EFFECT OF VARIABLE SUCTION ON MHD OSCILLATORY FLOW THROUGH A POROUS MEDIUM

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Abstract : In the present paper, we have studies the variable suction effect of MHD oscillatory flow through porous medium. Variations of velocity field with magnetic parameter are shown by the help of Tables and Graphs. The effect of Prandtl's number (Pr), Eckert number (Ec) and small parameter ($\in$) on temperature profile have also been shown by Tables and Graph, Skin friction and heat transfer rate have been obtained and discussed.

***Keywords**:* **Porous medium , Prandtl's number (Pr), Eckert number (Ec) , MHD oscillatory flow .**

I. INTRODUCTION

: An analysis of a two dimensional flow of water past an infinite vertical prorous plate is presented under the following conditions:

(i) The suction velocity oscillates in time about a constant non - zero mean.

(ii) The free stream velocity oscillates in time about a constant mean.

(iii) The plate temperature is constant.

(iv) The difference between the temperature of the plate and the free stream is moderately large causing free convection currents.

The flow past an infinite vertical porous and isothermal plate with constant suction was studied by Soundalgeker [6]. The effect of free convection currents on the mean flow as discussed and those on the unsteady flow were discussed. The plate was assumed to be stationary. Pop has discussed the hydromagnetic laminar periodic boundary layer flow for the suction velocity of the form $v' = v_0\,(1{+}\in Ae^{-nt})$

Sondalgeker[7] investigated free convection effects on oscillatory flow of a viscous incompressible fluid past an infinite plate with variable suction. Oscillatory flow through a porous medium in the presence of free convection flow was considered by Raptis[5]. Unsteady two dimensional free convective flow and heat transfer of dissipative incompressible visco-elastic fluid past an infinite porous plate with time dependent suction have been presented by Acharya et al.[1].

Oscillatory flow past a porous bed was discussed Chawla and Singh[4] Recently Acharya et al.[2] studied the free convection and mass transfer flow through porous medium with constant suction and constant heat flux. Magnetic field effects on the free convection and mass trnasfer flow through porous medium with time dependent suction and constant heat flux was studied by Balbir ef al.[3]

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II. ***Mathematical Analysis :*** Consider two dimensional oscillatory flow of an electrically conducting viscous, incompressible fluid past an infinite horizontal plate x-axis is taken along the plate in the direction of flow and y-axis is taken perpendicular to the plate. A uniformly distributed constant magnetic field B0 is acting perpendicular to the plate. Under the MHD boundary layer assumption, flow is governed by the following equations:

Momentum Equation

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$$\frac{\partial u'}{\partial t'} + \frac{\partial u'}{\partial y'} = \frac{\partial U}{\partial y'^2} + u\,\frac{\partial^2 u'}{\partial y'^2} - \delta\,\frac{B_0^2 u'}{p} - \frac{v}{k'}u' \tag{1}$$

Energy equation

$$pC_p\left\{\frac{\partial T'}{\partial t'} + \frac{\partial T'}{\partial y'}\right\} = k\left\{\frac{\partial^2 T'}{\partial y'^2}\right\} + u\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{2}$$

The boundary condition are :

$$\left.\begin{array}{llll} T' = T'_{\infty}, & u' = 0, & at & y = 0 \\ u' = U(t), & T' = T'_{\infty}, & as & y \to \infty \end{array}\right\} \quad (3)$$

On introducing the following non-dimensional quantities

$$u = \frac{u'}{\upsilon_0}, \quad y = \frac{y' v0}{\upsilon}, \quad t = \frac{t' v_0^2}{\upsilon}$$

$$\theta = \frac{T' - T'_{\infty}}{T_w' - T_w'}, \quad u = \frac{u'}{\upsilon_0}, \quad k_0 = \frac{k' v_0}{\upsilon^2} \quad (4)$$

$$P = \frac{pvC_p}{k}$$ (Prandtl Number)

$$E = \frac{v_0^2}{Cp\,(T_w' - T_{\infty}')}$$ (Eckert Number)

in equation (1) and (2), we get

$$\frac{\partial^2 u}{\partial y^2} - \left(1 + \in Ae^{-nt}\right)\frac{\partial u}{\partial y} + M\,(U - u) + \frac{dU}{dt} - \frac{\partial u}{\partial t} - \frac{u}{K_0} = 0 \quad (5)$$

$$P\frac{\partial \theta}{\partial t} - P\left(1 + \in Ae^{-nt}\right)\frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} + PE\left(\frac{\partial u}{\partial y}\right)^2 \quad (6)$$

