Reduction of PAPR in OFDM System Using Combined DCT/Hadamard Transform with Nonlinear Companding Transform

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Abstract: ***OFDM is familiar and the most useful multi-carrier modulation technique used in different modern high speed wireless communication applications like, 3GPP and LTE standards. High peak-to-average power ratio (PAPR) is the considerable problem in OFDM (Orthogonal frequency division multiplexing) system. In this paper, proposed a technique i.e combination of DCT/Hadamard transform with nonlinear Companding transform to reduce PAPR more effectively. Basically nonlinear Companding transform enlarges the small magnitude values and compresses the large magnitude values of OFDM signal. So that PAPR is minimized in OFDM signal. DCT/Hadamard transforms are data compression techniques which reduces the autocorrelation in input data sequence. These transforms are applied on data prior to IFFT operation in OFDM system. The simulation results shows that the proposed combined technique gives better PAPR reduction compared to their individual performances.***

Index Terms -Orthogonal frequency division multiplexing (OFDM), Discrete cosine transform (DCT), Hadamard Transform, Nonlinear Companding transform and Peak-to-average power ratio (PAPR).

I. INTRODUCTION

OFDM is a multi carrier modulation and multiplexing technique which is used in modern high data rate transmission wireless communication applications like 3GPP and 4G technology due to its spectral efficiency, more robustness to ISI (Inter symbol interference) effect and frequency selective fading etc[1],[2].But it has some limitations like High PAPR(Peak-to-Average Power Ratio) and ICI(Inter-Carrier Interference).Due to high PAPR in OFDM system, in-band distortion and out-of-band radiation occurs if the dynamic range of the high power amplifier (HPA) is low at transmitters. In literature, so many PAPR reduction techniques are proposed. Basically, PAPR reduction techniques are classified into three types [1], [2]. The first one is signal distortion technique, which reduces the peak amplitudes by using nonlinearly distorting the OFDM signal. For example, clipping and companding are two best distortion techniques to reduce PAPR more effectively by compromising in terms of BER. Second one is coding technique, which uses the special codes to reduce the PAPR in OFDM signal, but it needs more band width. And the third one is scrambling method, in which different scrambling sequences are used to minimize the PAPR of OFDM signal. For Example, SLM[4]-[6] and PTS[7] are two familiar techniques used under non distortion techniques. But, these are very complex and need more operations. But BER performance is better than distortion techniques. In this paper, we proposed a new non- linear companding transform combined with DCT [18]/Hadamard [19] Transform to reduce the PAPR value more effectively.

2 Theoretical Analysis of OFDM and Its PAPR

The general OFDM transmitter block diagram is shown in fig.1below.

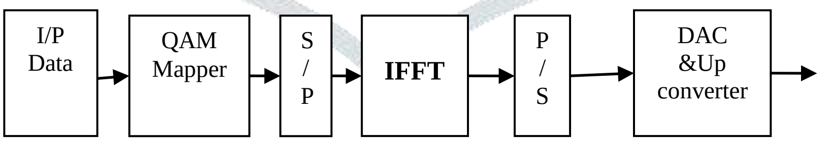

Fig1 Basic Block diagram of OFDM transmitter.

Generally in wireless communication systems the input binary data is modulated by either QAM or QPSK modulator to preserve the channel bandwidth. In OFDM, the modulated data is converted into serial to parallel data and then passed to IFFT system to generate the discrete-time OFDM signal.

Consider X (0), X (1), X (2) ………..X (N-1) are the modulated complex data sequence to be transmitted in an OFDM symbol with N number of sub carriers. The base band representation of discrete- time OFDM symbol is given by,

$$x(n) = \frac{1}{\sqrt{LN}} \sum_{k=0}^{N-1} X(k).\exp\left(\frac{j.2\pi nk}{LN}\right) \qquad (1)$$

Where 'L' is the Over sampling factor and generally it is selected as four or more than it for finding the more accurate peak values of OFDM signal.

and $n = 0,1,2,-------LN-1$.

