Some Product cordial families of double crown graphs of wheel.

Mukund V. Bapat[1]

Abstract: We discuss one point union families obtained by double crown graphs such as W_3^{++}, W_4^{++}, $W_5^{++}W_6^{++}$etc.We show that all these families are product cordial. We have taken one point union at different vertices of given graph to produce pairwise different structure (upto isomorphism) and we have obtained particular product cordial labeling of the families.

Key words: product, cordial, wheel, crown, fusion, labeling.

Subject Classification: 05C78

2. Introduction:

The graphs we consider are simple, finite, undirected and connected. For terminology and definitions we depend on Graph Theory by Harary [6], A dynamic survey of graph labeling by J.Gallian [7] and Douglas West.[8]. I.Cahit introduced the concept of cordial labeling [5].There are variety of cordial labeling available in labeling of graphs. Sundaram, Ponraj, and Somasundaram [9] introduced the notion of product cordial labeling. A product cordial labeling of a graph G with vertex set V is a function f from V to {0,1} such that if each edge uv is assigned the label f(u)f(v), the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1, and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph with a product cordial labeling is called a product cordial graph. We use $v_f(0,1) = (a, b)$ to denote the number of vertices with label 1 are a in number and the number of vertices with label 0 are b in number. Similar notion on edges follows for $e_f(0,1) = (x, y)$.

A lot of work is done in this type of labeling so far. One interested in survey may refer Dynamic survey in Graph labeling by J. Gallian. We mention some part of it. Sundaram, Ponraj, and Somasundaram have shown that trees; unicyclic graphs of odd order; triangular snakes; dragons; helms; Pm∪Pn; Cm∪Pn; Pm∪K1,n; Wm∪Fn (Fn is the fan Pn+K1); K1,m∪K1,n; Wm∪ K1,n; Wm∪Pn; Wm∪Cn; the total graph of Pn (the total graph of Pn has vertex set V (Pn)∪E(Pn) with two vertices adjacent whenever they are neighbors in Pn); Cn if and only if n is odd; $C_n^{(t)}$, the one-point union of t copies of C_n, provided t is even or both t and n are even; K2+mK1 if and only if m is odd; $C_m \cup P_n$ if and only if m+n is odd; $K_{m,n} \cup P_s$ if s >mn; Cn+2∪K1,n; Kn∪Kn,(n−1)/2 when n is odd; Kn∪Kn−1,n/2 when n is even; and P2 n if and only if n is odd. They also prove that $K_{m,n}$ (m,n> 2), $P_m \times P_n$ (m,n> 2) and wheels are not product cordial and if a (p,q)-graph is product cordial graph, then q 6 (p−1)(p + 1)/4 + 1.In this paper We show that one point union ofW_3^+, W_4^+,W_5^+,W_6^+ are families of product cordial graphs also the families obtained by shifting the common point in $G^{(K)}$.

3. Preliminaries:

3.1 Fusion of vertex. Let G be a (p,q) graph. Let u≠v be two vertices of G. We replace them with single vertex w and all edges incident with u and that with v are made incident with w. If a loop is formed is deleted. The new graph has p-1vertices and at q-1 edges.If $u \in G_1$ and $v \in G_2$, where G_1 is (p_1,q_1) and G_2 is (p_2,q_2) graph. Take a new vertex w and all the edges incident to u and v are joined to w and vertices u and v are deleted. The new graph has p_1+p_2-1 vertices and $q_1 + q_2$ edges. Sometimes this is referred as u is identified with v. The concept is well elaborated in D. West [9].

3.2Wheel graph Wn. It is obtained from a cycle C_n. Take a vertex other than cycle vertex and join each cycle vertex to this new vertex called as hub. The edges incident with hub are referred as pokes. Wn has 2n+1 vertices and 2n edges.

3.3Crown graphG^+. It is $C_n ʘ K_2$.At each vertex of cycle an edge was attached. We develop the concept further to obtain crown for any graph. Thus crown G is a graph $G ʘ K_2$.It has a pendent edge attached to each of it's vertex. If G is a (p,q) graph then crown(G) has q+p edges and 2p vertices. t-crown of G also denoted by $G^{+t}$is a graph G whose each vertex has fused t-pendent vertices .If G is a (p,q) graph then G^{+t} has p+pt vertices and q + pt edges. We discuss Wn^+ .Ithas 2n+1 vertices and 3n edges.

