ON MIXED INTEGER BO-OBJECTIVE PROGRAMMINGPROBLEMS

Manoj Kumar Rana,
University of Delhi.

ABSTRACT: This article deals with Mixed Integer Biobjective Programs. The biobjective programming problem is converted into a single level problem by making use of Karush Kuhn Tucker conditions which convert the lower(second) level problem into constraints thereby simplifying the biobjective program into a single level program which is then solved using LINGO 15,0 software. The proposed approach makes the problem handling and solving much simpler and user friendly.

Keywords: Biobjective programming; Positive Definiteness ; Quadratic Programming; Integer Proghramming.

1. *INTRODUCTION:*

This article focuses on non-linear multilevel programs.The algorithms for solving general mathematical programming problems approach systematically to a local optimal(maximum/minimum) solution. Under appropriate assumptions, a local optimai solution can be shown to be global optima. For instance, when the constraint set is a convex polyhedron,, the objective function is convex (concave), the local minimum (maximum) is also global minimum (maximum) [5].Single objective decision making method reflect an earlier and simpler era. The world has become more complex as oneenters the information age. One can find that almost everyimportant real world problem involves more than one objective. Since the goals or objectives might be conflicting with each other, no single optimal solution can be found and the optimization problem becomes finding the best compromise solutions [5].The general Multi-Objective Programming problem [9] isdefined as

(MPP) L1 max Z_1 (X)

L2 max Z_2 (X)

Li max Z_i(X)

:

:

Lr max Z_r (X)

subject to

$A_{i1}\bar{X}_1 + A_{i2}\bar{X}_2 + \ldots + A_{it}\bar{X}_t \ (\leq, =, \geq) \ b_i,$ 1≤i≤ r

$\bar{X}_1 \geq 0, \bar{X}_2 \geq 0 \ldots \bar{X}_t \geq 0$ and X_1,X_3intgers where Z_i $(1 \leq i \leq r)$ denotes the i-th level objective function, Z_1 being quadratic (quasiconcave), and $Z_i(2 \leq i \leq r)$ being linear fractional; the i-th level decision maker can have more than one decision maker ;

A bi-objective problem is a special class of multilevel programs with two decision making levels where $Z_1(X)$ and $Z_2(X)$ are the first and the second objective
functions respectively and S is constraint set.

1.1 Quadratic Programming Problem

A quadratic programming problem (QPP) is an optimization problem wherein one either minimizes or maximizes a quadratic objective function of a finite number of decision variables subject to finite number of linear inequality and/or equality
constraints.
Without loss of generality, we assume that the associated matrix is symmetric matrix
and so it is free to replace the matrix by the symmetric matrix
Henceforth, the matrix considered is symmetric.
If constant term exists, it is dropped from the model since it plays no role in optimization step. The decision variables are denoted by the n-dimensional column vector and the constraints are defined by an matrix G and an n-dimensional column vector b of right hand side.

1.2. Statement of the Problem

Different researchers made use of different methods for solving BOQP problems, but the method that they used is so lengthy and in addition to this, most of them use linearization technique, this creates an approximation error. So, the researcher was trying to find out if can we solve definite quadratic bi-objective programming problem by Karush Kuhn Tucker conditions easily.

1.3 *Significance of the Study*

This study will give a direction towards definite quadratic bi-objective programming problems to anybody or any organization to solve their real life problems (or whatever any problem) which are modeled as definite quadratic biobjective programming with constraint region determined by linear constraints.

1.4. Objective of the Study

The general objective of this study is to solve the definite BOQP problem with constraint region determined by linear constraints by KKT conditions.
The study is intended to explore the following specific objectives
a) discuss definite quadratic bi-objective programming problem;
b) discuss KKT conditions, and to solve the definite BOQP.

A matrix Q (symmetric) is said to be definite quadratic if it satisfies one of the following concepts
i. positive definite if $X^tQX > 0$ for all $X \neq 0$,
ii. positive semi definite if $X^tQX \geq 0$ for all $X \neq 0$
iii. negative definite if $X^tQX < 0$ for all $X \neq 0$,
iv. negative semi definite if $X^tQX \leq 0$ for all $X \neq 0$,

Instead of the above four concepts, we can use the following criteria to determine the definiteness of the quadratic program. i.e.
i. positive definite if and only if all principal minors are strictly greater than zero,
ii. negative definite if and only if all principal minors alternate in sign starting with negative one
iii. positive semi definite if and only if all principal minors are greater than or equal to zero,
iv. negative semi definite if and only if all principal minors of odd degree are less than or equal to zero, and all principal minors of even degree are greater than or equal to zero.

A bi-objective programming problem, which both the objective functions are definite quadratic is called definite quadratic bi-objective program.
A bi-objective programming problem, which both the objective functions are definite quadratic is called definite quadratic bi-objective program. Consider the following definite quadratic bi-objective programming problem.

$$\max_{x,y} Q(x,y) = (p_1x + q_1y) + \frac{1}{2} X^tQX$$

where y solves

$$\max_{y} f_2(x,y) = (p_1x + q_1y) + X^tQX \text{ for a given x}$$

subject to $A_1x + A_2y \leq b$

$x, y \geq 0$

SOLUTION METHODOLOGY

- The biobjective programming problem under consideration is converted into a single level program by making use of KKT conditions and the resulting problem which is a single level program is solved using LINGO software.

References

[1] Benoit C. Chachuat. 2007. Nonlinear and dynamic optimization, IC-32: Winter Semister.
[2] Etoa, J. B. E. 2011. Solving quadratic convex bi-level programming problems using a smooth method, *Applied Mathematics and Computation*, 217 (15): 6680-6690.
[3] Hosseini, E. and Isa Nakhai, I, K. 2014. Taylor approach for solving nonlinear bi-level programming, 3 (11): 2322-5157.
[4] Jin Hyuk Jung. 2008. Adaptive constraint reduction for convex quadratic programming and training support vector machines, University of Maryland.
[5] Kalyanmoy Deb. *Multi-objective optimization using evolutionary algorithm*, Indian Institute of Technology,Kanpur, India, 48-53.
[6] Liu, G. P., Yang, J. B. and Whidborne, J. F. 2003. *Multiobjective Optimization and Control*, *Research Studies Press LTD. Baldock*, Hertfordshire, England, 73-82.
[7] Maria M. Seron. 2004. Optimality condition, Center for Complex Dynamics Systems and Control, University of Newcastle, Australia.
[8] Rana Manoj Kumar. 2016. An Approach for Multilevel Programming Problems, *Journal of Emerging Technology and Innovative Research* , Volume 3, Issue 6 :277-280.
[9] Rana Manoj Kumar. 2017. A Solution Approach for Multilevel Programming Problems *Journal of Emerging Technology and Innovative Research,* Volume 4, Issue 3:319-322.
[10] Wang, Y. P. and Li, H. C. 2011. Agenetic algorithm forsolving linear-quadratic programming problems, *Advances Materials Research*, 186: 626-630.