RIPPLET TRANSFORM BASED COMPRESSIVE SENSING FOR IMAGES

[1]LAKSHMI.S.PANICKER,[2]SURUMY BACKER,[3]JAYADAS.C.K
[1]P.G.Scholar,[2]Assistant Professor,[3]Associate Professor
[1]Dept. of Electronics and Communication,
[1]College of Engineering Cherthala, Alappuzha, Kerala, India

Abstract : Compressive sensing is an emerging field of research which allows simultaneous sampling and compression of signals at a rate below the Nyquist rate by exploiting sparsity of signals. Ripplet transform is a higher dimensional generalization of curvelet transform designed to represent images, allows optimal sparse representation of objects with singularities along the smooth curves. In this paper we have implemented the application of ripplet transform in compressive sensing of images which proves to offer better results than the current state of art techniques using curvelet transform. Here the regularization term used for the reconstruction encompasses both local similarity and nonlocal self similarity priors, which further improves the performance. We have presented the experimental results and the comparison with the implementation using curvelet transform.

IndexTerms - Ripplet transform, Compressive sensing, Nonlocal Statistical similarity, Sparsity, Joint adaptive sparsity regularizer

I. INTRODUCTION

In signal processing Nyquist sampling has long been seen as the guiding principle of signal acquisition. The theory of compressive sensing offers a new framework that allows sampling at a rate lower than Nyquist sampling rate without any significant loss of information.The field of CS grew out of the work of Candès, Romberg, and Tao and of Donoho,[1][2][3] who showed that a finite-dimensional signal having a sparse or compressible representation can be recovered from a small set of linear, non-adaptive measurements.

Compressive sensing is an emerging technology which finds its applications in diverse fields ranging from medical imaging to cyber forensics. The basic theory predicts the reconstruction of signals from far fewer samples, by offering a framework for simultaneous sampling and compression of finite dimensional vectors, than that is required by Nyquist sampling theorem, thereby contradicting the traditional wisdom. The theory of compressive sensing offers the framework of reconstruction of original signal from its random projections onto a measurement matrix, provided the signal is sparse in some suitable transform domain. Sparsity plays an important role in CS as it is a necessary constraint for acquiring lesser number of samples.

Compressive sensing of a finite dimensional signal $x \in \mathbb{R}^n$ is done by computing m linear measurements of the image, where $m \ll n$ through following random linear projection

$$f = \Theta x \tag{1}$$

$f \in \mathbb{R}^m$is the observation/measurement vector and $\Theta \in \mathbb{R}^{m \times n}$ is the sparse sensing matrix. . Since the number of un s are higher than the observations the problem of recovery of x from f is considered as ill posed problem. The solution to such ill posed problems can be formulated as regularization based optimization problem. If x is sparse in some suitable transform domain,Ψ, then it is possible to have exact recovery of x if the matrix $\Theta\Psi$ satisfies the Restricted Isometry Property(RIP). Selection of suitable sparse domain is important. In this paper we consider Discrete Ripplet Transform (RT)[11],[12] as the transform matrix as they provide efficient representation of edges in images and are designed to represent images at different scales and directions.

The solution to the above ill posed problem can be regularized to the following minimization problem by employing prior information about the original signal.

$$\min_u \left\{ \frac{1}{2} \|f - \Theta x\|_{l_2}^2 + \lambda \mathcal{R}(x) \right\} \tag{2}$$

Here the first term is the $l_2$data fidelity term, represents the closeness of the solution to the observed scene and quantifies the prediction error with respect to the measurements. The second term represents the a priori information about the original scene. λ is a regularization term that balance the contribution of both the terms.

Designing an effective regularization term $\mathcal{R}(x)$ to reflect the image prior plays a key role in the quality of reconstructed image. Classical regularization terms utilize local structural patterns and are built on the assumption that images are locally smooth except at edges. However such terms smear out finer image details. In contrast, nonlocal regularizes can effectively model long range dependencies and yield improvement in reconstruction results. The performance of the reconstruction algorithm can greatly improve if the nonlocal self-similarity and local sparsity prior are both incorporated in a combinational regularization term.

