Stacked Ensemble Model in Predicting Stock Trend of Three Major Capitalization Companies of NSE

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Abstract: Predicting the stock trend movement is an enticing prospect to data scientists. Several Classification algorithms such as Decision tree, Naïve Bayesian, Neural network, k Nearest Neighbor and Support Vector machines are being used in predicting the stock trend. In this study, we presented ensemble based method that pick an optimal combination of multiple predictive models using a stack based ensemble method. Experimental study shows that accuracy of stacked ensemble model is performed better than individual predictive models.

***Index Terms* - stacking, ensemble methods, stock trend, bagging, boosting**

I. INTRODUCTION

The field of machine learning technique is concerned with the question of how to construct computer programs that automatically learn and improve the predictability of particular task with experience. The name *Machine learning* was coined in 1959 by Arthur Samuel[1] and developed checkers-playing program was among the world's first successful self-learning programs, and as such a very early demonstration of the fundamental concept of artificial intelligence. Also, Tom[2] presented a widely quoted, more formal definition of the machine algorithm: "A computer program is said to learn from experience *E* with respect to some class of tasks *T* and performance measure *P* if its performance at tasks in *T*, as measured by *P*, improves with experience *E*. Machine learning draws on concepts and results from many fields, including statistics, artificial intelligence, philosophy, information theory, biology, cognitive science, computational complexity, and control theory.

Machine learning generates a lot of buzz because it is applicable across such a wide variety of applications. In recent years many models based on machine learning techniques are proposed, such as neural networks, decision trees, genetic algorithms, rough set theory, support vector machines and hidden Markov model to build a financial prediction.

Moreover ensemble learning has also become a popular machine learning approach, and the series of workshops on Multiple Classifier System founded by Joseph Kittler and Fabio [3] is one of the most popular forums in the field of ensemble learning. Ensemble methods usually produce much improved solutions than a single model, especially in the field of stock forecasting.

Stock trend prediction is a process of predicting the future value of the security. Technical analysts use several algorithms to predict the stock trend such as Decision Tree, Naïve Bayesian, Neural network, Rough Fuzzy etc. Choosing an algorithm is a critical step in the machine learning process, so it's important that it truly fits the use of the problem at hand. In this study we used Stack ensemble method to forecast the stock trend. The idea of ensemble learning is to construct a pool of learners and combine them in a smart way into an overall system, rather than to construct a monolithic system.

II. LITERATURE REVIEW

Ifikhar ul and Khurum[4] in their study uses neural network(ANN) and linear regression(LR) to predict daily gold rate. They developed ANN and LR prediction models with the aid of Rapid Miner and studied the influence of 22 market variables for models prediction. They optimized both the models with Root Mean Square Error (RMSE) as performance measure. Subsequently, they improved their model predictability with ensemble learning, and deep learning schemes.

Kumbhar and Varsha [5] proposed a Ensemble Learning Model which predicts about daily trend of stock market, whether to take long position or short position. They used two-tier framework approach and they extracted various technical indicators based on open, high, low, close and volume in first tier. In the second tier, they applied different classification algorithms on the extracted feature set and combined them through Ensemble methods to achieve higher performance. Parameters like Kappa, Max dd, annualized return and sharp ratio are used as performance components.

Deepika and Saluja[6] compared the performance of three variant models viz: fundamental model, technical indicators model and hybrid models by using SVM, ANN, GA-SVM and GA-ANN algorithms. They compared these models with an evaluation measures using root mean square error (RMSE). They concluded that GA significantly increases the accuracy of ANN and SVM and ANN is well suited for Indian stocks and can help investors and traders maximize their quarterly profits.

Zhang, Li[7] developed an $\varepsilon-$SVM model ($\varepsilon-$ support vector machine) to build a stock price prediction model. They choose (opening prices, high prices, low prices, turnovers) of the 20 trading days from May 19, 2010 to June 18, 2010 as input vectors to SVM and predicted closing price for 20 trading days. Then they combined SVM and ARMA error correction model to predict the closing price. Moreover, after the ARMA error correction model is used, the prediction accuracy is further improved.

