Reverse-Magic Graphoidal Numbering of a Graphs

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Abstract: ***A Graphoidal cover is a partition of edge set of G into internally disjoint paths (not necessarily open). The minimum cardinality of such cover is known as graphoidal covering number of G.Let $G = (V, E)$ be a graph and let ψ be a graphoidal cover of G. A graph G is called magic graphoidal if there exists a minimum graphoidal cover ψ of G such that G admits ψ-magic graphoidal total labeling. In this paper we introduced a new labeling of reverse process of magic graphoidal called reverse-magic graphoidal labeling. In this paper we proved that Path, Star, Comb, $[P_n: S_1]$ are reverse magic graphoidal.***

Keywords: Graphoidal Constant, Graphoidal Cover, Magic Graphoidal, reverse magic graphoidal.

1.INTRODUCTION

A graph G is said to be magic if there exist a bijection $f: V \cup E \rightarrow \{1,2,3 \dots \dots m+n\}$; where '$n$' is the number of vertices and 'm' number of edges of a graph. Such that for all edges xy, $f(x) + f(y) + f(xy)$ is a constant. Such a bijection is called a magic labeling of G.
B.D. Acharya and E. Sampath Kumar [1] defined Graphoidal cover as partition of edge set of G in to internally disjoint paths.
Let $G = (V, E)$ be a graph and let ψ be a graphoidal cover of G. Define $f: V \cup E \rightarrow \{1,2 \dots\dots, m+n\}$ such that for every path $P = \{v_1, v_2, \dots\dots, v_n\}$ in ψ with $f^*(p) = f(v_1) + f(v_n) + \sum_{i=1}^{n-1} f(v_i v_{i+1}) = k$ is a constant, where f^* is the induced labeling on ψ. Then, we say that G admits ψ - magic graphoidal total labeling of G. A graph G is called magic graphoidal if there exists a minimum graphoidal cover ψ of G such that G admits ψ - magic graphoidal total labelling of G.
Here we introduced a new type of (ie. Reverse) magic graphoidal total labeling is called reverse magic graphoidal (rmg) total labeling.

Definition 1.1
The ***Path graph*** P_n is the n- vertex graph with $(n-1)$ edges, all on a single path.
Definition1.2
A complete bipartite graph $K_{1,n}$ is called a ***star*** and it has $(n+1)$ vertices and n edges
Definition 1.3
The ***Trivial graph*** K_1 or P_1 is the graph with one vertex and no edges.
Definition1.4
Let $P_n \Theta K_1$ be the ***Comb*** which is the graph obtained from a path P_n by attaching pendant edge at each vertex of the path .
Definition1.5
Let $S_1 = (v_0, v_1)$ be a star and let $[P_n : S_1]$ be the graph obtained from n copies of S_1 and the path $P_n = (u_1, u_2, u_3, \dots \dots \dots \dots, u_n)$ by joining u_j with the vertex v_0 of the j^{th} copy of S_1 by means of an edge, for $1 \leq j \leq n$

II.MAIN RESULTS

Definition 2. 1
A reverse-magic graphoidal labeling of a graph G is one-to-one map f from $V(G) \cup E(G) \rightarrow \{1,2,3, \dots, m+n\}$, where *'n'* is the number of vertices of a graph and *'m'* is the number of the edges of a graph, with the property that , there is an integer constant 'μ_{rmgc}' such that
$f^*(P) = \sum_{i=1}^{n-1} f(v_i\, v_{i+1}) - \{f(v_1) + f(v_n)\} = \mu_{rmgc}$, is a contant
Then the reverse methodology of magic graphoidal labeling is called reverse-magic graphoidal labeling (rmgl). Reverse process of magic graphoidal of a graph is called reverse-magic graphoidal graph.(rmgg).

