LAMINAR INCOMPRESSIBLE BOUNDARY LAYER ALONG A POROUS WALL IN DIVERGENT CHANNEL

Shanker Kumar[1] and Jalaj Kumar Kashyap[2]

[1]Dept. of Mathematics, T.M. Bhagalpur University, Bhagalpur,
[2]Dept. of Mathematics, T.M. Bhagalpur University, Bhagalpur.

ABSTRACT : In this Paper we have study the momentum integral equation, the kinetic energy integral equation and the wall compability condition in non- dimensional form have been derived and used to find the numerical solution for the flow of a viscous incompressible fluid in the laminar boundary layer along a wall in divergent channel. The parameter of the pohlhansen profile at disposal. The Range-kutta method has been used for the calculation at some initia points.

Keywords : Non-dimensional, divengent channel, momentum integral equation, Kinetic energy integral equation, wall compatibility condition, Runge-Kutta method.

INTRODUCTION

Schlichting was given one parameter system of velocity profile especially suited to the approximate calculation of the boundary layer with suchtion. Pohlhausen calculated the point of separation for the incompressible laminar boundary layer in a divergent channel with straight solid walls. Inpersent case we find the numerical solution for the flow of a viscous incompressible fluid along a porous wall in divergent channel.

NOTATIONS

x = radial distance from the source along the wall.

a = distance of the edge of wall from the source.

$\bar{x} = \frac{x}{a} =$ non-dimensional radial distance.

y = distance normal to the wall.

u = velocity in the boundary layer in x-direction.

v = velocity in y-direction

$U(x)$ = potential velocity.

U_0 = entrance velocity at the edge of the wall.

$\bar{U}(x) = \frac{U(x)}{U_0}$

DIVERGENT FLOW: SOLUTION WITH THE AID OF POHLHAUSEN'S PROFILE

In case of laminar incompressible flow indivergent channel potential velocity $U(x) = U_0 \frac{a}{x}$.

Let $\bar{x} = \frac{x}{a}$ the non dimensional radial distance

$$\bar{U} = \frac{U}{U_0} = \frac{U_0 \frac{x}{a}}{U_0} = \frac{a}{x} = \frac{1}{x} \tag{1}$$

$$t^* = \left(\frac{\theta}{a}\right)^2 = \frac{U_0 a}{\surd} \tag{2}$$

$$\wedge = \frac{\theta}{\surd}\frac{dU}{dx} = t^* \frac{d\bar{U}}{dx} = \frac{-t^*}{\bar{x}^2} \tag{3}$$

$$\bar{v}_s = \frac{v_s}{U_0}\sqrt{\frac{U_0 a}{v}} \tag{4}$$

and $$\lambda = \frac{v_s \theta}{v} = t^{*\frac{1}{2}} \bar{v}_s, \quad \lambda < 0, \text{ suction} \qquad \lambda > 0, \text{ injection} \tag{5}$$

The momentum integral equation in dimension less form

$$\frac{dt^*}{d\bar{x}} = \frac{2}{\bar{U}}\left[I - (2+H)\wedge + \lambda\right] \tag{6}$$

The Kinetic energy integral equation in dimension less. Form

$$\frac{dH_\in}{\partial \bar{x}} = \frac{1}{t^* \bar{U}}\left[2D - H_\in \left\{I - (H-1)\wedge + \lambda\right\}\lambda\right] \tag{7}$$

and wall Compatibility condition

$$m = -\wedge + \lambda I \tag{8}$$

where $m = \dfrac{\theta^2}{U} + \left(\dfrac{\partial^2 u}{\partial y^2}\right)_{y=\infty}$ (8)