Abubakar, Magaji[8] implemented Logic Function on Back-propagation algorithm on the WEKA platform to predict the Nigerian Stock Exchange Market(NSEM). They used 570 days data from January 4, 2010 - April 30, 2012 as input. They concluded that the Back-propagation model of Artificial Neural Network (ANN) performed very well with 99.3% correct classification and thus effectively and efficiently be used for predicting the Nigerian Stock Exchange Market.

In our previous study[9] the role of technical indicators in predicting the stock trend of six major high capitalization companies of NSE are investigated using tree based classifier algorithms such as C5.0, CART and ID3. Classification accuracy of classifier ID3, C5.0 and CART reveals that technical indicators contribute about 85% of accuracy in predicting the stock market behaviour. Performance of ID3, C5.0 and CART are analysed with many predictive measures – predictive accuracy, F-Measure, ROC curve and AUC value.

In another attempt [10] Deep Neural Network with different activation functions are used to predict the stock movement direction. We compare the results of six major capitalization companies' and underlined the fact that Rectifier and Maxout activation function provides better accuracy than Tanh activation function. The proposed model gives maximum accuracy of 87.76% with Maxout activation function in predicting stock trend.

In another study [11] a novel approach is proposed by combining both Support Vector Machine (SVM) and Artificial Neural networks (ANN) in predicting stock trend of six major capitalization companies of NSE, India with relevant feature subsets. SVM technique is introduced to remove irrelevant and redundant variables and subsequently neural network based classification technique is used to forecast stock trends with the reduced feature set. Importance of choosing correct input features using SVM before classification is reflected using evaluation measures accuracy, F-measure, ROC curves and AUC.

In this study, with the above experience, we are using Stack Ensemble method to combine five classification algorithms such as C50, Naïve Bayesian, k Nearest Neighbour, Support Vector Machine and Artificial Neural Network to forecast the stock trend.

III. MATERIALS AND METHODS

In this study we attempt to enhance the forecasting accuracy of stock trend prediction models using Stacked Ensemble method with the support of 22 two technical indicators. Four years trading details from January 2012 to December 2015 for three companies of National Stock Exchange (NSE), India were used for our research study. The values of technical indicators were obtained from intra-trading data - OHLCV (Open, High, Low, Close, Volume). Initially, Decision tree, Naïve Bayesian, Support Vector Machine, Nearest Neighbour, Neural network classification algorithms were used as baseline classifier to forecast the stock trend. Subsequently, a meta-learning approach was experimented using Stacking technique to forecast the stock trend. Experimental results reveal that stacking method yields better forecasting accuracy when compared to individual forecasting models. Furthermore a comparison was made to expose the advantage of using stacking technique over other individual classification algorithms with the aid of various quantitative evaluation measures like accuracy, kappa-statistic, Receiver Operating Characteristic (ROC) and Area under curve(AUC) values.

3.1 Research Data

Three most active companies of NSE, India from three different sectors were used for our research study. The Companies which were used for study includes i) Tata Consultancy Service (TCS), ii) Housing Development Finance Corporation Ltd (HDFC), and iii) Sun Pharmaceutical Industries Ltd. (SUNPHARMA)

3.2Features used

Technical analysis is a methodology for forecasting the upward and downward direction of share prices by analyzing historical trading details of share price and volume of transactions. The most fundamental part of technical analysis is estimating values of technical indicators, which are mathematical calculations involving price, volume, or open interest of a security or contract indices. Technical analysts use the calculated values of these indicators to predict future price movements. There are several technical indicators used by stock analyst to predict the stock trend. The most predominant indicators used in our study are listed in Table 1.

