D(F^*(s), F_{m+1}(s)) \le D^*(F^*, F_{m+1}) \le \frac{[L(b-a)(S-a)]^m}{1-L(b-a)(S-a)} D^*(F_1, F_2),$$

$$m = 1, 2, \cdots.$$

$$\begin{aligned} D^*(F_1, F_2) &= \sup_{a \le z \le b} D(F_0 \oplus \tilde{0}, F_0 \oplus \int_a^s f(z)dz \oplus \int_a^s \int_a^b K(z, t, F_0(t))dtdz) \\ &\le \sup_{a \le z \le b} [D(\tilde{0}, \int_a^s f(z)dz) + D(\tilde{0}, \int_a^s \int_a^b K(z, t, F_0(t))dtdz)] \\ &\le \sup_{a \le z \le b} [\int_a^s \|f(z)\|_E dz) + (\int_a^s \int_a^b \|K(z, t, F_0(t))\|_E dtdz)] \\ &\le M_1(S-a) + M_2(S-a)(b-a). \end{aligned}$$

In this way, we obtain the inequality (5).

Theorem 3.2 states the existence and uniqueness of the solution to Eq. (1) and the sequence of successive approximations $(F_m)_{m \in \mathbb{N}}$, converges to this solution in (X, D^*). To approximate, this solution by terms of the sequence of successive approximations must compute the integral and differential.

IV. The Numerical Approaches

We replace the interval [a, b] by a set of discrete equally spaced grid points

$$a = s_0 < s_1 < \cdots < s_{n-1} < s_n = b$$

At which the exact solution F*(s) is approximated by some x(s).The exact and approximate solutions at S_i , The grid point at which the solution is calculated are

$$s_i = s_0 + ih, \quad h = \frac{(b-a)}{n}; \quad 1 \le i \le n.$$

The first-order approximation of F'(s) is given by

$$F'(s) \approx \frac{F(s+h) \ominus F(s)}{h}.$$

$$x_{m+1}(s_{i+1}) = x_{m+1}(s_i) \oplus h[f(s_i) \oplus \sum_{j=0}^{n-1} \frac{b-a}{2n} [K(s_i, s_j, x_m(t_j)) \oplus K(s_i, s_{j+1}, x_m(t_{j+1}))]];$$

$$x_1(s_i) = x_{m+1}(s_0) = F_0; \; i = 0, 1, \cdots, n; \; m = 1, 2, \cdots.$$

By theorem 3.2

$$\underline{x}_{m+1}(s_{i+1}; r) = \underline{x}_{m+1}(s_i; r) + h[\underline{f}(s_i; r) + \sum_{j=0}^{n-1} \frac{b-a}{2n} [\underline{K}(s_i, s_j, \underline{x}_m(t_j), \overline{x}_m(t_j)) + \underline{K}(s_i, s_{j+1}, \underline{x}_m(t_{j+1}), \overline{x}_m(t_{j+1}))]];$$

$$\overline{x}_{m+1}(s_{i+1}; r) = \overline{x}_{m+1}(s_i; r) + h[\overline{f}(s_i; r) + \sum_{j=0}^{n-1} \frac{b-a}{2n} [\overline{K}(s_i, s_j, \underline{x}_m(t_j), \overline{x}_m(t_j)) + \overline{K}(s_i, s_{j+1}, \underline{x}_m(t_{j+1}), \overline{x}_m(t_{j+1}))]];$$

$$\underline{x}_1(s_i; r) = \underline{x}_{m+1}(s_0; r) = \underline{F}_0; \overline{x}_1(s_i; r) = \overline{x}_{m+1}(s_0; r) = \overline{F}_0; \quad i = 0, 1, \cdots, n; \; m = 1, 2, \cdots.$$

Let $\underline{K}(s, t, u, v)$ and $\overline{K}(s, t, u, v)$ be functions $\underline{K}$ and $\overline{K}$ of (12) where u and v are constants and $u \le v$. In other words, $\underline{K}(s, t, u, v)$ and $\overline{K}(s, t, u, v)$ are obtained by substituting $x = (u, v)$ in (12). The domain where $\underline{K}$ and $\overline{K}$ are defined

$$B = \{(s, t, u, v) \mid a \le s, t \le b, -\infty < v < +\infty, -\infty < u \le v\}.$$