As per central limit theorem, OFDM signal x_n from antenna can be approximated as complex Gaussian process for a large value of N (e.g. $N \geq 64$). Thus, with zero mean and a constant variance $\sigma^2 = E\{|X(k)|^2\}/2$, the real and imaginary parts of $x(n)$ become Gaussian distribution with probability density function (PDF) as follows.

$$f_{|x(n)|} = \frac{2x}{\sigma^2}.\exp(-\frac{x^2}{\sigma^2}), \text{ where } x \geq 0 \qquad (2)$$

As a result, it is sure about that the OFDM signal maximal amplitude may exceed its average signal amplitude. The probability distribution function (CDF) of $|x(n)|$ is given by

$$F_{|x(n)|}(x) = \Pr ob\{|x(n)| \leq x\}$$

$$= \int_0^x \frac{2y}{\sigma^2}.\exp(-\frac{y^2}{\sigma^2})dy = \left(\frac{\exp\left(-y^2/\sigma^2\right)}{-1}\right)_0^{x^2/\sigma^2}$$

$$= 1 - \exp(-\frac{x^2}{\sigma^2}),\ x \geq 0 \qquad (3)$$

PAPR is defined as the ratio of maximum instantaneous power to its average power.

$$PAPR = \frac{\max\left\{|x(n)|^2\right\}}{E\left\{|x(n)|^2\right\}} \qquad (4)$$

PAPR value is treated as a random variable and use a statistical description given by the complementary cumulative distribution function (CCDF) defined as the probability that exceeds a particular threshold $PAPR_0$, i.e.

$$CCDF_x(PAPR_0) = \Pr ob\{PAPR_x > PAPR_0\} \qquad (5)$$

INTRODUCTION OF CMPANDING TRANSFORM AND IT'S ROLE IN OFDM SYSTEM TO MITIGARE PAPR.

At OFDM transmitter, companding transform [8]-[17] is applied in order to reduce the high peaks and amplify the low magnitudes of the OFDM signal. So that the PAPR will decrease. At receiver, inverse companding transform is used to recover the original OFDM signal. Companding transform is more popular and attractive technique in order to reduce the PAPR of OFDM signal due to its low complexity and flexibility. Companding transform design is very flexible in that we can change the transform parameters easily as per requirements. The companding transform design doesn't depend on number of sub carriers, frame format and band width etc [8]-[17].While designing companding transform caution must be taken about its average power. The average transmitted power should maintain constant even after comanding process. Otherwise, if the average power altered then the BER performance will degrade at OFDM receiver. By designing a proper companding transform, it reduces PAPR more effectively and improves the Bit error rate performance. Number of companding transforms have been proposed and studied in the literature. Firstly, Wang proposed the well-known scheme named µ-law companding technique [21]. This gives better performance than clipping [3] process. But the average power of the signal will increases after companding process and exhibit non-uniform distributions. Later, a new companding technique is introduced, namely exponential companding transform [9]. This transform keeps the average power as constant and also transforms the original OFDM signal into uniformly distributed signal. Exponential companding transform [9] gives superior performance than u-law companding in terms of PAPR, BER and side-lobe generation. The main advantage of Companding techniques compared to other traditional techniques like clipping & filtering [3], SLM and PTS is flexible in design and has lowest computational complexity. In general companding transforms are used in speech signal processing. Speech signal and OFDM signal have the similar characteristics in time domain interms of occurrence of peaks. So, these companding transforms can also be used to reduce the OFDM signal PAPR.One more important feature of companding is, its complexity is not affected by the number of sub-carries, and it doesn't require any side information at receiver and also it does not reduce bit rate. Due to all these advantages companding transforms become more attractive interms of PAPR reduction in OFDM systems.

PROPOSED COMPANDING TRANSFORM:

In this paper, a nonlinear companding transform is proposed using hyperbolic tangent function, which compresses the small signals and expands the large signals of OFDM signal. The block diagram of OFDM transmitter and receiver adopting the proposed method is shown in figure 2.

The proposed nonlinear companding transform is defined by,

$$y_n = h(x) = \begin{cases} \operatorname{sgn}(x).\rho.\tanh(k.|x|) & if\ |x| \leq cA \\ \operatorname{sgn}(x).\rho.\tanh(k.c.A) & if\ |x| > cA \end{cases} \qquad (6)$$

Where 'cA' is the inflection point.sgn(x) is the sign function. The hyperbolic tangent function tanh(x) is defined by,

$$\tanh x = \left(\frac{e^x - e^{-x}}{e^x + e^{-x}} \right)$$

In this proposed companding transform, two parameters ‘ρ’ and ‘k’ plays very key role in order of to maintain the average output power level and the degree of companding form. Under a certain constraint with respect to the average power before and after the companding operation, we let

$$E\left\{|y_n|^2\right\} = E\left\{|x_n|^2\right\} = \int_0^\infty x^2 f_{|x_n|}(x).dx \qquad (7)$$

Where $f_{x_n}(x)$ is the probability density function.