In this paper we have taken $\mathcal{R}(x)$ as Joint Adaptive SpasityRegularisation(JASR) term, which is capable of characterizing structured sparsities of natural image. Finally the resulting optimization problem is solved using Split BregmanIterative(SBI) algorithm

1.1Ripplet Transform

Efficient representation of images play a key role in image processing tasks. Edges and curves can cause singularities in the image intensity. Traditional wavelet system can efficiently represent 1D singularities with minimum number of wavelet coefficients. However they suffer in case of higher dimensional singularities due to lack of anisotropic properties. In order to overcome its drawbacks ridgelets and curvelets were proposed.

Ripplet transform is a generalization of curvelet transform. Curvelet transform uses parabolic scaling law to achieve anisotropic property. However curvelet transforms can resolve only 2D singularities along C^2 singularities. To address this issue ripplet transform generalizes the

scaling law by adding two parameters support c and degree d. This results in the anisotropy of ripplet functions which guarantees to capture singularities along arbitrarily shaped curves.
Ripplet functions can be represented as

$$\rho_{a,\vec{b},\theta}(\vec{x}) = \rho_{a,\vec{0},0}\left(R_\theta(\vec{x}-\vec{b})\right) \tag{3}$$

where $\rho_{a,\vec{0},0}$ is the ripplet element function and $R_\theta = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$ is the rotation matrix.
For digital image processing discrete ripplet transform implementation is used. Scale parameter is sampled at dyadic scale, position parameter $\vec{b}$ and angle parameter θ are sampled at equal spaced interval, a, $\vec{b}$, and θ are substituted with discrete parameters $a_j, \overrightarrow{b_k}$ and θ_l which satisfy that $a_j = 2^{-j}$,$\overrightarrow{b_k} = [c \cdot 2^{-j} \cdot k_1, 2^{-j/d} \cdot k_2]$and $\theta_l = \frac{2\pi}{c} \cdot 2^{-\lfloor j(1-1/d)\rfloor} \cdot l$
The discrete ripplet transform of M×N image $f(n_1, n_2)$ is of form

$$R_{j,\vec{k},l} = \sum_{n_1=0}^{M-1}\sum_{n_2=0}^{N-1} f(n_1,n_2)\overline{\rho_{j,\vec{k},l}(n_1,n_2)} \tag{4}$$

where $R_{j,\vec{k},l}$ is the ripplet coefficients
The image can be reconstructed through inverse discrete ripplet transform

$$\tilde{f}(n_1,n_2) = \sum_j \sum_{\vec{k}} \sum_l R_{j,\vec{k},l}\, \rho_{j,\vec{k},l}(n_1,n_2) \tag{5}$$

II. BACKGROUND

Due to the drawbacks of basic CS techniques Gan[4] proposed block based compressive sensing(BCS) which is capable of reducing memory requirement at the same time speed of reconstruction is still maintained. Motivated by the success of BCS, Eslahi et al. [6] combined BCS and iterative curvelet thresholding (ICT) to further improve the performance of the restoration. They compared the performance of two accelerated iterative shrinkage techniques, that is, fast iterative shrinkage algorithm (FISTA) and two-step iterative shrinkage algorithm (TwIST). Incorporation of prior information of the original signal in the estimation problem can improve the quality of reconstruction. The prior knowledge about the image signals can be characterized by local smoothness and nonlocal similarity. Several techniques have been proposed for combining local smoothness and nonlocal self similarity into a unified manner[8],[9],[10].

III. PROPOSED METHODOLOGY

In this paper we have tried to implement the application of recently developed ripplet transform (RT) in the recovery of compressive sensing of images by using ripplet transform as the useful local smoothness regularity prior. Our experimental results has proven to offer better performance results when compared with other transform results.
According to the theory of compressive sensing original signal can be reconstructed from small set of random measurements by finding the solution to the ill posed problem using sparsity promoting nonlinear recovery algorithm. The performance of the reconstruction can be improved by incorporating the image prior knowledge. In this paper we have tried to incorporate nonlocal sparsity in addition to local sparsity constraint in the regularization parameter. The scheme that we have used to enforce both sparsity constraints into the regularization term is JASR.
Mathematically JASR can be expressed as