Now we introduce these dimensionless quantity momentum integral equation (6), Kinetic energy equation (7) and wall compatibility condition (8) for the boundary layer in a divergent channel with porous wall reduce to

$$\frac{dt^*}{d\bar{x}} = f\left(\bar{x}, H_\in, t^*\right) = 2\bar{x}\left[I + \frac{(2+H)t^*}{\bar{x}^2} + t^{*\frac{1}{2}}\bar{v}_s\right] \tag{9}$$

$$\frac{dH_\in}{d\bar{x}} = g\left(\bar{x}, H_\in, t^*\right) = \frac{\bar{x}}{t^*}\left[2D - H_\in\left\{I + \frac{(H-1)t^*}{\bar{x}^2} + t^{*\frac{1}{2}}\bar{v}_s\right\} + t^{*\frac{1}{2}}\bar{v}_s\right] \tag{10}$$

and $m = \dfrac{t^*}{\bar{x}^2} + t^{*\frac{1}{2}}\bar{v}_s I$ (11)

At the starting point the boundary layer parameters are

$\bar{x} = 1, \bar{U} = 1, t^* = 0, \lambda = 0, \wedge = 0$

Hence $m = 0$ from equation (11), we have

$H_\in = 1.571$, $H = 2.554$, $I = 0.235$, $D = 0.1745$.

NUMERICAL SOLUTION OF THE MOMENTUM AND KINETIC ENERGY

The momentum integral equation (9) and Kinetic energy equation (10) have been solved with the help of compatibility condition (10) by applying the Runge-Kutta Method.

The Runge-Kutta method for the two ordinary first order differential equation is

$$K_1 = f\left(\bar{x}_0, t_0^*, H_{\in 0}\right)\Delta\bar{x}$$

$$K_2 = f\left(\bar{x}_0 + \frac{\Delta\bar{x}}{2}, t_0^* + \frac{K_1}{2}, H_{\in 0} + \frac{I_1}{2}\right)\Delta\bar{x}$$

$$K_3 = f\left(\bar{x}_0 + \frac{\Delta\bar{x}}{2}, t_0^* + \frac{K_2}{2}, H_{\in 0} + \frac{I_2}{2}\right)\Delta\bar{x}$$

$$K_4 = f\left(\bar{x}_0 + \Delta\bar{x}, t_0^* + K_3, H_{\in 0} + I_3\right)\Delta\bar{x}$$

$$K_5 = \frac{1}{6}\left(K_1 + 2K_2 + 2K_3 + K_4\right)$$

The calculation produces step-by-step up to the point of separation.

RESULT

Calculation have been made for three different constant value of $\bar{v}_s = 0, -0.2, -0.3$ when solved with the aid of Pohlhausen's profile. For, $\bar{v}_s = 0$ the equation reduce to the equations for the solid wall problem. The point of separation for $\bar{v}_s = 0$ is found to be at $\bar{x} = 1.1612$ which is in close agreement with the value $\bar{x} = 1.2130$ obtained by Pohlhausen. In this case point of separation shifts further down steam.

Methods	Suction parameter $\bar{v}_s$	Points of separation $\bar{x}$
1. Pohlhausen	0.0	1.2130
2. Present method with the	0.0	1.1612
aid of Pohlhausen's profile	-0.2	1.1947
	-0.3	1.2115

From above we observe that with increasing rate of suction parameter $\bar{v}_s$ the point of separation moves further down stream.

References

Ahmad, S. and Pop, I (2010). Mixed convechtion Boundary layer flow from a vertical flate plate Embedded in a porous medium. International communication in Heat and mass transfer. 37 (2010), 987-991.

Devi Anjali, S.P and Andrew, J. (2011). Laminar Boundary layer flow of nanofluid over a flat plate. International Journal of applied Mathematics, p. 52-71.

Choudhary R.C. (1966). Steady laminar boundary layer of a viscous in compressible fluid in convergent channel. Ind. Acad. Sci, Lxlll, p. 91-104.

Awad, F.G. Sibanda and Khidir, A. (2013). Thermo diffusion effect on magnet flued over a stretching sheet. Ind. Jour. of Heat and Mass Transfer, 105:508-521.