The corresponding boundary conditions

$$\left.\begin{array}{llll} u = 0, & \theta = 1, & at & y = 0 \\ u \to U(t), & \theta = 0, & as & y \to \infty \end{array}\right\} \quad (7)$$

We assume the solution of equation (5) and (6), as

$$u(y,t) = u_0 + Eu_1 e^{-nt}$$

$$U(t) = 1 + Ee^{-nt}$$

$$\theta(y,t) = \theta_0 + E\,\theta_1 e^{-nt} \quad (8)$$

III- where E is small.
Using equation (8) in equations (5) and (6), we get

$$u_0'' + u_0' + M\,(1 - u_0) - \frac{u_0}{K_0} = 0 \quad (9)$$

$$u_1'' + u_1' + \left(n - \frac{1}{K_0}\right)u_1 = -u_0' A - n \quad (10)$$

$$\theta_0'' + P\,\theta_0' = 0 \quad (11)$$

$$\theta_1'' + P\,\theta_1 n + P\,\theta_1' = P\,u_0'^2 \quad (12)$$

Now the corresponding boundary conditions are:

$$\begin{array}{llllll} u_0 = 0, & u_1 = 0, & \theta_0 = 1, & \theta_1 = 1 & at & y = 0 \\ u_0 = 1, & u_1 = 1, & \theta_0 = 1, & \theta_1 = 1 & as & y \to \infty \end{array} \quad (13)$$

On solving the equations (9), (10), (11) and (12), we get the values of u and θ as:

$$u = -e^{-A_1 y} - \frac{M\,e^{-A_1' y}}{\frac{1}{K_0} + M} + 1 + \frac{M}{\frac{1}{K_0} + M} + E$$

$$\left\{-(S_1+1)e^{-B_1 y}+(S_2+N)e^{B_1' y}+S_1 e^{-A_1 y}-S_2 e^{A_1' y}-N\right\}e^{-nt} \quad (14)$$

$$\theta = e-Py+E\left[e-M2y-1\left\{\frac{PA_1^2}{(4A_1^2-2PA_1+Pn)}-\frac{PA_1^2M^2}{\left(\frac{1}{K_0}+M\right)\{4A_1^2-2PA_1'+Pn\}}-\frac{2PA_1A_1'M}{\left(\frac{1}{K_0}+M\right)\{(A_1+A_1')^2-P(A_1+A_1')+Pn\}}\right\}+\frac{PA_1^2e^{-2A_1y}}{(4A_1^2-2PA_1+Pn)}+\frac{PA_1'^2M^2e^{-2A_1'y}}{\left(\frac{1}{K_0}+M\right)\{4A_1'^2-2PA_1'+Pn\}}+\frac{2PA_1A_1'Me^{-(A_1+A_1')y}}{\left(\frac{1}{K_0}+M\right)\{(A_1+A_1')^2-P(A_1+A_1')+Pn\}}\right] \quad (15)$$

$$A_1=\frac{1+\sqrt{1+4\left(\frac{1}{K_0}+M\right)}}{2},\qquad A'_1=\frac{1-\sqrt{1+4\left(\frac{1}{K_0}+M\right)}}{2}$$

$$S_1=\frac{AA_1}{A_1^2-A_1+\left(n-\frac{1}{K_0}\right)},\qquad S_2=\frac{AA_1'}{A_1'^2-A'_1+\left(n-\frac{1}{K_0}\right)}$$

$$B_1=\frac{1+\sqrt{1-4\left(n-\frac{1}{K}\right)}}{2},\qquad B'_1=\frac{1-\sqrt{1-4\left(n-\frac{1}{K_0}\right)}}{2}\qquad N=\frac{n}{n-\frac{1}{K_0}}$$

Skin Friction and Nusselt Number - The non-dimensional skin friction at the plate is given by

$$r=\left(\frac{\partial u}{\partial y}\right)_{y-0}=A_1+\frac{MA'_1}{\frac{1}{K_0}+M}$$

$$+E\{(S_1+1)B_1+(S_2+N)B'_1-S_1A_1-S_2A'_1\}e^{-nt}$$

The non-demensional Nusselt Number at the plate is given by:

$$N_u=\left(\frac{\partial\theta}{\partial y}\right)_{y-0}$$

$$=-P+E\left[-M_2\left\{1-\frac{PA_1^2}{(4A_1^2-2PA_1+Pn)}-\frac{PA_1^2M^2}{\left(\frac{1}{K_0}+M\right)\{4A_1^2-2PA'_1+P_n\}}-\frac{2PA_1A_1'M}{\left(\frac{1}{K_0}+M\right)\{(A_1+A'_1)^2-P(A_1+A'_1)+P_n\}}\right\}-\frac{2PA_1A_1^2}{(4A_1^2-2PA_1+P_n)}\right.$$

$$-\frac{2A'_1 P A_1'^2 M^2}{\left(\frac{1}{K_0}+M\right)\left\{4A_1'^2 - 2PA_1'^2 + P_n\right\}} + \frac{\left(A1 + A_1'\right) 2A1A_1' M}{\left(\frac{1}{K_0}+M\right)\left\{\left(A_1 + A_1'\right)^2 - P\left(A_1 + A_1'\right) + P_n\right\}}$$