Here ρ can be determined by choosing the proper value of k.

At the receiver, the original OFDM signal is recovered by using the corresponding decompanding function as follows.

$$h^{-1}(x) = \operatorname{sgn}(x).\frac{1}{k}.a\tanh\left(\frac{|x|}{\rho}\right), \qquad (8)$$

Where atanh(x) is inverse hyperbolic tangent function expressed as,

$$a\tanh(x) = \frac{1}{2}.\ln\left(\frac{1+x}{1-x}\right)$$

5. Discrete Cosine Transform (DCT) and Hadamard transform:

DCT [18] has almost similar properties of DFT but DCT has high energy compaction compared to DFT.Due to this advantage DCT plays a key role in different signal coding & image processing applications to compress the data. For example MP3 and JPEG are two familiar techniques involved with DCT to compress the data.DCT is the summation of cosine functions oscillating at different frequencies. The main advantage of the DCT is it reduces the autocorrelation of the data, so that by applying DCT operation before generating OFDM signal we can reduce the PAPR of the system.

The 1-D DCT of length N is expressed as,

$$C[k] = \alpha(k) \sum_{n=0}^{N-1} x[n] \cos\left(\frac{\pi(2n+1)k}{2N}\right), \qquad (9)$$

for $k = 0,1,2,--------N-1.$

Similarly the inverse Discrete Cosine transform is defined as,

$$x[n] = \sum_{k=0}^{N-1} \alpha(k).C[k] \cos\left(\frac{\pi(2n+1)k}{2N}\right), \qquad (10)$$

for $n = 0,1,2,-----N-1.$

Where $\alpha(k)$ is defined as,

$$\alpha(k) = \begin{cases} \frac{1}{\sqrt{N}}, \; for\ k = 0, \\ \frac{2}{\sqrt{N}}, \; for\ k \neq 0, \end{cases} \qquad (11)$$

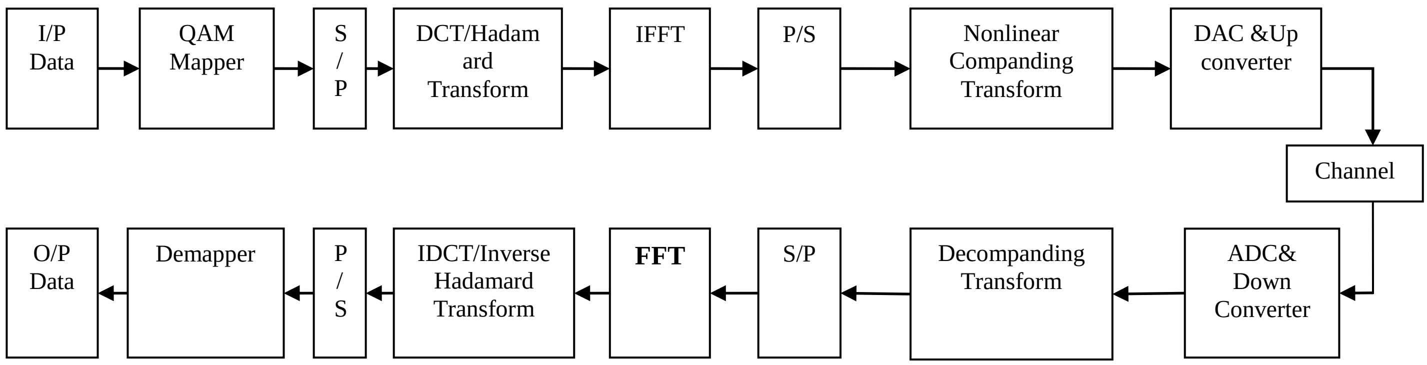

Fig 2.OFDM system using proposed Companding transform with DCT/Hadamard transforms.

Hadamard transform is another type of data compression transform which reduces the autocorrelation of the input data sequence and thereby reduces the peak to average power in OFDM. One of the features in Hadamard transform is Hadamard matrix is a standard orthogonal matrix. Let H denotes the Hadamard transform matrix of order N. In Hadamard matrix every element is either 1 or -1. Hadamard matrix of order 2x2 and 4×4 are defined below. Here we should notice that the order of Hadamard matrix must be a power of 2.

$$H_{2X2} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}; H_{4X4} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix} \quad \text{-----------------------------------} \quad (12)$$

For an input of Hadamard matrix X, the output Y can be expressed as **Y**=H.**X**

V. SIMULATION RESULTS

The overall OFDM system performance for the proposed companding transform is evaluated through MATLAB simulations by considering 1024 subcarriers using with 16-QAM (Quadrature amplitude modulation).For better PAPR estimation, oversampling ratio is considered as L=4.Here,the proposed method is compared with the u-law companding transform.