Theorem 4.1 *Let $\underline{K}(s,t,u,v)$ and $\overline{K}(s,t,u,v)$ belong to $C^1(B)$. Let the partial derivatives of $\underline{K}, \overline{K}$ be bounded over B also*

$$D(F_m(s_p), x_m(s_p)) = \max_{0\le i\le n}\{D_m(F_m(s_i), x_m(s_i))\},$$

$$D(F_{m+1}(s_{k+1}), x_{m+1}(s_{k+1})) = \max_{0\le i\le n}\{D_{m+1}(F_{m+1}(s_i), x_{m+1}(s_i))\}.$$

Then, for arbitrary fixed $r: 0 \le r \le 1$,

$\lim_{h\to 0} D(F_{m+1}(t_k), x_{m+1}(t_k)) = 0.$

Proof Let

$$\begin{aligned}\underline{F}_{m+1}(s_{k+1}) &= \underline{F}_{m+1}(s_k) + h[\underline{f}(s_i;r) + \sum_{j=0}^{n-1}\frac{b-a}{2n}[\underline{K}(s_i,s_j,\underline{F}_m(t_j),\overline{F}_m(t_j))+\\ &\underline{K}(s_i,s_{j+1},\underline{F}_m(t_{j+1}),\overline{F}_m(t_{j+1}))]] + O(h^2),\\ \overline{F}_{m+1}(s_{k+1}) &= \overline{F}_{m+1}(s_k) + h[\overline{f}(s_i;r) + \sum_{j=0}^{n-1}\frac{b-a}{2n}[\overline{K}(s_i,s_j,\underline{F}_m(t_j),\overline{F}_m(t_j))+\\ &\overline{K}(s_i,s_{j+1},\underline{F}_m(t_{j+1}),\overline{F}_m(t_{j+1}))]] + O(h^2),\end{aligned}$$

and

$$\begin{aligned}\underline{x}_{m+1}(s_{k+1}) &= \underline{x}_{m+1}(s_k) + h[\underline{f}(s_i;r) + \sum_{j=0}^{n-1}\frac{b-a}{2n}[\underline{K}(s_i,s_j,\underline{x}_m(t_j),\overline{x}_m(t_j))+\\ &\underline{K}(s_i,s_{j+1},\underline{x}_m(t_{j+1}),\overline{x}_m(t_{j+1}))]],\\ \overline{x}_{m+1}(s_{k+1}) &= \overline{x}_{m+1}(s_k) + h[\overline{f}(s_i) + \sum_{j=0}^{n-1}\frac{b-a}{2n}[\overline{K}(s_i,s_j,\underline{x}_m(t_j),\overline{x}_m(t_j))+\\ &\overline{K}(s_i,s_{j+1},\underline{x}_m(t_{j+1}),\overline{x}_m(t_{j+1}))]].\end{aligned}$$

Consequently,

$$\begin{aligned}\underline{F}_{m+1}(s_{k+1}) - \underline{x}_{m+1}(s_{k+1}) &= \underline{F}_{m+1}(s_k) - \underline{x}_{m+1}(s_k) + h[\sum_{j=0}^{n-1}\frac{b-a}{2n}\\ &[\underline{K}(s_i,s_j,\underline{F}_m(t_j),\overline{F}_m(t_j)) - \underline{K}(s_i,s_j,\underline{x}_m(t_j),\overline{x}_m(t_j))\\ &+\underline{K}(s_i,s_{j+1},\underline{F}_m(t_{j+1}),\overline{F}_m(t_{j+1})) - \underline{K}(s_i,s_{j+1},\\ &\underline{x}_m(t_{j+1}),\overline{x}_m(t_{j+1}))]] + O(h^2),\\ \overline{F}_{m+1}(s_{k+1}) - \overline{x}_{m+1}(s_{k+1}) &= \overline{F}_{m+1}(s_k) - \overline{x}_{m+1}(s_k) + h[\sum_{j=0}^{n-1}\frac{b-a}{2n}\\ &[\overline{K}(s_i,s_j,\underline{F}_m(t_j),\overline{F}_m(t_j)) - \overline{K}(s_i,s_j,\underline{x}_m(t_j),\overline{x}_m(t_j))\\ &+\overline{K}(s_i,s_{j+1},\underline{F}_m(t_{j+1}),\overline{F}_m(t_{j+1})) - \overline{K}(s_i,s_{j+1},\underline{x}_m(t_{j+1}),\\ &\overline{x}_m(t_{j+1}))]] + O(h^2).\end{aligned}$$