IV. RESULTS AND DISCUSSION

: The velocity profile are tabulated in table -1 For K_0= 15, P = 0.71, n = 0.1, t = 1, E = 0.01 the Graphs 1- 5 are drawn for different values of M and A.

	M	A
Graph-1	0	0.5
Graph-2	2	0.5
Graph-3	4	0.5
Graph-4	2	1.0
Graph-5	2	0.0

From graphs 1 - 5 it is found that velocity decreases with increase in y. On Comparing the graphs each other it is observed that velocity decreases with increases in M and increases with increase in A.
The temperature profiles are tabulated in Table - 2. For K_0 = 15, p = 0.71, n = 0.1, E = 0.01 the graphs 1 - 5 are drawn for different values of M. It is found that temperature Qincreases with increase in M.
The skin friction profiles are tabulated in Table - 3. For K0 = 15, n = 0.1 and E = 0.01, the graphs 1 - 4 are drawn for different values of M and A. It is found that skin friction decreases with increase in M and A both.
The heat transfer rates (Nu) are tabulated in Table - 4, graphs 1 - 5 are drawn for different values of M.
From graphs 1 - 5 it is found that on increasing the values of P, the heat transfer rate (Nu) decreases and increasing the values of M for lower Prandtl number and heat transfer rate increases but for higher Prandtl number i.e. P > 1 heat transfer rate decreases slightly.

Table-1***: Velocity Profile***

Y	**Graph -**	**Graph - 2**	**Graph – 3**	**Graph – 4**	**Graph -5**
0	-0.00905	-0.01344	-0.00905	-0.00905	-0.00905
1	0.65687	-2.18170	-2.86010	-0.84680	-0.85610
2	0.87800	-5.53280	-21.06370	-5.50570	-5.50970
3	0.95908	-18.9620	-109.4470	-19.04150	-18.7228
4	0.98801	-55.40540	132.7350	-55.40750	-55.40230

Table - 2*:* **Temperature Profile**

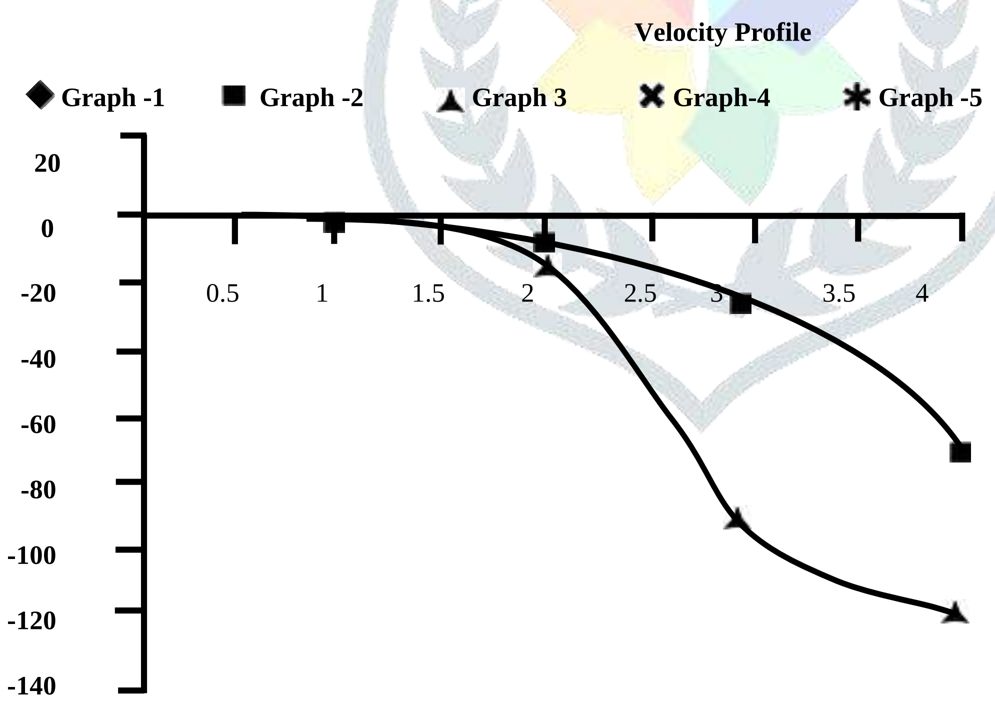
Y	**Graph - 1**	**Graph - 2**	**Graph - 3**	**Graph – 4**	**Graph -5**
0	1.0099	1.0093	1.0092	1.0092	1.0091
0.5	0.7089	0.7931	0.7658	0.8113	0.9609
1.0	0.0655	0.5231	0.7073	0.8627	4.3411
1.5	0.3509	0.4505	1.1226	2.1009	53.5725
2.0	0.2501	0.4400	3.4499	100.9298	797.6523