Theorem 2.1
Every Path P_n is reverse-magic graphoidal for $n \geq 2$.
Proof :
Let G be the graph P_n .
Let $V(G) = v_i$; $1 \leq i \leq n$
and $E(G) = v_i v_{i+1}$; $1 \leq i \leq n-1$
Define $f: V(G) \cup E(G) \rightarrow \{1,2, \dots, m+n\}$ by

$f(v_1) = 1$
$f(v_n) = m+n$

Here, $P_n = m+n = 2n-1$

$f(v_i v_{i+1}) = n-1+i;$ $1 \leq i \leq n-1$

Let $\psi = \{P = (v_1 v_2 v_3 \dots v_n)\}$

$f^*(P) = f(v_1\, v_2) + f(v_2\, v_3) + \cdots + f(v_{n-1} v_n) - \{f(v_1) + f(v_n)\}$

$$= \sum_{i=1}^{n-1}(n-1+i) \; - \{1+2n-1\}$$

$$f^*(P) = \sum_{i=1}^{n-1}(n-1+i) \; - (2n)$$

$$= \sum_{i=1}^{n-1}(n+(i-1)) \; - (2n)$$

$$= \sum_{i=1}^{n-1} n + \sum_{i=1}^{n-1}(i-1) \; - (2n)$$

$$= n(n-1) + \frac{(n-1)(n-2)}{2} - (2n)$$

$$= n^2 - n + \frac{n^2-3n+2}{2} - (2n)$$

$$= \frac{2n^2-2n+n^2-3n+2-4n}{2}$$

$$= \frac{3n^2-9n+2}{2} = \mu_{rmgc} \qquad (1)$$

From (1) , we conclude that G admits $\psi-$ reverse magic graphoidal total labeling. The reverse- magic graphoidal constant μ_{rmgc} of a path P_n is $\frac{3n^2-9n+2}{2}$. Hence every P_n is reverse-magic graphoidal

Example 2.1

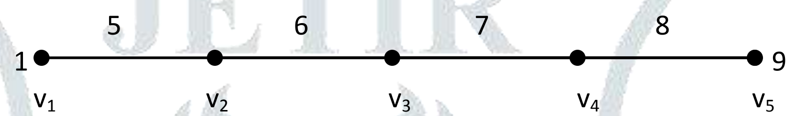

Figure 2.1: reverse-magic graphoidal of P_5

Theorem 2.2

Every Star $K_{1,n}$ is reverse-magic graphoidal for $n \geq 1$

Proof :

Let G be the graph $K_{1,n}$.

Let $V(G) = v, v_i$; $1 \leq i \leq n$

$E(G) = v\,v_i$; $1 \leq i \leq n$

Define $f: V \cup E \rightarrow \{1,2,\dots,m+n\}$ by

$f(v) = m+n = 2n+1$

$f(v_i) = i$; $1 \leq i \leq n$

$f(vv_i) = n+i$; $1 \leq i \leq n$

Let $\psi = \{ P = (vv_1, vv_2, vv_3, \dots, vv_n)$

So, $f^*(P_i) = f(vv_i) - \{f(v) + f(v_i)\}$

$= n+i-\{2n+1+i\}$; $1 \leq i \leq n$

$= n+i-2n-1-i$

$= -n-1 = -(n+1) = \mu_{rmgc}$ (1)

From equation (1) , we conclude that G admits $\psi -$ reverse magic graphoidal . The reverse-magic graphoidal constant μ_{rmgc} of a star is $-(n+1)$. Hence the theorem.

Example 2.2

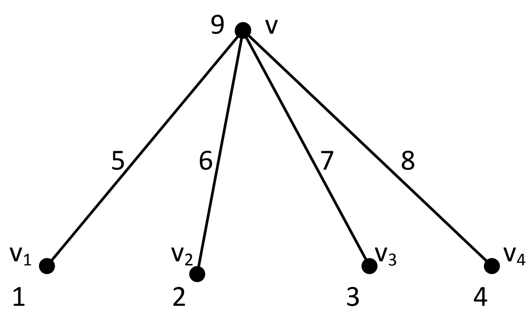

Figure 2.2: reverse- magic graphoidal of $K_{1,4}$

Theorem 2.3
Every Comb $P_n \odot K_1$ is reverse-magic graphoidal for $n > 2$
Proof :
Let G be the graph of $P_n \odot K_1$ for $n > 2$
Let $V(G) = v_i, u_i$; $1 \le i \le n$
and $E(G) = \{(v_i u_i);\ 1 \le i \le n\} \cup \{(v_i v_{i+1});\ 1 \le i \le n-1\}$
Define $f: V \cup E \to \{1,2,\ldots,m+n\}$ by