3.4One point union of graphs $G^{(K)}$ is obtained by fusing a same fix vertex from two copies of G.When k such copies are fused together we get $G^{(K)}$.It has k.q edges and k(p-1) + 1 vertices. If point of union on G is changed the different structure is (up to isomorphism) obtained.For given graph we have tried all such pairwise non isomorphic structure and have proved that all are product cordial graph.

4. Main Results.

Theorem 4.1The one point union double crown of W3 given by G = $(W_3^{++})^{(k)}$ is product cordial iff k is an even number. Proof:
Define i^{th} copy of W_3^{++} in $(W_3^{++})^{(k}$ as :w_i be the hub. The C_3 cycle be given by $(v_{i,1}, e_{i,1}, v_{i,2}, e_{i,2}, v_{i,3}, e_{i,3}, v_{i,1})$.The two pendent vertices $w_{i,j}$, and $u_{i,j}$ at each cycle vertex are attached to $v_{i,j}$ by edges $(v_{i,j}w_{i,j})$, j = 1,2,3 and $(v_{i,j}u_{i,j})$;j =1, 2, 3.The two pendent vertices at w_i be $w_{i,4}$ and $u_{i,4}$.By an edge (referred as pokes) each the w_i is adjacent to each of cycle vertex. Note that W_3^{++} has 12 vertices and 14 edges .G has 11k + 1 vertices and 14k edges.

Define f : V(G)→ {0,1} as :f gives us two type of labeling Type A and Type B as follows. Both are not product cordial.

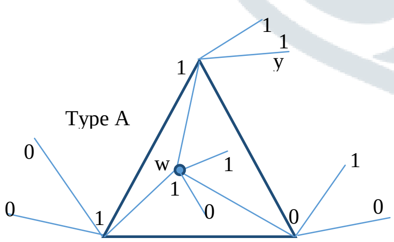

Fig. 4.1$W_3^+ v_f(0,1)$ = (5,7) , $e_f(0,1)$ = (8,6)

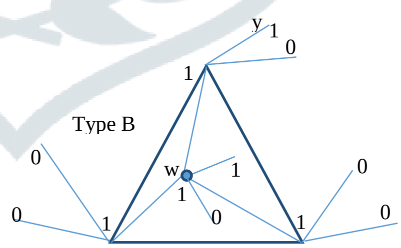

Fig. 4.2$W_3^+ v_f(0,1)$ = (6,6) , $e_f(0,1)$ = (6,8)

One can take one point union at any pendent vertex or at the 5- degree vertex like w. We choose points 'y' and 'w' as shown in diagrams above. To obtain $(W_3^{++})^{(2)}$ fuse Type A label and Type B label at point 'y' (point 'w').to obtain = $(W_3^{++})^{(2x)}$ repeat the procedure for x times; x= 1,2,..The label number distribution is given by $v_f(0,1)$ = (11x,11x+1), $e_f(0,1)$= (14x, 14x); k = 2x ; x= 1, 2, .

When k =1 we have to label the copy of W_3^{++} such that there will be exact 6 vertices and exact 7 edges.To do this we must label all 5-degree vertices as '1'.This produces all vertices to which pendent edges are attached has label '1'.This prohibits giving label '1' to pendent vertices as it produces edge label as ‘1’.This happens for all k as even number. Therefore when k is odd number we can’t get product cordial ;labeling.

Theorem 4.2 G = $(W_4^{+t})^{(k)}$ the one point union of k copies of t- crown of W_4 is a product cordial graph. Proof: We define i^{th} copy of W_4^{+t}as follows. It has a C_4 cycle given by =($v_{i,1}$, $e_{i,1}$, $v_{i,2}$, $e_{i,2}$, $v_{i,3}$, $e_{i,3}$, $v_{i,4}$, $e_{i,4}$, $v_{i,1}$). To each of vertices v_i there are t number of pendent edges attached. The corresponding pendent vertices are $u_{i,j,}$ j = 1, 2, ..t. and i = 1, 2, …k. Each of the cycle vertices is attached with a new vertex say $w_i$by an edge each called as pokes. The vertex w has t number of pendent edges corresponding to which are t vertices given by y_i;i = 1, 2, .., t .

Define a function f:V(G)→{0,1} given as follows:

$f(w_i)$=1; i = 1, 2, …k;

$f(v_j)$ = 1, j = 1, 2, 3.