Denote $W_{m+1}(s_{k+1}) = \underline{F}_{m+1}(s_{k+1}) - \underline{x}_{m+1}(s_{k+1})$, $V_{m+1}(s_{k+1}) = \overline{F}_{m+1}(s_{k+1}) - \overline{x}_{m+1}(s_{k+1})$. Then

$$\begin{aligned}|W_{m+1}(s_{k+1})| &\le |W_{m+1}(s_k)| + h[\sum_{j=0}^{n-1}\frac{b-a}{2n}[2L\max\{|W_m(t_j)|,|V_m(t_j)|\}\\ &+2L\max\{|W_m(t_{j+1})|,|V_m(t_{j+1})|\}] + O(h^2),\\ |V_{m+1}(s_{k+1})| &\le |V_{m+1}(s_k)| + h[\sum_{j=0}^{n-1}\frac{b-a}{2n}[2L\max\{|W_m(t_j)|,|V_m(t_j)|\}\\ &+2L\max\{|W_m(t_{j+1})|,|V_m(t_{j+1})|\}] + O(h^2),\end{aligned}$$

where $L > 0$ is a bound for the partial derivatives of $\underline{K}, \overline{K}$. Thus, we have

$$|W_{m+1}(s_{k+1})| \le |W_{m+1}(s_k)| + 2nLh\frac{b-a}{n}D(F_m(t_p), x_m(t_p) + O(h^2),$$

$$|V_{m+1}(s_{k+1})| \le |V_{m+1}(s_k)| + 2nLh\frac{b-a}{n}D(F_m(t_p), x_m(t_p)) + O(h^2).$$

Since $W_{m+1}(t_0) = V_{m+1}(t_0) = 0$, we obtain

$$|W_{m+1}(s_{k+1})| \leq 2(k+1)nLh\frac{b-a}{n}D(F_m(t_p), x_m(t_p) + O(h^2),$$

$$|V_{m+1}(s_{k+1})| \leq 2(k+1)nLh\frac{b-a}{n}D(F_m(t_p), x_m(t_p) + O(h^2)$$

and if $h \to 0$, we get $D(F_{m+1}(t_k), x_{m+1}(t_k)) \to 0$.

V. Numerical Example

Consider the following nonlinear fuzzy Fredholm integro-differential equation

$$F'(s) = (r - \frac{s^2r^2}{40}, 2 - r - \frac{s^2(2-r)^2}{40}) \oplus \int_0^1 (\frac{s^2t}{10}) \odot F^2(t)dt,$$

$$F(0) = 0; \quad 0 \leq r \leq 1.$$

The exact solution in this case is given by

$F(s) = (r, 2-r)s.$

By using Adomian decomposition method we obtain

$$x_{m+1}(s_{i+1}) = x_{m+1}(s_i) \oplus h[(r - \frac{s_i^2r^2}{40}, 2 - r - \frac{s_i^2(2-r)^2}{40}) \oplus \sum_{j=0}^{29}\frac{1}{60}[(\frac{s_i^2s_j}{10} \odot x_m^2(s_j)) \oplus (\frac{s_i^2s_{j+1}}{10} \odot x_m^2(s_{j+1}))]].$$

Also by using Adomian polynomials

$$x_1(s_i) = x_{m+1}(s_0) = 0; \; s_i = ih, i = 0, 1, \cdots, 30; \; m = 1, 2, \cdots, 10,$$

Where

$$x_m^2(s_j) = (\min\{\underline{x_m}^2(s_j;r), \overline{x_m}^2(s_j;r), \underline{x_m}(s_j;r)\overline{x_m}(s_j;r)\},$$
$$\max\{\underline{x_m}^2(s_j;r), \overline{x_m}^2(s_j;r), \underline{x_m}(s_j;r)\overline{x_m}(s_j;r)\}).$$

Comparison between the exact solution and the approximate solution of nonlinear fuzzy Fredholm integro-differential equation in the example given by the numerical solution.

