M/M/C/K QUEUEING PROCESS AND ITS DESIGNING SELECTION OF ACCEPTANCE SAMPLING PLANS

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Abstract : A Queueing system comprises of batches or lots arriving for service, waiting for service, and then leaving the system after service. In Quality Control literature, the group of lots/batches arrive for testing, wait for testing, and then leave the system after acceptance or rejection of the entire batches or lots. Hence the arrival of incoming lots can be considered as Poisson input and the service time can be considered as exponential output. In this article a new designing algorithm is given for the selection of acceptance sampling plans from a M/M/C/K Queuing process. Lots come in a Queueing policy and waits for the service. Here service is meant only for testing the lot and making the decision about the lot. A novel algorithm is given for sentencing the number of lots in a production process. The sampling plans are developed based on the maximum number of arrivals (K) in the Queueing system. The parameters of the Queueing policy are derived and hence the parameters of the sampling plans are determined. Tables are constructed to select the appropriate sampling plans.

***IndexTerms* - Expected number of arrivals, M/M/C/K Production process, Sampling Plans, Traffic intensity.**

I. INTRODUCTION

A Queueing system comprises of batches or lots arriving for service, waiting for service, and then leaving the system after service. In Quality Control literature, the group of lots/batches arrive for testing, wait for testing, and then leave the system after acceptance or rejection of the entire batches or lots. Hence the arrival of incoming lots can be considered as Poisson input and the service time can be considered as exponential output. Many authors have contributed towards Queueing theory. But the literature is not sufficient in applying the Queueing theory into Quality control practice. Hence an attempt has been made to develop quality and efficiency domain for a production process with a M/M/C/K Queueing system. The main advantage of combining the Queueing theory and Quality Control concept is the determination of the minimum number of servers and hence the decision on the lots whether to accept or reject it. This is a novel method which combines the Queueing and Quality control theory to control the product quality. In managing a production process with queuing system, the important and desirable event is to determine the minimum number of Quality control inspectors (C). The main objective of the paper is to find the number c such that it balances the quality and cost of inspection. In a steady state queuing system, $C = r + \Delta$, where $r = \frac{\lambda}{\mu}$, Δ = number of additional inspectors or servers needed. The constant c mainly depends on the offered load r. Hence one has to find the smallest c such that $1 - W_q(0) \le \alpha$.Many authors have contributed towards Queuing theory and Sampling plans. Donald Gross et.al(2008) have published a collection of Queuing theory and models in their text Fundamentals of Queuing theory. Huei-Wen Ferng and Jin-Fu Chang (2000) have discussed the departure process of discrete-time Queueing systems with Markovian type inputs. Murali and Ramaswami (2014) have contribute *A* Numerical frame work for Solving Discrete time Queues for the specific model D/G(A,N/A,Q)/ $1/Q_{max}$. Devaarul S and Santi T (2014) have contributed towards Group Sampling Plans for a special type of production process. Ramaswami (2005) has studied Bulk Queueing models for various measures . Martin J. Beckmann(1994) have explained the optimal number of servers in a Queueing system. Dieter Fiems And Herwig Bruneel (2002) have discussed Discrete-Time Queueing System with Timed Vacations. Hideaki Takagi and Kin K. Leung(1994) have published discrete-time Queueing system with time-limited service. Igor N. Kovalenko (1993) have contribute towards the Rare events in queueing systems. Kiyoshi Muto et.al (1995) discussed the new type of counting for M/M/C Queueing systems. Schilling (1967) has constructed tables by unity value approach in order to determine the various parameters of the sampling plans and their associated measures of the plans.

II. Definition and Algorithm for clearing the lots in an M/M/C/K system

Determine the parameters n, c and K by using Operating characteristic and Non-Delayed functions given in

1. Determine the truncation number K using the measures of Queuing system from the equation (1) thru (3) given in the designing procedure.
2. Draw a random sample of size n from a lot or a batch.
3. Find the number of defective items in the sample. Let it be D.
4. If D $\leq d$ in each lot accept the lots in the system.
5. If D $> d$ reject the lot and screen the entire lot.
6. Proceed steps 2 to 4 as long as the number of lots reached the truncation level K in the Queuing system.
7. After Kth lot is inspected, the lots in the entire system is either accepted or rejected. The rejected lots are scrapped or sent to rework. Now proceed from step (1) onwards.