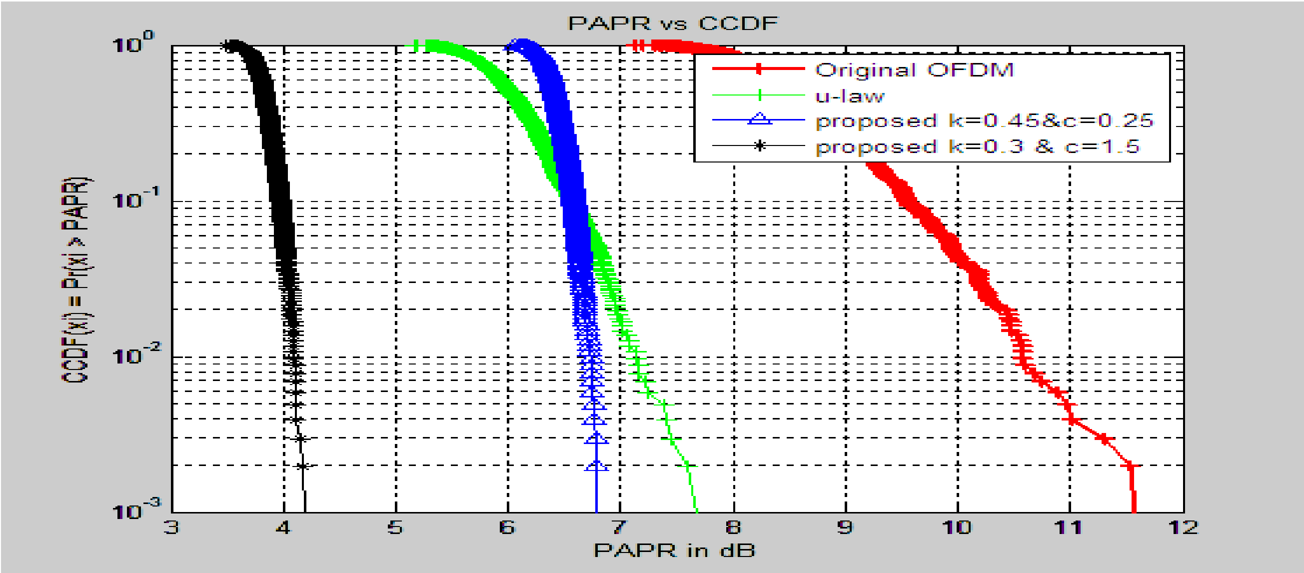

Fig 3. CCDF for PAPR of OFDM system with proposed nonlinear Companding transform without DCT/Hadamard Transform.(N=1024,16-QAM,L=4)