Table - 3: **Skin Friction Profile**

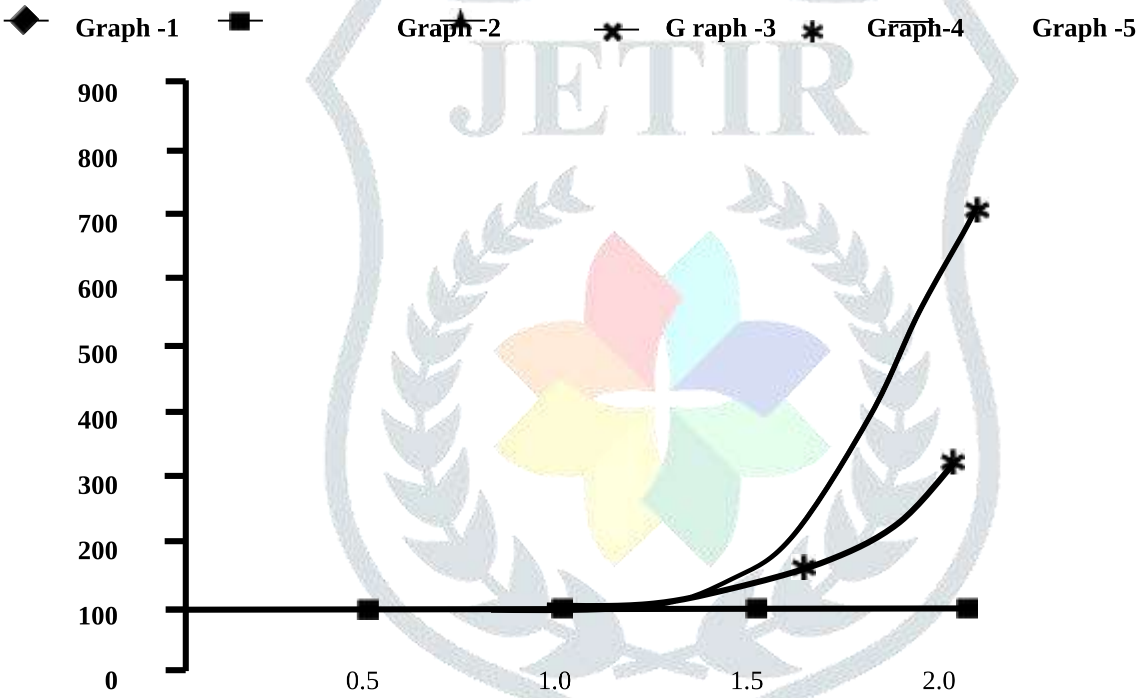
A	Graph – 1	Graph – 2	Graph – 3	Graph – 4
0	1.0663	1.0328	1.0310	1.0258
0.25	1.0618	1.0301	1.0284	1.0233
0.50	1.0573	1.0275	1.0258	1.0213
0.75	1.0529	1.0248	1.0233	1.0187
1.00	1.0474	1.0222	1.0207	1.0160

Table - 4 : Heat Transfer Rate (Nu)