$$\mathcal{R}_{JASR} = \Psi_{RT}(x) + \tau\Psi_{NSSM} \tag{6}$$

Here τ is the tradeoff between the local and nonlocal statistical sparsity terms. The first and second terms indicate image local smoothness prior and nonlocal self similarity prior. Nonlocal self-similarity characterizes the repetitiveness of the texture or structure in natural images within nonlocal area. This can effectively retain sharpness and edges of natural images to maintain image non local consistency.
The first term corresponds to regularization term that is a measure of sparsity of transform coefficients.
NSSM characterizes nonlocal self-similarity by means of distribution of transform coefficients by transforming 3D array generated by stacking similar image patches. Initially, for an image x, the image is partitioned into n overlapped patches of equal size and then the set including the c best matched patches is searched and stacked into a 3D array to form a group. The transform coefficients of each group is found by applying 3D transform to the group, followed by arranging them in lexicographic order to form Θ_x. The number of non-zero coefficients gives the nonlocal 3D sparsity. So NSSM can be mathematically represented as

$$\Psi_{NSSM} = \|\Phi_x\|_{l_1} \tag{7}$$

Here the 3D transform used in the NSSM is composed of 3Dwavelet transform.
So the final optimization problem is of form

$$\min_x \left\{\frac{1}{2}\|f-\Theta x\|_{l_2}^2 + \lambda\Psi_{RT}(x) + \tau\|\Phi_x\|_{l_1}\right\} \tag{8}$$

The optimization problem is solved using the framework of Split Bregman Iteration (SBI) algorithm [7]. Computation of nonlocal self similarity of an image requires estimate of original image, which is computed using Fast Iterative Shrinkage Thresholding (FISTA) algorithm [14].

Algorithms

TABLE I

ALGORITHM OF FISTA

Input: f,Θ_B,σ_0, B: block size, k_{max}: maximum iteration number,
TOL: tolerance.
Output: x: Reconstructed image;

Initialization: Set k $= 0; x^{-1} = x^0$;
$t^0 = 1; x_i^0 = \Theta_B^T f_i$
repeat
1. x^k = smoothing filter(x^k);
2. for each block i do
$\hat{x}_i^k \leftarrow x_i^k + \left(\frac{t^k - 1}{t^{k+1}}\right)(x_i^k - x_i^{k-1})$
$\hat{\hat{x}}_i^k \leftarrow \hat{x}_i^k + \Theta_B^T(f_i - \Theta_B \hat{x}_i^k)$
end for
3. $\hat{v}^k \leftarrow \Psi \hat{\hat{x}}^k$
4. $\sigma^k \leftarrow \sigma_o(1 - k/k_{max})$
5. $v^k \leftarrow \mathcal{H}_{\sigma^k}(\hat{v}^k)$
6. $\breve{x}^k \leftarrow \Psi^T v^k$
7. for each block i do
$\tilde{x}_i^{k+1} \leftarrow \breve{x}_i^k + \Theta_B^T(f_i - \Theta_B \breve{x}_i^k)$
end for

Note: Here x_i^k denotes i^th block of x^k in k^th iteration,
$\mathcal{H}_{\sigma^k}(\hat{v}^k)$ denotes hard shrinkage function.

TABLE II
ALGORITHM OF JASR

Input: $f, \Theta_B, \mu_1, \mu_2, J_0$: inloop iteration number
k'_{max}: maximum iteration number, TOL: Tolerance
Output: x^*: Recovered image;
Initialization: Set k = 0; $(b^0, c^0) = (0,0)$
$x_{init} = x^0 =$ BCS-FICTA(f, Θ_B)
While a stop criterion is not satisfied do
1. $x^{k+1} = \tilde{x} = x^k$

$$\hat{v}^{k+1} = \Psi x^{k+1} + b^k$$
$$z^{k+1} = x^{k+1} - c^k$$

2. for each element of v do
compute $v_{j,l,k}^{k+1}$
end for
3. update v^{k+1} by concatenating all $\{v_{j,l,k}^{k+1}\}$
4. for each patch $z_{p_{l'}}$ do
find $G_{z_{p_{l'}}}$, compute $T^{3D}\left(G_{z_{p_{l'}}}\right)$
end for
5. arrange all $T^{3D}\left(G_{z_{p_{l'}}}\right)$ in lexicographic order to form Φ_z
6. update w^{k+1}
7. for i = 1 : J_0