Case t is odd number given by t = 2x+ 1. $f(y_i)$ = 1 for all i = 1, 2, ..t; $f(v_{i,j})$ = 1 for all j = 1, 2, 3..t. and i = 1,2 $f(v_{3,j})$= 1 for j = 1, 2, ..(x-1). (If t = 1 x = 0, vertex incident to w_i will be labeled as 1 and all other pendent vertices incident with cycle vertices will receive label 0) $f(v_{i,j})$ = 0 for all j> x-1.when i = 3 and when i>3 and j = 1,2,..t (i>2) The label number distribution is $v_f(0,1)$ = (5x+5,5x+5) and $e_f(0,1)$= (5x+7, 5x+6) Case t = 2x;.$f(y_i)$ = 1 for all i = 1, 2, ..t; $f(v_{i,j})$ = 1 for all j = 1, 2, 3..t. and i = 1,2 $f(v_{3,j})$= 1 for j = 1, 2, ..(x-1). $f(v_{i,j})$ =0, i>2; i = 3 then j = x,x+1,…t; i>3 then for all j = 1, 2,..t

The label number distribution is $v_f(0,1)$ = (5x+3,5x+2) and $e_f(0,1)$= (5x+4, 5x+4)

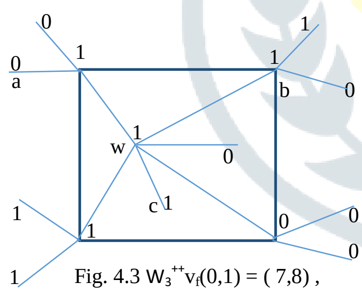

Fig. 4.3 $W_3^{++}v_f(0,1)$ = (7,8) , $e_f(0,1)$ = (9,9)

Theorem 4.3 G = $(W_5^{++})^{(k)}$ is product cordial for all values of k.

Proof:Define i^{th} copy of W_5^{++} in $(W_5^{++})^{(k}$ as : w_i be the hub. The C_5 cycle be given by ($v_{i,1}$, $e_{i,1}$, $v_{i,2}$, $e_{i,2}$, $v_{i,3}$,.. , $e_{i,5}$, $v_{i,1}$).The two pendent vertices $w_{i,j}$, and $u_{i,j}$(j=1,2) are attached to $v_{i,j}$ (at each vertex on cycle) by edges ($v_{i,j}w_{i,j}$), j = 1,2,3,4,5 and ($v_{i,j}u_{i,j}$) ;j =1, 2, 3,4,5.The two pendent vertices at w_i be $w_{i,6}$and $u_{i,6}$.By an edge (referred as pokes) each the w_i is adjacent to each of cycle vertex. Note that W_5^{++} has 18 vertices and 22 edges .G has 17k + 1 vertices and 22k edges.

Define f : V(G)→ {0,1} as :f gives labeling Type A and Type B as follows:

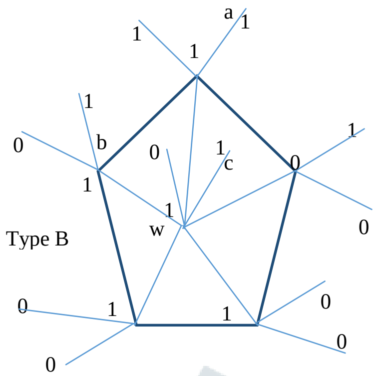

Fig. 4.4 $W_3^{+}v_f(0,1) = (8,10)$, $e_f(0,1) = (11,11)$

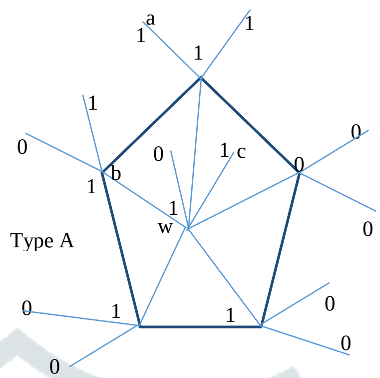

Fig. 4.5$W_3^{+}v_f(0,1) = (9,9)$, $e_f(0,1) = (11,11)$

One can take one point union at any pendent vertices attached to cycle C_5 vertex or pendent vertex at hub wi or at the five degree vertex like 'b' or at hub. When one point union is taken at these points we get structure 1, structure 2, structure 3, and structure 4 respectively. These four structures are pairwise non isomorphic.

Case k = 2x.

To obtain $(W_5^{++})^{(2)}$ fuse Type A label and Type B label at point 'a' (point 'b'), (point 'c'), (point 'w').To obtain $(W_5^{++})^{(2x)}$ repeat the procedure for x times; x= 1,2,..The label number distribution is given by $v_f(0,1) = (17x,17x+1)$, $e_f(0,1)= (22x, 22x)$; k = 2x ; x= 1, 2, .

casek is of type 2x+1; x= 0,1,2..First obtain structure $(W_5^{++})^{(2x)}$ and at it's appropriate point. Then fuse Type A label at the appropriate vertex.The label number distribution is given by $v_f(0,1) = (17x+9,17x+9)$, $e_f(0,1)= (22x+11, 22x+11)$; k = 2x+1 ; x= 0,1, 2, .