Table 1. Technical Indicators

SMA	Simple Moving Average
EMA	Exponential Moving Average
WMA	Weighted Moving Average
DEMA	Double Exponential Moving Average
VAMA	Volume Adjusted Moving Average
MACD	Moving Average Convergence/Divergence
ADX	Average Directional Movement Index
TDI	Trend Detection Index
Aroon	Aroon Indicator
VHF	Vertical Horizontal Filter
RSI	Relative Strength Index
STOCH	Stochastic Oscillator
SMI	Stochastic Momentum Index
WPR	William%R– Williams Percentage Range
CMO	Chande's Momentum Oscillator
CCI	Commodity Channel Index
BBands	Bollinger Bands
DC	Donchain channel
ATR	Average True Range
CMF	Chaikin Money Flow

OBV	On Balance Volume
MFI	Money Flow Index

3.3Classification Algorithms

Our research study focused on the studying optimal combination of multiple predictive models using a stack based ensemble method. In this study, five classification algorithms such as decision tree, naïve bayesian, neural network, k nearest neighbor and support vector machines are used as base line classifier in predicting the stock trend.

3.3.1 Decision Tree

Decision tree methodology[15] is used to establish classification systems based on multiple covariates or for developing prediction algorithms for a target variable. The algorithm is non-parametric and can efficiently deal with large, complicated datasets without imposing a complicated parametric structure. Decision Tree employs a top-down, greedy search through the space of possible branches with no backtracking [15]. It uses entropy and information gain to construct a decision tree. Quinlan in 1980 developed a decision tree algorithm known as ID3 (Iterative Dichotomiser). Then he presented C4.5, which was the successor of ID3. In this algorithm, there is no backtracking; the trees are constructed in a top-down recursive divide-and-conquer manner. Quinlan created C5.0 which offers a number of improvements on C4.5 in terms of speed, memory usage, smaller decision trees, boosting, weighting and winnowing.

3.3.2 Naïve Bayesian

The Naive Bayesian classifier[18] is based on Bayes' theorem with independence assumptions between predictors [18]. A Naive Bayesian model is easy to build, with no complicated iterative parameter estimation which makes it particularly useful for very large datasets. Despite its simplicity, the Naive Bayesian classifier often does surprisingly well and is widely used because it often outperforms more sophisticated classification methods. Naive Bayesian classifier assumes that the effect of the value of a predictor (x) on a given class (c) is independent of the values of other predictors. This assumption is called class conditional independence.

3.3.3 Support Vector Machine

SVM is a supervised learning technique that performs classification tasks by constructing hyper planes in a multidimensional space that separates cases of different class labels. Vapnik and Chervonenkis[19] originally invented support vector machine. In early stages algorithm was developed only for linear classifiers. Later in 1992, Vapnik, Boser & Guyon[19] supplements a way for building a non-linear classifier. Vapnik suggested creating non-linear classifiers by applying the kernel trick to maximum-margin hyper planes. In non-linear SVM Classification, data points are plotted in a higher dimensional space. Some standard kernels are linear kernel, polynomial kernel, RBF kernel and sigmoid kernel. In these popular kernel functions, RBF is the main kernel function because of following reasons i) The RBF kernel nonlinearly maps samples into a higher dimensional space unlike to linear kernel ii) The RBF kernel has less hyper parameters than the polynomial kernel iii) The RBF kernel has less numerical difficulties.

3.3.4 Neural Network

Neural Networks are considered a robust classifier. A neural network is a machine learning algorithm based on the model of a human neuron. An Artificial Neural Network is an information processing technique. It works like the way human brain processes information. ANN includes a large number of connected processing units that work together to process information [20]. They also generate meaningful results from it. Classification is not only the application of neural network. ANN finds its application in regression of continuous target attributes. Neural networks find great application in data mining used in sectors. For example economics, forensics, pattern recognition. It can be also used for data classification in a large amount of data after careful training.