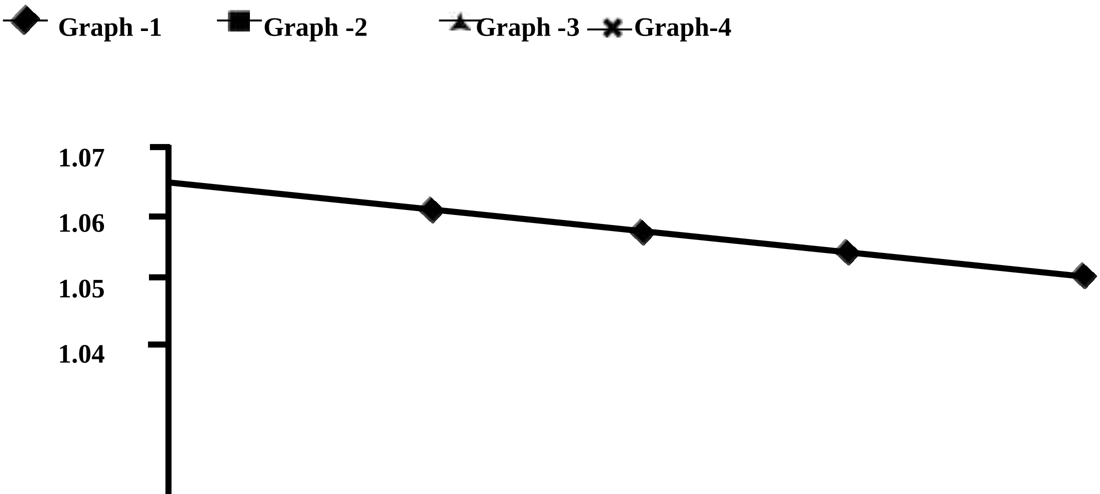
Pr	M = 0	M = 2	M = 4	M = 5	M = 10
	Graph – 1	Graph - 2	Graph – 3	Graph – 4	Graph -5
1	-1.0101	-0.55837	-0.58282	-0.10224	0.85307
2	-2.1024	-2.11636	-2.19032	-2.20412	-2.28989
3	-2.9587	-3.16372	-3.16372	-3.16773	-3.19730
4	-3.9788	-4.26329	-4.26329	-4.2323	-4.33408

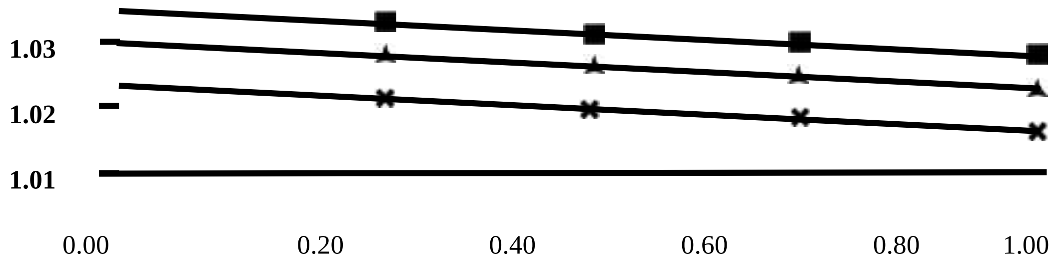


Temperature Profile

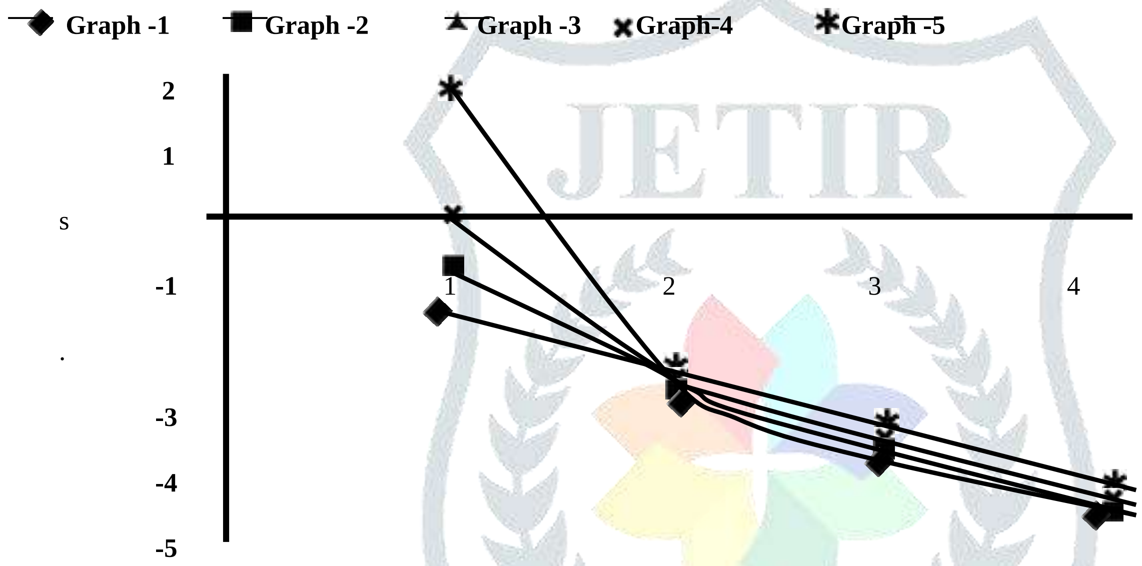


Skin Friction Profile





A Heat Transfer Profile



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