Table 5.1

Comparison between the exact solution and the approximate solution for t =0.1

	Present Result		Exact Result	
	FADM		Parandin (2013)	
r	$x(t,r)$	$\underline{x}(t,r)$	$x(t,r)$	$\underline{x}(t,r)$
0	4.7635	2.5649	4.7635	2.5649
0.1	4.6535	2.6749	4.6535	2.6749
0.2	4.5436	2.7848	4.5436	2.7848

0.3	4.4337	2.8947	4.4337	2.8947
0.4	4.3238	3.0047	4.3238	3.0047
0.5	4.2138	3.1146	4.2138	3.1146
0.6	4.1039	3.2245	4.1039	3.2245
0.7	3.9940	3.3344	3.9940	3.3344
0.8	3.8841	3.4444	3.8841	3.4444
0.9	3.7741	3.5543	3.7741	3.5543
1	3.6642	3.6642	3.6642	3.6642

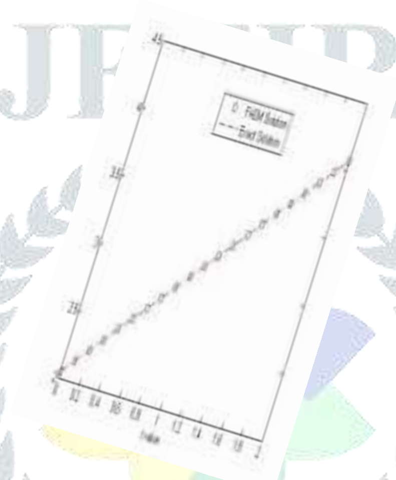
Table 5.2
Comparison between the exact solution and ADM solution for t =0.5

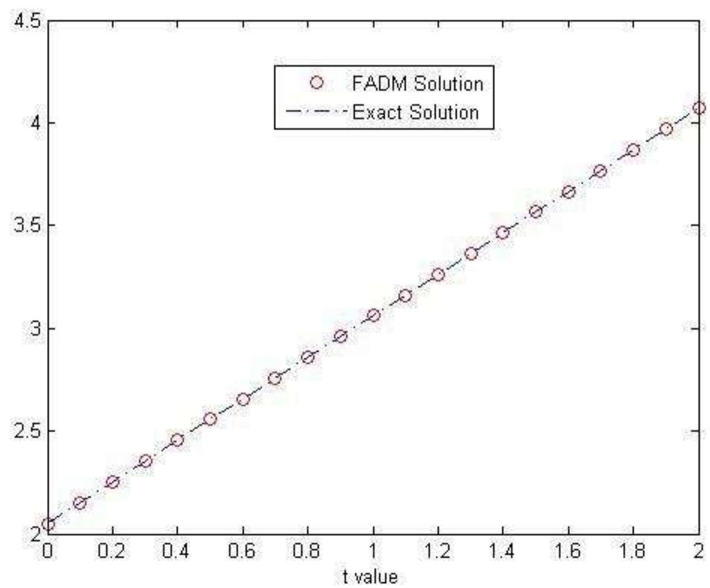
r	Present Result FADM		Exact Result Parandin (2013)	
	$x(t,r)$	$\underline{x}(t,r)$	$x(t,r)$	$\underline{x}(t,r)$
0	9.5103	6.7918	9.5140	6.7957
0.1	9.3744	6.9278	9.3781	6.9316
0.2	9.2384	7.0637	9.2422	7.0675
0.3	9.1025	7.1996	9.1062	7.2034
0.4	8.9666	7.3355	8.9703	7.3394
0.5	8.8307	7.4714	8.8344	7.4753
0.6	8.6947	7.6074	8.6985	7.6112
0.7	8.5588	7.7433	8.5626	7.7471
0.8	8.4229	7.8792	8.4267	7.8830
0.9	8.2870	8.0151	8.2908	8.0189
1	8.1511	8.1511	8.1548	8.1548

Table 5.3
The Comparison between the error of the FADM and existing method in t = 0.5

R	Error FADM		Error Parandin (2013)	
	$x(t,r)$ _	$\underline{x}(t,r)$	$\underline{x}(t,r)$	$x(t,r)$
0	0.003712797	0.003872731	0.00040369	0.0040166
0.1	0.003720793	0.003864735	0.00036342	0.0036139
0.2	0.003728791	0.003856738	0.00032316	0.0032113

0.3	0.003736787	0.003848741	0.00028289	0.0028086
0.4	0.003744784	0.003840744	0.00024262	0.0024059
0.5	0.003752781	0.003832748	0.00020235	0.0020032
0.6	0.003760777	0.003824751	0.00016209	0.0016006
0.7	0.003768774	0.003816754	0.00012182	0.0011979
0.8	0.003776771	0.003808757	0.00008155	0.0079520
0.9	0.003784767	0.003800761	0.000041283	0.0039253
1	0.003792764	0.003792764	0.000001015	0.0010151