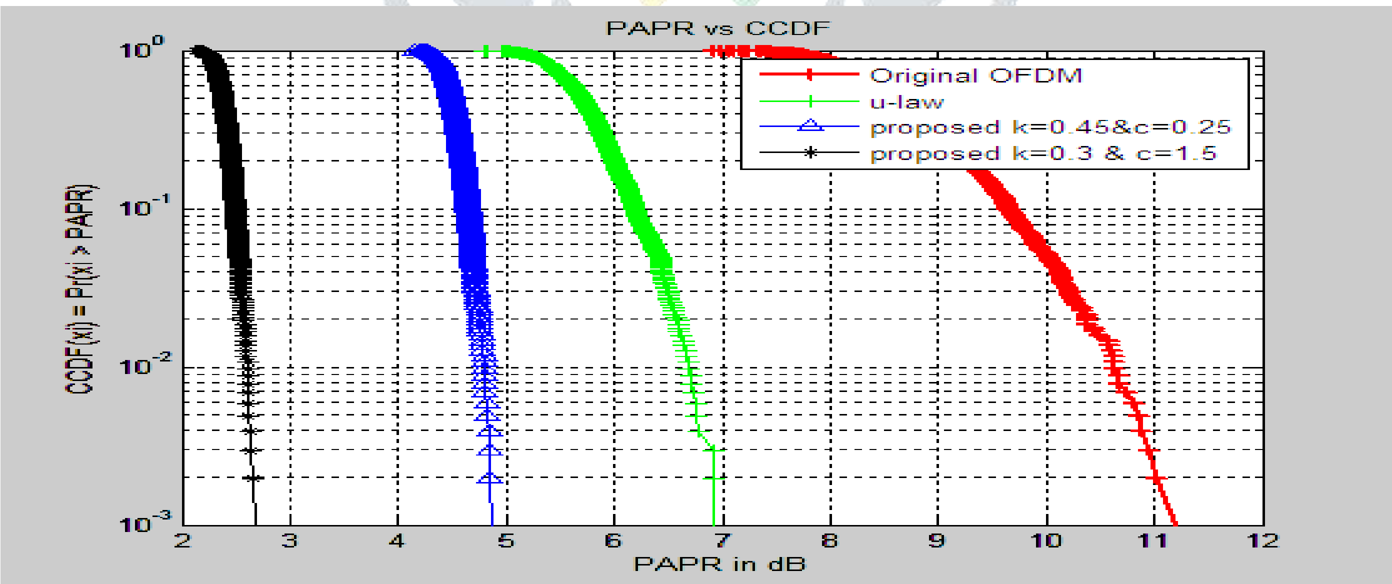

Fig 4. CCDF for PAPR of OFDM system with combined DCT and proposed nonlinear Companding transform. (N=1024, 16-QAM, L=4)

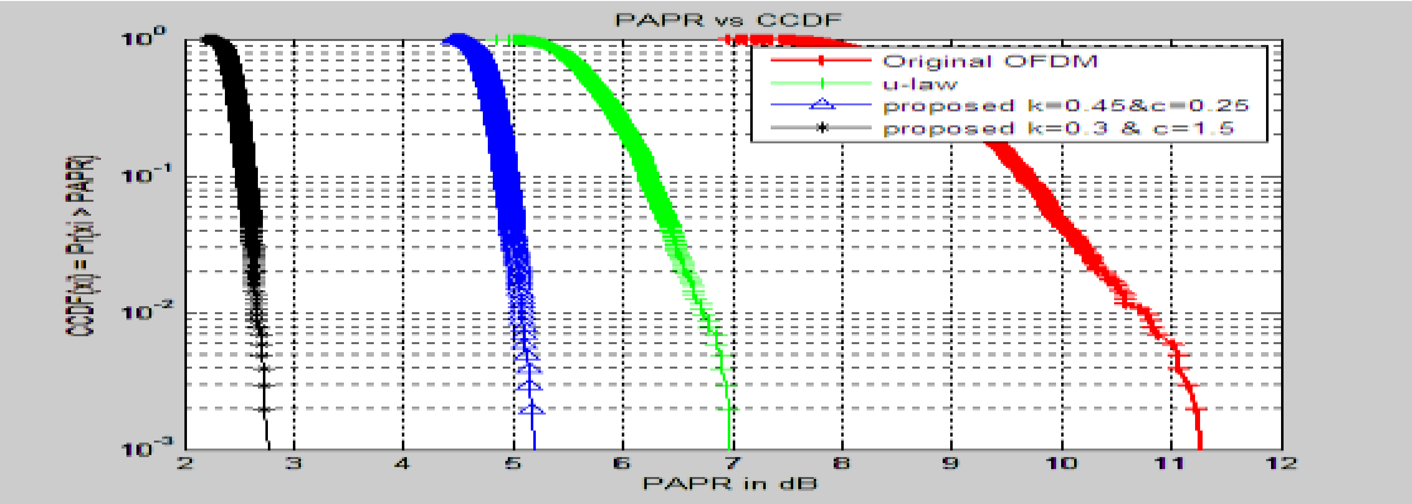

Fig 5. CCDF for PAPR of OFDM system with combined Hadamard transform and proposed nonlinear companding transform. (N=1024, 16-QAM, L=4).

7 Conclusion

OFDM is the best solution to face the multipath propagation effects in the channel. Due to several advantages of OFDM it is used in different wireless communication systems.But, PAPR is the major issue of OFDM, because it reduces the efficiency of the power amplifiers at transmitter. In literature number of solutions found to reduce the PAPR in OFDM system. Companding transform is one of the solution to reduce the PAPR.Due to its simplicity, flexibilty and efficiency it is more popular compared to other techniques. In this paper we introduced one nonlinear Companding transform using hyperbolic tangent to reduce PAPR of OFDM signal.Additionaly DCT and Hadamard Transforms are introduced in the transmitter for better reduction of PAPR.

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