$$g_i^k \leftarrow \left(\Theta^T(\Theta x^{k+1} - f) + \mu_1 \Psi^T(\Psi x^{k+1} - v^{k+1} + b^k) + \mu_2(x^{k+1} - w^{k+1} - c^k)\right)$$
$$\eta_i^k \leftarrow diag\left(abs\left(\frac{{g_i^k}^T g_i^k}{{g_i^k}^T(\Theta^T\Theta + (\mu_1 + \mu_2)I_n)g_i^k}\right)\right)$$
$$x^{k+1} \leftarrow x^{k+1} - \eta_i^k g_i^k$$

end for
8. compute $s^{k+1} = SSIM(x^{k+1}, \tilde{x})$
9. compute $diff = abs(s^{k+1} - s^k)$
10. update b^k, c^k
11. $k \leftarrow k + 1$
end While
stopping criterion: $k = k'_{max}$ or $diff \leq TOL$

IV. EXPERIMENTAL RESULTS

In this section the performance of the restoration implemented with the ripplet transform is compared with that implemented with curvelettansform. The performance of our experiments are evaluated on gray scale test images (256×256). All experiments were performed using MATLAB 2013a, on a computer equipped Intel® Core(TM) i5-4210U,2.4 GHz processor with 4 GB of RAM and running on Windows 8.1. To evaluate the simulation results two quality assessors are, Peak Signal to Noise Ratio (PSNR) and Structural Similarity (SSIM) are used.To evaluate the performance the results are compared with that of implementation with curvelet.

Table 4.1 shows the comparison of PSNR and SSIM of different images at different measurement ratios along with the comparison with the implementation using curvelet transform.

Fig 4.1 demonstrates the restoration performance of proposed method at different measurement rate. Fig 4.2 shows the comparison of PSNR performance of proposed method with that of curvelet implementation.

(a) (b) (c) (d)

Fig 4.1: Restoration of Barbara image using ripplet transform at different measurement ratio (a)MR=0.05 (b)MR=0.1 (c)MR=0.15 (d)MR=1.2

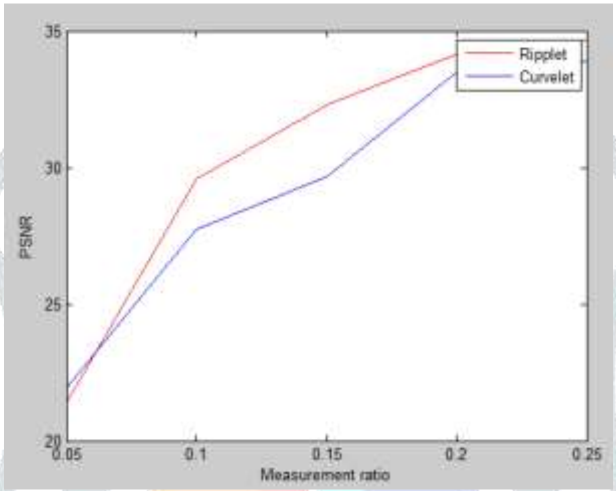

Fig 4.2: Performance comparison of proposed method with that of curvelet implementation.

Table 4.1: Comparison of PSNR and SSIM with curvelet and ripplet transform implementation