3.3.5 Nearest Neighbour Classification

K-Nearest Neighbor (KNN) is a simple algorithm that stores all available cases and classifies new cases based on a similarity measure using various distance functions[21]. KNN has been used in statistical estimation and pattern recognition already in the beginning of 1970's as a non-parametric technique. A case is classified by a majority vote of its neighbors, with the case being assigned to the class most common amongst its K nearest neighbors measured by a distance function.

3.3.6 Ensemble Modelling

Ensemble learning involves combining multiple predictions derived by different techniques in order to create a stronger overall prediction. Some commonly used ensemble learning techniques are

- Bagging
- Boosting
- Stacked Ensemble

Bagging is also known Bootstrap Aggregation [12]. It creates samples from the dataset with replacement; that is, any instance that is already selected may repeat in the same sample many times. Training data is increased by bootstrap; each created and then used a classifier model. The final prediction is the average of all prediction models. The most popular bagging algorithm used by data scientists, is random forest.

Boosting[12] is a process in which multiple weak classifiers are trained and their results are combined to create a strong classifier. Boosting algorithms are primarily used to prevent under fitting (high bias) and over fitting (high variance) of the classification model. AdaBoost, XGBoost are popular boosting algorithms used by data scientists.

Stacking [13] also called stacked generalization involves training a learning algorithm to combine the predictions of several other learning algorithms.

The steps involved in stacked ensemble technique are

- Set up the ensemble
  - Specify a list of N different base algorithms
  - Specify a meta-learning algorithm
- Train the ensemble
  - Train each of the n different base algorithms on the training set
  - Perform k-fold cross-validation on each of these learners and collect the cross-validated predicted values from each of the n algorithms
  - The M cross-validated predicted values from each of the n algorithms can be combined to form new M X N matrix. This matrix, along with the original response vector, is called the level-one data. (N=number of rows in the training set)
  - Train the meta-learning algorithm on the level-one data. The "ensemble models" consists of the M base learning models and the meta-learning model, which can then be used to generate predictions on a test set.
- Predict on new data
  - To generate ensemble predictions, first generate predictions from the base learners.
  - Feed those predictions into the meta-learner to generate the ensemble prediction. Stacking is composed of two phases.

Wolpert's stacked generalization [14] is illustrated in Fig. 1. It first creates Tier-1 classifiers, $C_1, \ldots, C_T$, based on a cross-validation partition of the training data, ie. the entire training dataset is divided into B blocks. Then each Tier-1 classifier is first trained on B-1 blocks of the training data. Each classifier is then evaluated on the Bth block which is not seen during training. The output of the base classifiers on their pseudo-training blocks constitute the training data for the Tier-2 (meta) classifier, which effectively serves as the combination rule for the Tier-1 classifiers. Meta-classifier is not trained on the original feature space, but rather on the decision space of Tier-1 classifiers. Once the meta-classifier is trained, all Tier-1 classifiers (each of which has been trained B times on overlapping subsets of the original training data) are discarded, and each is retrained on the combined entire training data. The stacked generalization model is then ready to evaluate previously unseen field data.