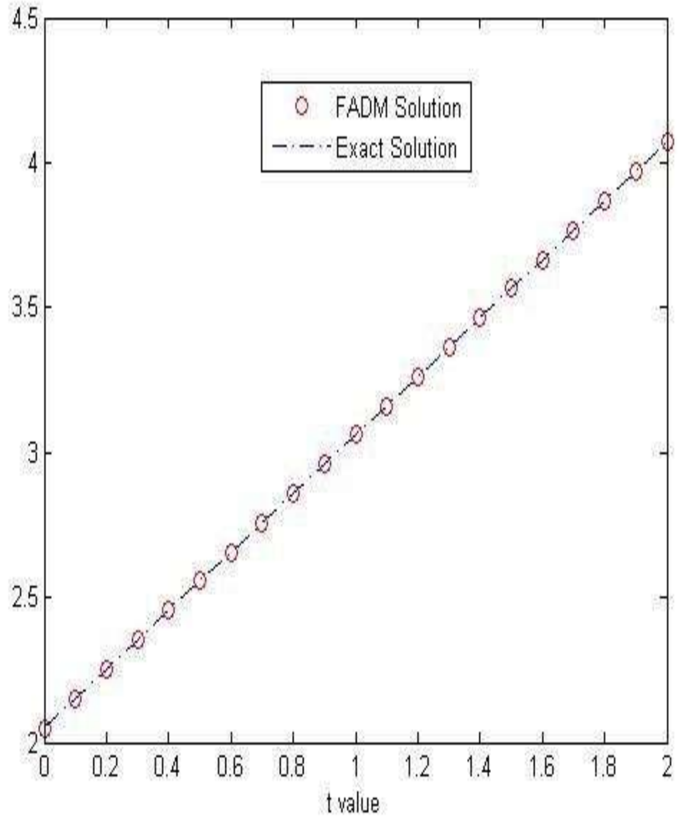


Figure 5.1 Solution of $x(t,r)$for variousvalues of r at t=0.02 using FADM with

Exact

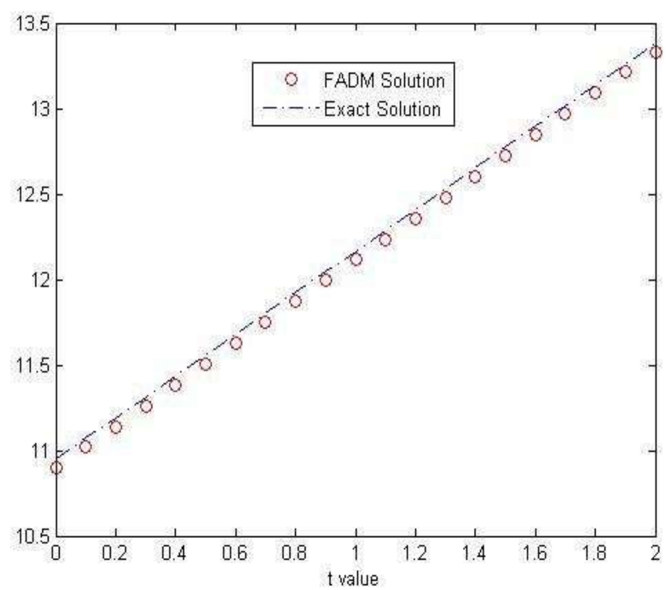
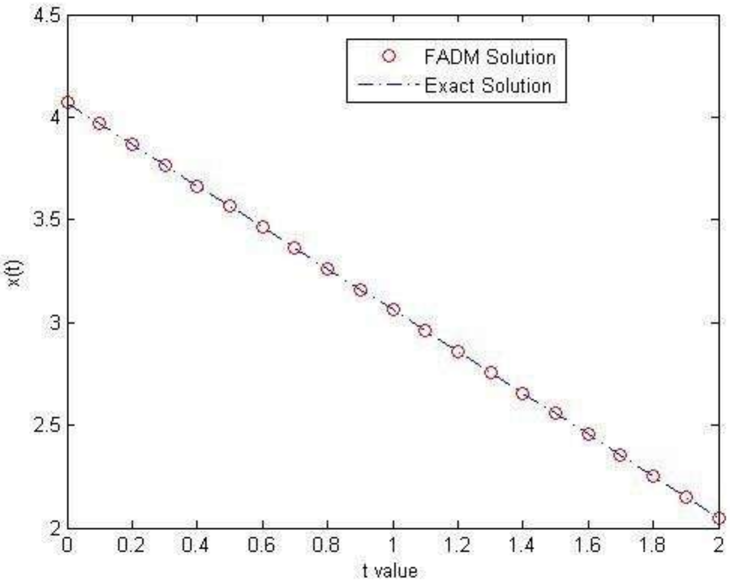