Using the above algorithm the operating characteristic function is derived for Kth level truncation and is given in the following theorem. The main advantages of the above algorithm are the entire lot in a system is sentenced based on the expected number of arrivals of the lot and the sampling plans are used for K number of lots in succession. The number of lots in the system is decided based upon the traffic intensity. Even though several literatures are available on Group Sampling Plans, this is new method in combining the Queueing theory models and acceptance sampling plans.

2.1Theorem:

The Probability of acceptance of an M/M/C/K model is defined to be a function of np, c and K. $Pa\ (p) = \left\{\sum_{x=0}^{d} \frac{e^{-np}\,(np)^x}{x!}\right\}^K$

Proof:

Let 'd' be the number of defectives found in each lots.Let 'n' be the sample size of each lot.Let 'D' be the acceptance constant. Then by using the given algorithm,A single lot is accepted if d ≤ D. The entire system is accepted if d≤ D for K such lots in the system.A lot can be accepted if either the number of defectives is zero or one or two ... or D. Therefore the events P(d=0) or P(d=1).... are mutually exclusive events. Hence the Probability of the acceptance can be written using addition theorem on Probability.

$$Pa\ (p) = \{P(d=0)\ or\ P(d=1)\ or \ldots\ldots P(d=D)\} \ldots\ldots\ldots \text{ up to K lots}$$

$$= \{P(d \leq D)\}^K$$

Therefore $Pa\ (p) = \left\{\sum_{x=0}^{d} \frac{e^{-np}\,(np)^x}{x!}\right\}^K$ Where, K = Truncation level in the system.

For a M/M/C/K model, the parameter K can be determined such that

$$W = \{r + \left(\frac{r^C \rho}{C!\,(1-\rho)^2}\right) p_0\ [\,1 - \rho^{k-C+1} - (1-\rho)(k-C+1)\,\rho^{k-C}]\ \}\ /\ \lambda \leq \alpha$$

$$p_0 = \left[\sum_{n=0}^{C-1} \frac{\lambda^n}{n!\,\mu n} + \sum_{n=C}^{k} \frac{\lambda^n}{C^{n-C} C!\mu n}\right]^{-1} \quad \text{Where,} \quad r = \frac{\lambda}{\mu}\ ,\ \rho = \frac{r}{C} = \frac{\lambda}{C\mu}$$

Hence the proof.

Important Note:

d-represents number of defective components in the lot and D is the acceptance constant for each lots.

C – Represents minimum number of Quality Control Inspectors.

Conditions for application:

1. The production process should be stable and steady.
2. The production process should be able to deliver products in batches or lots.

3. The testing is done by the quality control inspectors.
4. Lots arrive according to the Queueing discipline.

2.2 Measure of Sampling Plans for the Queuing System

2.2.1 Operating Characteristics function:

The Operating Characteristics function is a measure of Probability of acceptance of the entire system for the known traffic intensity.

$$Pa\ (p) = \left\{\sum_{x=0}^{d} \frac{e^{-np}\ (np)^{x}}{x!}\right\}^{k}$$

where K is the truncation level and can be determined with the help of 1- $W_q(0) \leq \alpha$.
α = Probability that the lots are delayed in the Queue.
$W_q(0)$ = Probability that the lots are inspected without time delay.

The Standard definitions for AOQ, ASN, etc... when the lot is very large is found in Schilling (1982). The same is utilized in this research work except the $P_a(p)$ which is found using the novel algorithm.

2.2.2 Average Outgoing Quality of each lot in the system

$${P_a}^{(p)} = \{P(d=0)\ or\ P(d=1)\ or\ \ldots \ldots P(d=D)\} \ldots \ldots \ldots \text{ up to K lots}$$

$$= \{P(D \leq d)\}^{K}$$

AOQ = $p\,{P_a}^{(p)}$ where p = fraction defectives of the system

2.2.3 Average Sample Number of each lot

$$ASN = n$$

Proof:

The average sample number is the expected value of the sample size required for making a decision on the lot. Since only one sample is considered the ASN = n.