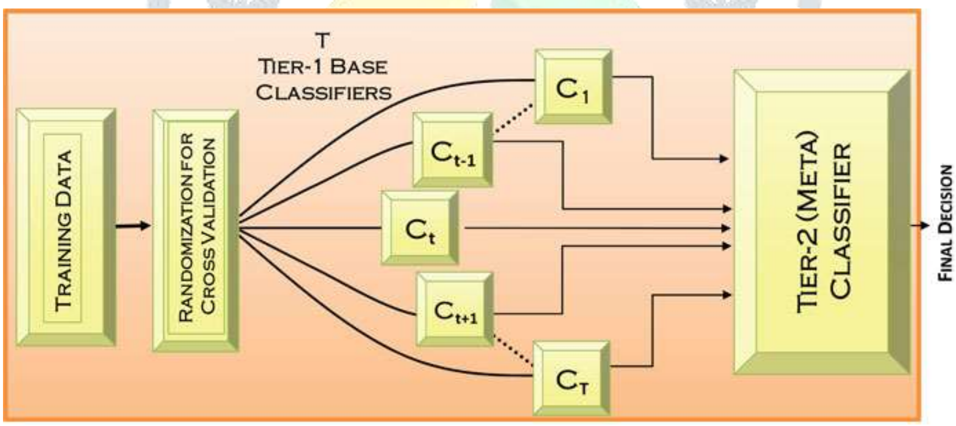

Figure 1: Stacking Classifier

IV. BUILDING THE MODEL

The daily stock price data of three major capitalization companies from three different sectors were used in this study. Four years data from January 2012 to December 2015 were used for determining the stock trend. Dataset was partitioned into70% as training set and 30% as testing set. Technical indicators were derived from OHLCV data using TTR package in R[17]. Decision variable for predicting the stock trend was evaluated using the difference of *open price* and *close price*. Experimental study was carried out in two phases.

In the first phase, five classification algorithms C50, Naive Bayesian, Nearest Neighbour, Support Vector Machine and Neural network are used as base classifiers for predicting the stock trend. Five-fold cross validation was carried out on each of the base classifier and cross-validated predicted values are collected from each of the five classification algorithms. A level-one vector was generated by combining the cross-validated predicted output of five base classifiers and the response vector.

In the second phase, Gradient Boosting Machine (GBM) was used as meta-learner on level one data. Prediction from the five base classifiers and meta-classifier was predicted on the test set. GBM yields better accuracy than individual classification algorithms. Furthermore we proceed to repeat the meta-learning with the same five different base classifier algorithms. A comparison was made among six different classification algorithms that were used as meta-classifier and the best among them is chosen to predict the stock trend. The forecast accuracy of base classifier and meta-classifier was compared and depicted using evaluation measures accuracy, kappa, ROC and AUC. Finally the probability of the stock trend of meta-classifier is shown with the help of decision tree.

V. RESULTS AND DISCUSSION

First we build the model using five classifiers- C5.0, Naïve Bayesian(NB), Nearest Neighbour(KNN), Neural Network(NNET) and Support Vector Machine(SVM). The stock trend is forecasted for the three companies. Fig 2 reveals the evaluation measures of base classifiers using dot plot.