$f(u_1) = 1$
$f(u_2) = m + n = 4n - 1$
$f(u_{i+2}) = 4n - 1 - i$; $1 \le i \le n-2$
$f(v_{i+1}) = i + 1$; $1 \le i \le n-2$
$f(v_1 u_1) = n$
$f(v_1 v_2) = n + 1$
$f(v_2 u_2) = 2n + 2$
$f(v_{i+1} v_{i+2}) = 2n + 1 - i$; $1 \le i \le n-2$
$f(v_{i+2} u_{i+2}) = 2n + 2 + i$; $1 \le i \le n-2$

Let $\psi = \{ P_1 = (u_1 v_1 v_2 u_2)$;
$P_2 = (v_{i+1} v_{i+2} u_{i+2})$; $1 \le i \le n-2\}$

So, $f^*(P_1) = \Sigma f(xy) - \{f(x) + f(y)\}$
$= f(v_1 u_1) + f(v_1 v_2) + f(v_2 u_2) - \{f(u_1) + f(u_2)\}$
$= n + n + 1 + 2n + 2 + - \{1 + 4n - 1\}$
$= 3 = \mu_{rmgc}$ ____________ (1)

$f^*(P_2) = f(v_{i+1} v_{i+2}) + f(v_{i+2} u_{i+2}) - \{f(v_{i+1}) + f(u_{i+2})\}$; $1 \le i \le n-2$
$= 2n + 1 - i + 2n + 2 + i - \{(i+1) + 4n - 1 - i\}$
$= 4n + 3 - \{i + 1 + 4n - 1 - i\}$
$= 4n + 3 - 4n$
$= 3 = \mu_{rmgc}$ ____________ (2)

from (1) & (2); we conclude that G admits $\psi -$ reverse magic graphoidal labeling. The reverse- magic graphoidal constant of a comb is always 3. Hence every $P_n \odot K_1$ is reverse-magic graphoidal.

Example

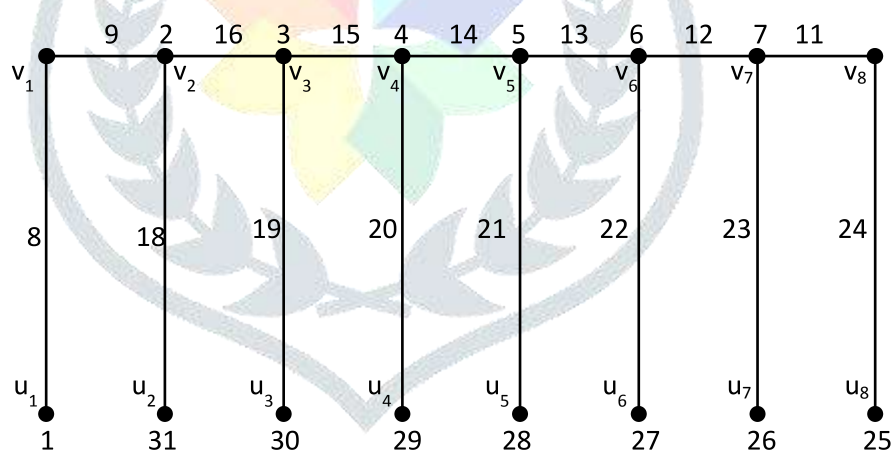

Figure 2.3: reverse-magic graphoidal of $P_8 \odot K_1$

Corollary 2.1
;For $n = 2, P_n \odot K_1$ is a path with $n = 4$; from theorem (2.1) P_4 is a reverse magic graphoidial. So the reverse magic graphoidal constant of $P_n \odot K_1 = P_4$ is not 3, it is -2.

Theorem 4
The graph $[P_1 : S_1]$.is reverse magic graphoidal.
Proof :
Let G be a graph $[P_1 : S_1]$.
Let $V(G) = v_i, w_i, u_i$; $1 \le i \le n$
And $E(G) = \{(v_i w_i, w_i u_i;\ 1 \le i \le n) \cup (v_i v_{i+1})\}$; $1 \le i \le n-1$
Define $f: V \cup E \to \{1,2,3,\ldots,m+n\}\ by$
$f(u_1) = 1$