ASN = E(n)
$= \sum n\, {P_a}^{(p)}$
$= n \sum {P_a}^{(p)}$
$= n\,.\,1$
$= n$

2.2.4 Average Sample Number of entire Queuing system

$$ASN = nK$$

Proof:

We know that the required sample size to make a decision in n units.
The average sample number of entire Queueing system is maximum number of lots in the system at any time epoch t is K.

ASN = E(n) K
$= K \sum n\, {P_a}^{(p)}$
$= Kn \sum {P_a}^{(p)}$
$= K\, n\,.\,1$
$= n\, K$

III. METHOD OF DESIGNING

3.1 Designing M/M/C/K Queueing system Sampling Plans

Let α be the probability of lots are delayed in the queue. To determine the smallest c, equation (1) is mandatory which is found in Donald Gross(2008).Then one must have $1 - W_q(0) \leq \alpha$ (1)

Or equivalently $W = (L / \lambda) \leq \alpha$ (2)

$$W= \{r + \left(\frac{r^c \rho}{c!\,(1-\rho)^2}\right) p_0\ [\ 1 - \rho^{k-c+1} - (1-\rho)(k-c+1)\,\rho^{k-c}]\ \} / \lambda \quad \leq \alpha \qquad (3)$$

$$p_0 = \left[\sum_{n=0}^{c-1} \frac{\lambda^n}{n!\,\mu n} + \sum_{n=c}^{k} \frac{\lambda^n}{c^{n-c} c! \mu n}\right]^{-1}$$

Now our aim is to find minimum number of Quality control inspectors (c) for the known α. For the known values of α, tables are constructed for easy selection of number of inspectors. Similarly for the given values of Acceptable Quality Levels AQL, tables are constructed for easy selection of parameters of the sampling plans. Using equations (1) and (2), tables (1) and (2) are constructed.

IV.RESULTS AND DISCUSSION

Table 1: Probability of Acceptance of lots in the system for the known values of k, r, D and np.

	k=10, r=0.6					
np D	0	1	2	3	4	5
1	0.55	0.83	0.95	0.99	1	1
1.5	0.41	0.7	0.88	0.96	0.99	1
2	0.3	0.58	0.79	0.91	0.97	0.99
2.5	0.22	0.47	0.69	0.85	0.93	0.97
3	0.17	0.38	0.6	0.77	0.88	0.95
3.5	0.12	0.3	0.51	0.69	0.82	0.91
4	0.09	0.24	0.42	0.61	0.76	0.86
4.5	0.07	0.19	0.35	0.53	0.68	0.81
5	0.05	0.15	0.29	0.45	0.61	0.75

Table 2: Probability of Acceptance of lots in the system for the known values of k, r, D and np.

	k=20, r=0.6					
np D	0	1	2	3	4	5
1	0.55	0.83	0.95	0.99	1	1
1.5	0.41	0.7	0.88	0.96	0.99	1
2	0.3	0.58	0.79	0.91	0.97	0.99
2.5	0.22	0.47	0.69	0.85	0.93	0.97
3	0.17	0.38	0.6	0.77	0.88	0.95
3.5	0.12	0.3	0.51	0.69	0.82	0.91
4	0.09	0.24	0.42	0.61	0.76	0.86
4.5	0.07	0.19	0.35	0.53	0.68	0.81
5	0.05	0.15	0.29	0.45	0.61	0.75

Table 3: Probability of Acceptance of lots in the system for the known values of k, r, D and np.

	k=10, r=0.65					
np D	0	1	2	3	4	5
1	0.52	0.82	0.95	0.99	1	1
1.5	0.38	0.68	0.87	0.96	0.99	1
2	0.27	0.56	0.78	0.9	0.97	0.99
2.5	0.2	0.44	0.67	0.83	0.93	0.97
3	0.14	0.35	0.57	0.75	0.88	0.94
3.5	0.1	0.27	0.48	0.67	0.81	0.9
4	0.07	0.21	0.39	0.58	0.74	0.85
4.5	0.05	0.16	0.32	0.5	0.66	0.8
5	0.04	0.12	0.26	0.42	0.59	0.73

Table 4: Probability of Acceptance of lots in the system for the known values of k, r, D and np.