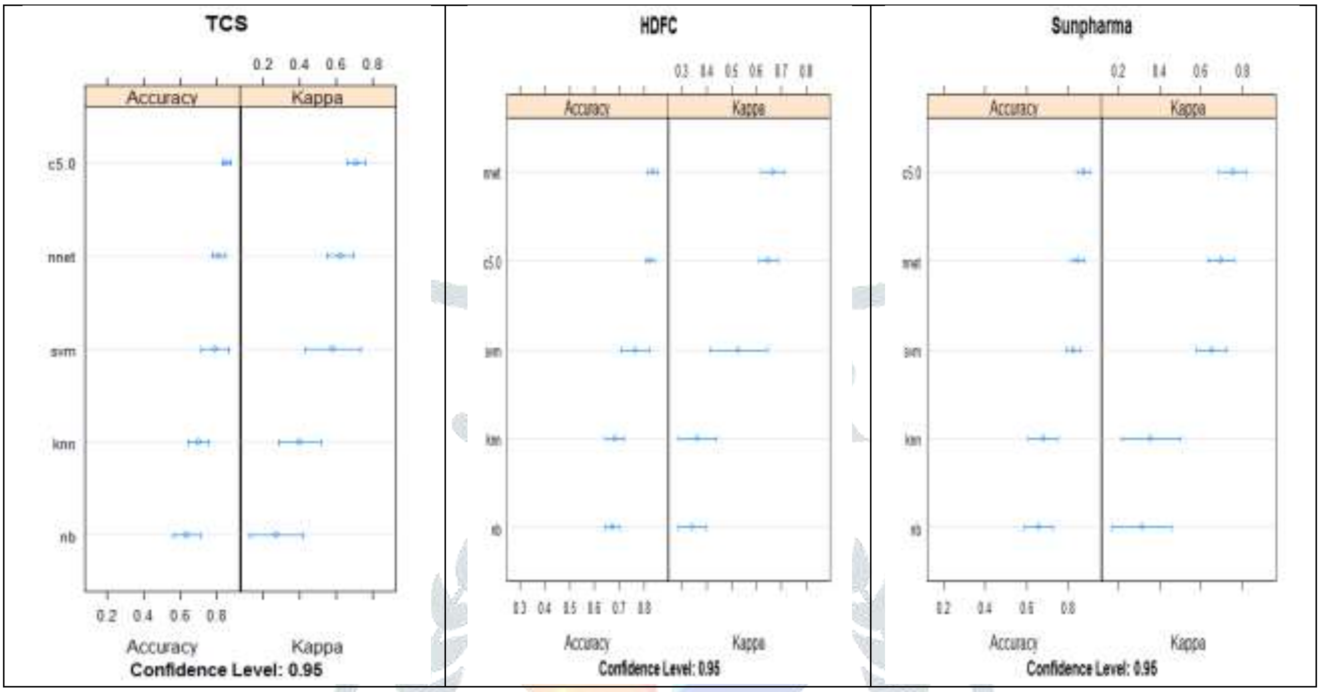

Figure 2: Dot plot of accuracy and kappa kappa for TCS, HDFC and SUNPHARMA

The accuracy of base classifiers is shown in Table 2. From the table it is evident that C50 provides accuracy 85.5% and 87.6% for TCS and SUNPHARMA and neural network gives 83.3% accuracy for HDFC.

Table 2: Tier -1 Accuracy of Base Classifier

Script	C5.0	NB	SVM	KNN	NNET
TCS	85.5	63.6	79.1	70	81.25
HDFC	82.3%	67%	76.4%	68.2%	83.3%
SUNPHARMA	87.6%	65.9%	82.6%	68%	84.8%

In the second phase level one matrix is constructed with the prediction output of five base classifiers and the response vector. Then the level-one data is trained with meta-classifier using GBM. Furthermore the meta-learning is repeated with same five base classifier algorithms. The prediction accuracy of six meta-classifiers is shown in Table 3.

Table 3: Tier-2 Accuracy of meta-classifier

Script	C50	NB	SVM	KNN	NNET	GBM
TCS	88.6	87.3	89%	86.6%	89%	88.6
HDFC	87.6	87.3	88%	87.6	87.6%	87.6%
SUNPHARMA	87.6%	84%	88.6%	87.6%	88.3%	88.3%

For the script TCS, NET and SVM meta-classifier gives an accuracy of 89% which is more than C5.0 base classifier accuracy 85.5%. And for HDFC the base learner classification accuracy is 83.3% using NNET and meta-learner classification accuracy is 88% using SVM. In SUNPHARMA, NB base classifier accuracy is 65.9% but when NB used as meta-classifier its accuracy is 84%. It is apparent from the study that stack ensemble improves forecasting accuracy. There is an increase of nearly 4% of prediction accuracy for TCS and HDFC and for SUNPHARMA 1% increase in forecasting accuracy is visible. Experimental study shows a clear picture that the accuracy of SUNPHARMA is increased by 1% using stacked ensemble technique. Among these three companies SVM plays major role as meta-classifier when compared to other classification algorithm by yielding higher classification accuracy. The difference in accuracy using stacked ensemble technique and base classifiers is depicted in Fig. 3