Figure 5.2 Solution of $x(t,r)$ for various values of r at t=0.7 using FADM with E



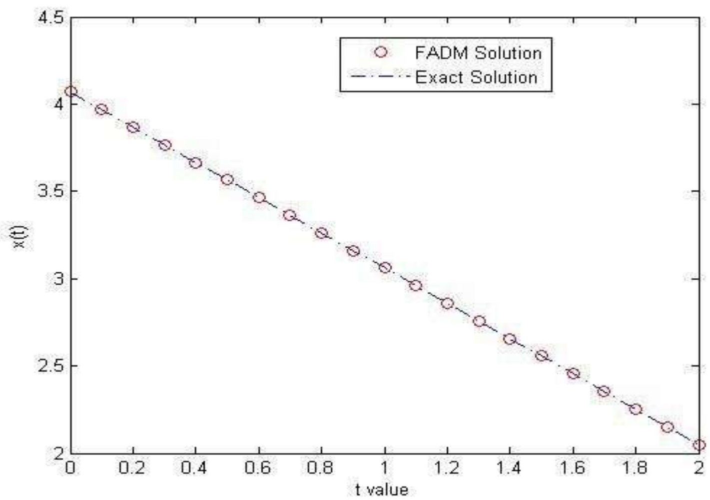


Figure 5.3 Solution of$\underline{x}(t,r)$ for various values of r at t=0.02 using FADM with Exact

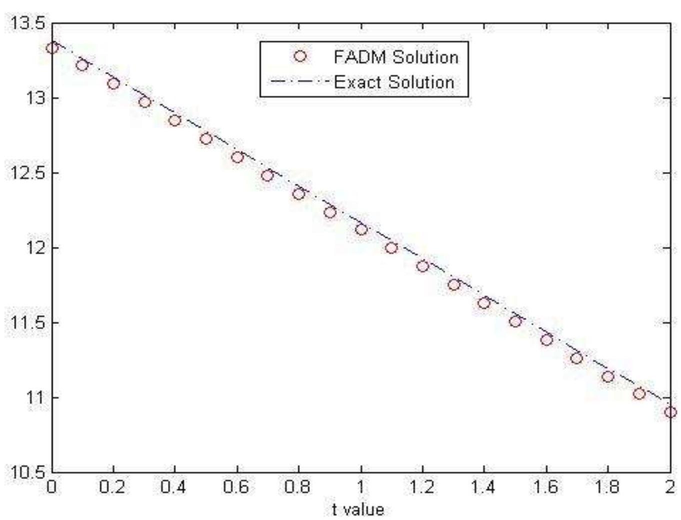


Figure 5.4 Solution of $\underline{x}(t,r)$ for various values of r at t=0.8 using FADM with Exact