	k=20, r=0.65					
np c	0	1	2	3	4	5
1	0.52	0.82	0.95	0.99	1	1
1.5	0.38	0.68	0.87	0.96	0.99	1
2	0.27	0.56	0.78	0.9	0.97	0.99
2.5	0.2	0.44	0.67	0.83	0.93	0.97
3	0.14	0.35	0.57	0.75	0.88	0.94
3.5	0.1	0.27	0.48	0.67	0.81	0.9
4	0.07	0.21	0.39	0.58	0.74	0.85
4.5	0.05	0.16	0.32	0.5	0.66	0.8
5	0.04	0.12	0.26	0.42	0.59	0.73

Table 5: The body of the table give Values of np for the known L and D

L D	0	1	2	3	4	5	6	7	8	9	10
1	0.06	0.36	0.81	1.36	1.96	2.56	3.25	3.91	4.61	5.36	6.11
2	0.02	0.23	0.6	1.06	1.58	2.15	2.76	3.39	4.05	4.72	5.41
3	0.02	0.19	0.51	0.93	1.42	1.96	2.54	3.14	3.77	4.42	5.08
4	0.01	0.16	0.46	0.86	1.32	1.84	2.39	2.98	3.59	4.22	4.87
5	0.01	0.14	0.42	0.8	1.25	1.75	2.29	2.86	3.46	4.08	4.71

Table 6: The body of the table gives the sample size for the fraction defectives p and known constants D & L

AQL					L=5						
p D	0	1	2	3	4	5	6	7	8	9	10
0.001	10	140	420	800	1250	1750	2290	2860	3460	4080	4710
0.002	5	70	210	400	625	875	1145	1430	1730	2040	2355
0.003	3	47	140	267	417	583	763	953	1153	1360	1570
0.004	3	35	105	200	313	438	573	715	865	1020	1178
0.005	2	28	84	160	250	350	458	572	692	816	942
0.006	2	23	70	133	208	292	382	477	577	680	785
0.007	1	20	60	114	179	250	327	409	494	583	673
0.008	1	18	53	100	156	219	286	358	433	510	589
0.009	1	16	47	89	139	194	254	318	384	453	523
0.01	1	14	42	80	125	175	229	286	346	408	471

4.2 Designing the Group Sampling Plans indexed through AQL for M/M/C/K model:

The Probability of acceptance of the system is $Pa\ (p) = \left\{\sum_{x=0}^{D} \frac{e^{-np}\ (np)^x}{x!}\right\}^k$ Where K is the truncation level.

Now, one has to find the parameters n, D, and K for the Known P_a(p) and fraction defectives p such that $Pa(p) \geq 1 - \alpha$. A computer program is written to solve the above non-linear equation and hence the parametric values are found and tabulated in tables (1) to (8).

4.3 Illustration 1:

A Quality Control system with M/M/C/K model consists of the traffic intensity r = 0.7 and K = 20 and for each lot, the system should consist of atleast 95% acceptance at AQL=1%. Determine the parameters of the Sampling Plans.

Solution:

The system has $r = 0.7$ and $1 - \alpha = 0.95$, therefore by using table (6) we get D= 3, $np = 1.5$. It is given that $p = 1\%$. Therefore when $np = 1.5$, the sample size is $n = 150$.

IV. RESULTS AND DISCUSSION

In this article acceptance sampling plans are developed for a M/M/C/K production queueing process. A novel algorithm is given for sentencing the lots which comprises K lots in a system. The sampling plans are developed based on the truncation level in the Queuing system. The measures of sampling plans are given for M/M/C/K type of production process. Tables are constructed to select the probability of acceptance and hence the required sampling plans for the known Queueing parameters. The results show that there is a decrease in the Probability of acceptance whenever the truncation level is increased. Hence it is advisable to keep K at minimum level. For the fixed value of the acceptance constant D, the sample size decreases as the fraction defective p increases. However for fixed p, the sample size increases whenever the acceptance constant increases. Whenever K increases the Probability of acceptance decreases.It is advisable to keep K at minimum level.

IV. ACKNOWLEDGMENT

Thepreferredspellingoftheword “acknowledgment” inAmericaiswithoutan “e” afterthe “g”.Avoidthestiltedexpression, “Oneofus(R.B.G.)thanks...” Instead,try“R.B.G.thanks”.Putapplicablesponsoracknowledgmentshere;DONOTplacethemonthefirstpageofyourpaperorasafootnote.

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