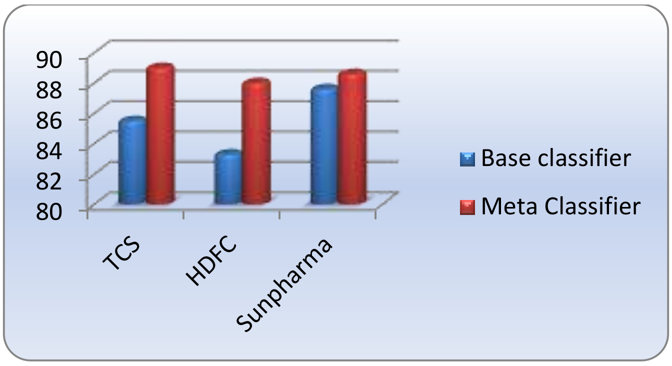

Figure 3: Classification Accuracy of Base and Meta Classifier

Receiver Operating Characteristics (ROC)[16] graphical plot is another performance metrics that are used to study the efficacy of different classifier models for two class problem. The best possible prediction method would yield a point in the upper left corner or in the co-ordinates (0,1) of the ROC space, representing 100% sensitivity (no false negatives) and 100% specificity (no false positives). The point (0, 1) is also called a perfect classification. Area under curve (AUC)[16] is another evaluation metrics which is measured by using the trapezoidal rule and high AUC value among them indicates the corresponding classification method is close to (0,1). The ROC graph and AUC values of five different base classifier and meta classifier stack ensemble technique is shown in Fig 4.

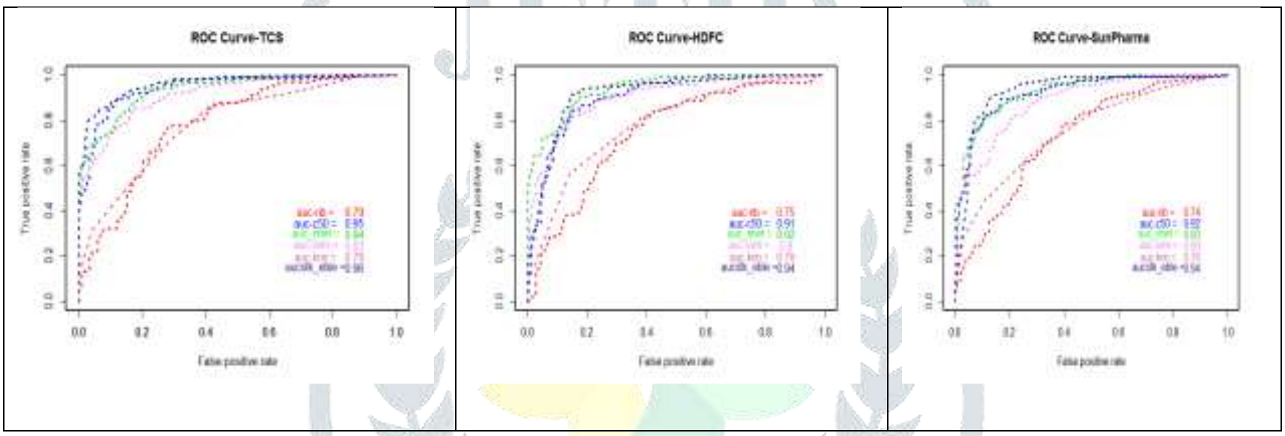

Figure 4: Receiver Operating Characteristics(ROC) for C50, NB, KNN, SVM, NNET and Stacked Ensemble

Area under curve for the five different classification algorithms and stacked ensemble technique is shown in Table 4. For all the three companies AUC value is 0.94 and above which indicates the classification using stacked ensemble gives perfect classification. Fig 5 depicts AUC of five base classifiers and the stacked ensemble model.