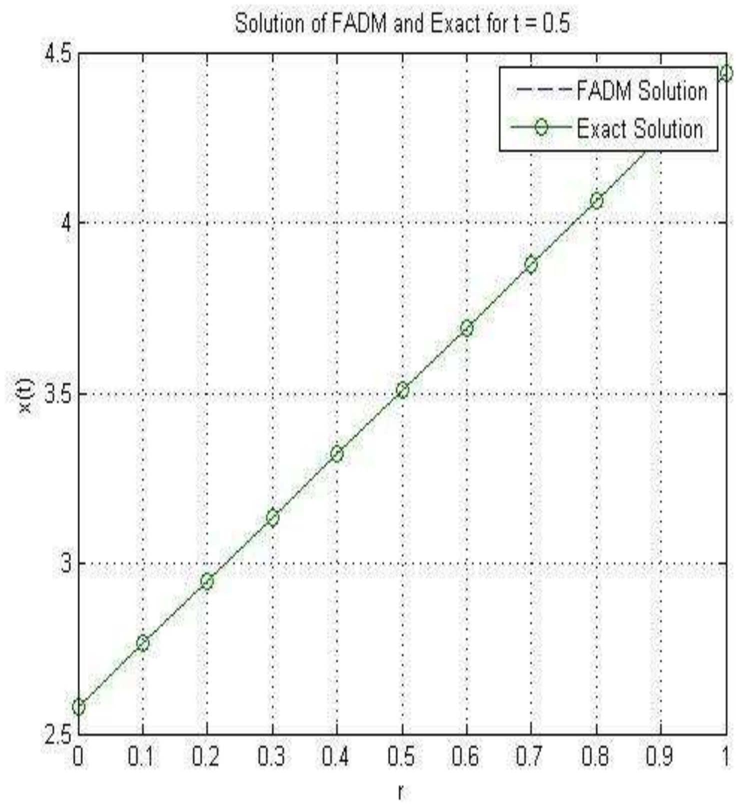


Figure 5.5 Solution of $\underline{x}(t,r)$ for various values of r at t=0.5 using FADM with Exact

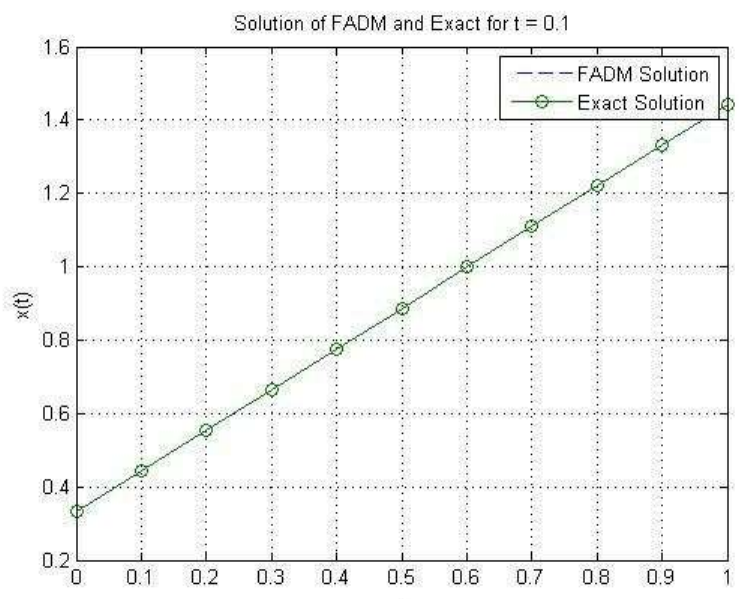


Figure 5.6 Solution of $\underline{x}(t,r)$ for various values of r at t=0.1 using FADM with Exact

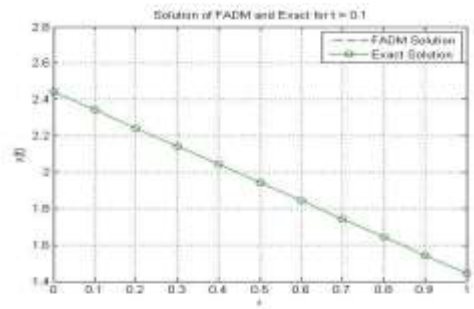


Figure 5.7 Solution of $x(t,r)$ for various values of r at t=0.1 using FADM with Exact

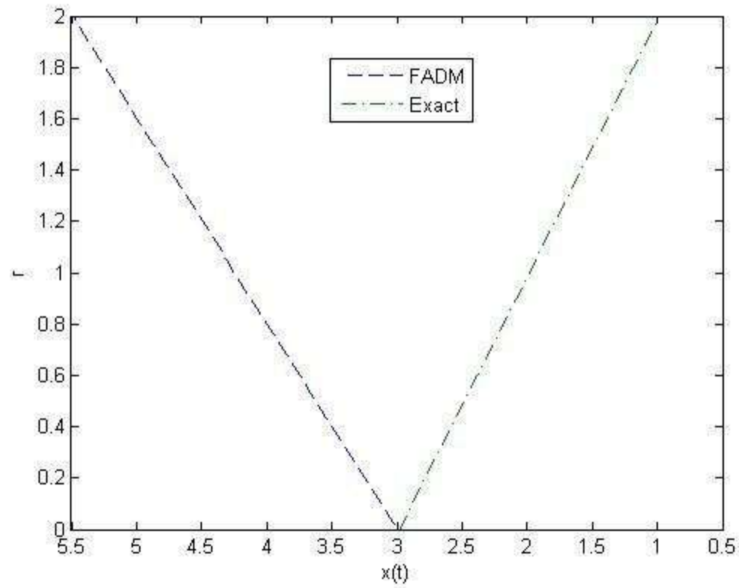


Figure 5.8 Solution of $\overline{x}(t,r)$ for various values of r at t = 0.2 using FADM with Exact