Theorem 4.4 G = $(W_6^{++})^{(k)}$ is product cordial for all values of k.

Proof:Define i^{th} copy of W_6^{++} in $(W_6^{++})^{(k}$ as : w_i be the hub. The C_6 cycle be given by ($v_{i,1}$, $e_{i,1}$, $v_{i,2}$, ..,$e_{i,6,}$ $v_{i,1}$).The two pendent vertices $w_{i,j}$, and $u_{i,j}$ are attached to $v_{i,j}$ (at each vertex on cycle) by edges ($v_{i,j}w_{i,j}$), j = 1,2,…,6 and ($v_{i,j}u_{i,j}$) ;j =1, 2,…,6.The two pendent vertices at w_i be $w_{i,7}$and $u_{i,7}$.By an edge (referred as pokes) each the w_i is adjacent to each of cycle vertices. Note that W_6^{++} has 21 vertices and 26 edges .G has 20k + 1 vertices and 26k edges.

Define f : V(G)→ {0,1} as :f gives labeling as follows:One can take one point union at any pendent vertices attached to cycle C_6 vertex or pendent vertex at hub w_i or at the five degree vertex like 'b' or at hub. When one point union is taken at these points we get structure 1, structure 2, structure 3, and structure 4 respectively. These four structures are pairwise non isomorphic. To obtain $(W_6^{++})^{(k)}$ fuse Type A label at point 'a' (point 'b'), (point 'c'), (point 'w').repeatedly.

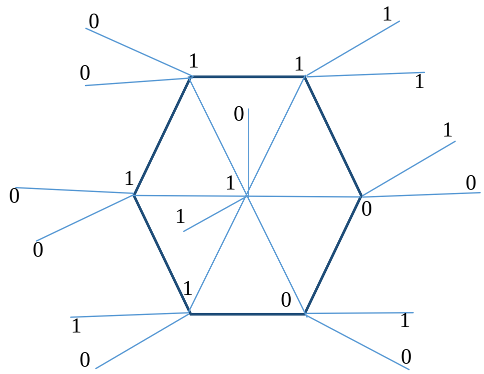

Fig. 4.6$W_3^{+}$$v_f(0,1)$ = (10,11) ,
$e_f(0,1)$ = (13,13)

The label number distribution is given by $v_f(0,1) = (10k,10k+1)$, $e_f(0,1)= (13k,13k)$; $k = 2x$; $x= 1, 2, .$

Conclusions: In this paper we have discussed one point union families obtained on double Crown graph of wheel, W_3^{++}, W_4^{+t}, W_5^{++}, W_6^{++}.We have shown that all these families are product cordial. Not only that but for each of these families we have obtained new one point unions by changing the point common to k copies in $G^{(k)}$. We have shown that there are different pairwise non isomorphic structures possible and all of them are shown to be product cordial. It is necessary to investigate G^{+t} for G is wheel graph on C_n.

References:

[1] Bapat M.V. Some new families of product cordial graphs, Proceedings, Annual International conference, CMCGS 2017, Singapore, 110-115

[2] Bapat M.V. Some vertex prime graphs and a new type of graph labelling Vol 47 part 1 yr. 2017 pg 23-29 IJMTT

[3] Bapat M. V. Some complete graph related families of product cordial graphs. Arya bhatta journal of mathematics and informatics vol 9 issue 2 july-Dec 2018.

[4] Bapat M.V. Extended Edge Vertex Cordial Labelling Of Graph ",International Journal Of Math Archieves IJMA Sept 2017 issue

[5] Bapat M.V. Ph.D. Thesis, University of Mumbai 2004.

[6] Harary, Theory, Narosa publishing, New Delhi

[7] J. Gallian Electronic Journal Of Graph Labeling (Dynamic survey)2016

[8] M. Sundaram, R. Ponraj, and S. Somasundaram, "Product cordial labeling of graph," Bulletin of Pure and Applied Science, vol. 23, pp. 155–163, 2004.

[9] Douglas West, Introduction to graph Theory, Pearson Education, Singapore.

[1]Bapat Mukund V.At and Post: Hindale, Tal.:Devgad, Dist.: Sindhudurg, Maharashtra. India 416630.