		Curvelet transform		Ripplet transform	
Image	MR	PSNR(dB)	SSIM	PSNR(dB)	SSIM
Barbara	0.05	21.94	0.621	21.42	0.689
	0.1	27.75	0.849	29.58	0.886
	0.15	29.66	0.892	32.31	0.931
Boat	0.05	25.03	0.733	25.50	0.794
	0.1	28.59	0.846	28.59	0.844
	0.15	31.14	0.900	31.18	0.897
Fence	0.05	17.28	0.510	17.90	0.535
	0.1	25.69	0.791	26.80	0.811
	0.15	28.28	0.852	28.63	0.857
House	0.05	29.94	0.838	29.98	0.898
	0.1	33.20	0.875	33.49	0.872
	0.15	34.99	0.908	35.29	0.896
Leaves	0.05	17.85	0.668	17.82	0.652
	0.1	25.84	0.917	26.62	0.977
	0.15	27.17	0.939	27.60	0.938
Monarch	0.05	21.38	0.733	22.98	0.768
	0.1	26.01	0.866	26.83	0.869
	0.15	28.81	0.911	28.91	0.915
Starfish	0.05	22.10	0.615	22.82	0.659
	0.1	25.40	0.737	25.66	0.743
	0.15	27.72	0.818	27.81	0.828
Straw	0.05	19.22	0.385	19.71	0.384
	0.1	21.27	0.633	21.92	0.657
	0.15	22.78	0.742	23.26	0.784
Texture	0.05	10.57	0.352	11.39	0.397
	0.1	13.36	0.664	13.50	0.661
	0.15	15.62	0.766	15.67	0.763

CONCLUSION

In this work we have depicted the application of ripplet transform in restoring compressive sensed images. Our results have shown that by choosing ripplet transform as the sparsifying domain offers superior reconstruction quality when compared to that of curvelet transfom. This work can be extended by incorporating the algorithms that ensure faster reconstruction without compromising the quality of reconstruction.

REFERENCES

[1] E. Candès. “Compressive sampling”. *ProcIntCongre Math*, Madrid, Spain, 2006.

[2] D. Donoho. “Compressed sensing”. *IEEE Trans Inform Theory*, 52(4):1289–1306,2006.

[3] E. Candès, J. Romberg, and T. Tao, “Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information,” *IEEE Trans.Inform. Theory*, vol. 52, no. 2, pp. 489–509, Feb. 2006.

[4] L. Gan, “Block compressed sensing of natural images,” in Proc. Int. Conf.Digital Signal Process., Cardiff, UK, Jul. 2007, pp. 403-406

[5] J. Zhang, D. Zhao, R. Xiong, S. Ma, and W. Gao, “Image restoration using joint statistical modeling in space-transform domain,” IEEE Trans. Circuits Syst. video Technol., vol. 24, no. 6, pp. 915-928, Jun. 2014.

[6] N. Eslahi, A. Aghagolzadeh, and S. M. H. Andargoli, “Block compressedsensing images using curvelet transform,” in Proc. 22nd IranianConf. Electr. Eng. (ICEE), Tehran, Iran, May 2014, pp. 1581-1586.

[7] T. Goldstein and S. Osher, “The split Bregman method for l_1 regularized problems,” SIAM J. Imag. Sci., vol. 2, no. 2, pp. 323-343, Apr. 2009..

[8] J. Zhang, D. Zhao, R. Xiong, S. Ma, and W. Gao, “Image restorationusing joint statistical modeling in space-transform domain,” IEEE Trans. Circuits Syst. video Technol., vol. 24, no. 6, pp. 915-928, Jun. 2014.

[9] S. Lefkimmiatis and S. Osher, “Non-local structure tensor functionalsfor image regularization,” IEEE Trans. Comput. Imag., vol. 1, no. 1, pp.16-29, Mar. 2015.

[10] X. Li “Image recovery via hybrid sparse representation: a deterministicannealing approach,” IEEE J. Sel. Topics Signal Process., vol. 5, no. 5,pp. 953-962, Sep. 2011.

[11] N. Eslahi and A. Aghagolzadeh, ”Compressive sensing image restoration using adaptive curveltthresholding and nonlocal sparse regularization” IEEE Trans. Image Processing, vol. 25, no.7, pp. 3126-3140, July 2016.

[12] .J. Xu, L. Yang and D. O. Wu, “Ripplet: A new transform for image processing”, Journal of Visual Communication and Image Representation,21(7):627-639, 2010.

[13] Xu, J., Wu, D.: “Ripplet-II transform for feature extraction”. Proc. SPIE Visual Communications and Image Processing (VCIP), 2010.Sujitha JulietDevaraj, Kirubakaran Ezra, Elijah Blessing Rajsingh.

[14] A. Beck, and M. Tebulle, “A fast iterative shrinkage-thresholding algorithm for linear inverse problems,” SIAM J. Imaging Sci., vol. 2, no. 1, pp. 183-202, Jan. 2009.