$f(u_2) = m+n = 6n-1$
$f(v_{i+1}) = i+1; \quad 1 \leq i \leq n-2$
$f(u_{i+2}) = 6n-1-i; \quad 1 \leq i \leq n-2$
$f(u_1, w_1) = n$
$f(w_1, v_1) = n+1$
$f(v_1 v_2) = n+2$
$f(v_{i+1} w_{i+1}) = 3n+2+i; \quad 1 \leq i \leq n-1$
$f(w_{i+1} u_{i+1}) = 4n+1+i; \quad 1 \leq i \leq n-1$
$f(v_{i+2} w_{i+2}) = 3n+3+i; \quad 1 \leq i \leq n-1$
$f(w_{i+2} u_{i+2}) = 4n+2+i; \quad 1 \leq i \leq n-1$
$f(v_{i+1} v_{i+2}) = 3n+3-2i; \quad 1 \leq i \leq n-2$

Let $\psi = \{P_1 = (u_1 w_1 v_1 v_2 w_2 u_2);\ P_2 = (v_{i+1} v_{i+2} w_{i+2} u_{i+2}); \quad 1 \leq i \leq n-2)\}$

So, $f^*(P_1) = \Sigma f(xy) - \{f(x) + f(y)\}$
$= f(u_1 w_1) + f(w_1 v_1) + f(v_1 v_2) + f(v_2 w_2) + f(w_2 u_2) - \{f(u_1) + f(u_2)\}$
$= n + n + 1 + n + 2 + 3n + 2 + 1 + 4n + 1 + 1 - (1 + 6n - 1)$
$= 10n + 8 - 6n$
$= 4n + 8 = \mu_{rmgc}$ ______________ (1)

$f^*(P_2) = \Sigma f(xy) - \{f(x) + f(y)\}$
$= f(v_{i+1} v_{i+2}) + f(v_{i+2} w_{i+2}) + f(w_{i+2} u_{i+2}) - \{f(v_{i+1}) + f(u_{i+2})\}$
$= 3n + 3 - 2i + 3n + 3 + i + 4n + 2 + i - \{i + 1 + 6n - 1 - i\}$
$= 10n + 8 - 6n$
$= 4n + 8 = \mu_{rmgc}$ ______________ (2)

From (1) &(2) , we conclude that G admits ψ - reverse magic graphoidal labeling. The –reverse- magic graphoidal constant of $[P_n : S_1]$ is $4n+8$. Hence the theorem.

Example:

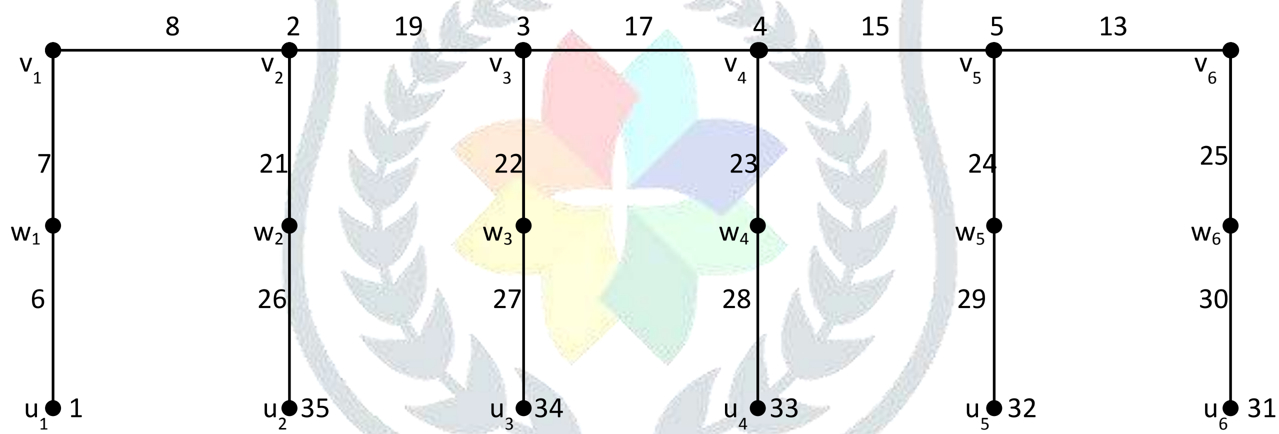

Figure 2.4 : reverse-magic graphoidal of $[P_6 : S_1]$

III. CONCLUSION

The graphoidal labeling is one the most important techniques in graph theory. As all the graphs graphoidal techniques is very interesting to investigate graphs or graph families which admit reverse graphoidal labeling. We have reported reverse graphoidal labeling of various graphs.

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