Table 4: Area under curve for C50, NB, KNN, SVM, NNET and Stacked Ensemble

Script	NB	C50	NNET	SVM	KNN	STACK
TCS	0.79	0.95	0.94	0.92	0.79	0.96
HDFC	0.75	0.91	0.92	0.9	0.79	0.94
SUNPHARMA	0.74	0.92	0.93	0.89	0.76	0.94

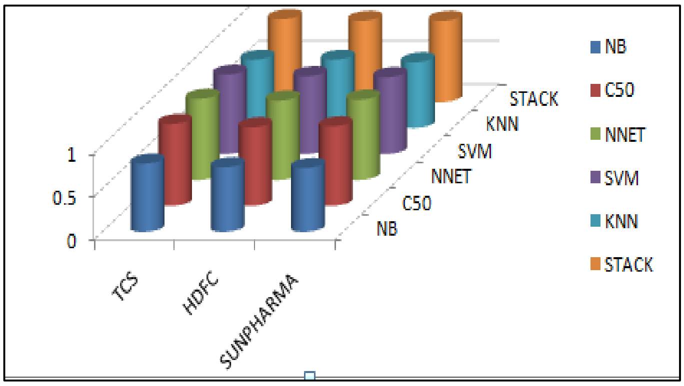

Figure 5: Area under curve of five classification algorithm and Stacked ensemble

The prediction probability of meta-classifier is depicted as decision tree as in Fig 6. From the decision tree we can see that the nearest neighbor (KNN) was omitted when stacking which has least accuracy of 86.6% as shown in Table 3.

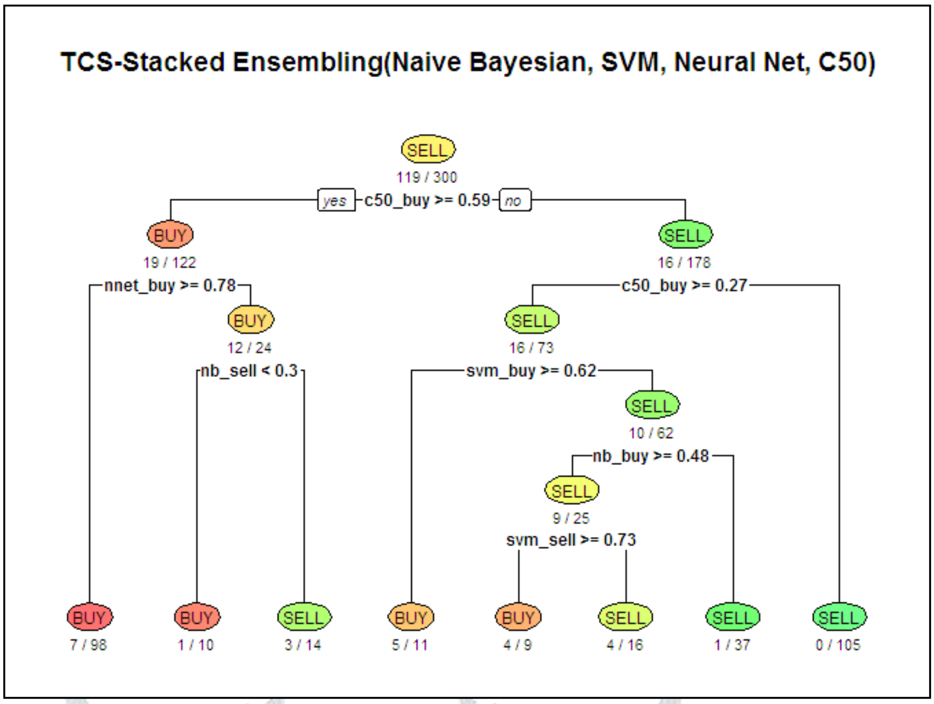

Figure 6: Prediction probabilities using stacked ensemble

VI CONCLUSION

The stock prediction algorithm using stacked ensemble model was presented in this study. In our previous study[9][10][11] we experimented with various classification algorithms using decision tree, Naive Bayesian, SVM, neural network etc., Different classification technique suits for each types of stock data in forecasting the stock trend in providing better results. To overcome this drawback a combination of classification algorithms is used in this study using stacked ensemble model. Experimental results also shows that accuracy in predicting the stock trend is increased using stacking when compared to individual classification algorithm. The investor can make use of stacked ensemble technique with high confidence to make optimal decision in deciding the market trend (bull/bear).

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