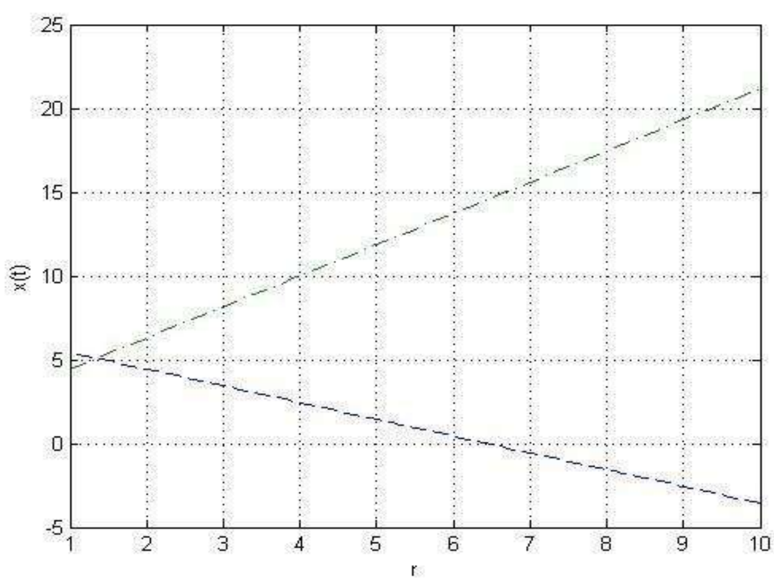


Figure 5.9 Solution of $\overline{x}(t,r)$ for various values of r at t=0.5 using FADM with Exact

3.6 CONCLUSION

Modified Adomian decomposition method is a powerful technique which is capable of handling higher order fuzzy Integro- differential equations. The methods have been successfully employed to higher order fuzzy Integro differential equations. When the approximation results are found by using Modified Adomian decomposition method and compared to the exact solutions with existing results. Also it is seen that the convergent are quite close.

REFERRENCES

[1] Abassy, T.A. (2010), Improved Adomian decomposition method, *Computers and Mathematics with Applications*, Vol. 59 No. 1, pp. 42-54.

[2] Abbaui, K.and Y. Cherruault, (1995) New ideas for proving convergence of decomposition method, comput. Math. Apple. 29 103-108.

[3] Abbasbandy, S. (2007), A numerical solution of Blasius equation by Adomian decomposition method and comparison with homotopy perturbation method. Chaos, Solitons and Fractals 31 (1), 257–280.

[4] Abdou, M.A. (2005) Adomian decomposition method for solving the telegraph equation in charged particle transport, J. Quant. Spectrosc. Radiat.Transfer 95 407-
414.

[5] Abramowitz, M. and Stegun I.A. (1965) Handbook of Mathematical Functions, Dover, New York.

[6] Abbasbandy, S and T. Allahviranloo, (2002) Numerical solution of fuzzy differential equation by Taylor method, Journal of Computational Methods in Applied mathematics, 2 113-124.

[7] Abbasbandy, S. T. Allahviranloo, (2002) Numerical solution of fuzzy differential equation, Mathematical and Computational Applications, 7 41-52

[8] Abbasbandy, S., Allahviranloo, T., Lopez-Pouso, and Nieto, J.J. (2004). Numerical methods for fuzzy differential Inclusions. *Journal of Computer and Mathematics with Applications* 48: 1633-1641.

[9] Abbasbandy, S. and T. Allahviranloo,(2002) Numerical solution of fuzzy differential equation by Runge-Kutta method, J. Sci. Teacher Training University,1(3)

[10] Abdou, M.A. (2005) Adomian decomposition method for solving the telegraph equation in charged particle transport, J. Quant. Spectrosc. Radiat. Transfer 95 407–414.

[11] Adomian, G. (1983), *Stochastic Systems*, Academic Press, New York, NY.

[12] Adomian, G. (1986), *Nonlinear Stochastic Operator Equations*, Academic Press, Orlando, FL.