Cordial Labeling Of One Point Union Of Double - Tail C_5 Garphs and their invariance.

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1. Abstract: We discuss graphs of type $G^{(k)}$ i.e. one point union of k-copies of G for cordial labeling. We take G as double-tail graph. A double-tail graph is obtained by attaching a path P_m to apair of adjacent vertices of given graph. It is denoted by double-tail(G,P_m) where G is given graph. We take G as C_5 and restrict our attention to m = 2 in P_m and consider upto three pendent edges attached at a pair of adjacent vertices . Further we consider all possible structures of $G^{(k)}$ by changing the common point in one point union and obtain non-isomorphic structures. We show all these structures as cordial graphs. This is called as invariance of different structures of $G^{(k)}$ under cordial labeling.

Key words: cordial, one point union, double-tail graph, cycle, labeling

Subject Classification: 05C78

2. Introduction

The graphs we consider are simple, finite, undirected and connected. For terminology and definitions we depend on Holton [6] Graph Theory by Harary [7], A dynamic survey of graph labeling by J.Gallian [9] and Douglas West.[10].I.Cahit introduced the concept of cordial labeling[6]. f:V(G)→{0,1} be a function. From this label of any edge (uv) is given by |f(u)-f(v)|. Further number of vertices labeled with 0 i.e $v_f(0)$ and the number of vertices labeled with 1 i.e.$v_f(1)$ differ at most by one .Similarly number of edges labeled with 0 i.e.$e_f(0)$ and number of edges labeled with 1 i.e.$e_f(1)$ differ by at most one. Then the function f is called as cordial labeling. Cahit has shown that : every tree is cordial; Kn is cordial if and only if $n \leq 3$; $K_{m,n}$ is cordial for all m and n; the friendship graph $C_3^{(t)}$ (i.e., the one-point union of t copies of C_3) is cordial if and only if t is not congruent to 2 (mod 4); all fans are cordial; the wheel W_n is cordial if and only if n is not congruent to 3 (mod 4).A lot of work has been done in this type of labeling. One may refer dynamic survey by J. Gallian [9].

Our focus of attention is on one point unions on different graphs. For a given graph there are different one point unions (upto isomorphism) structures possible. It depends on which point on G is used to fuse to obtain one point union. We have shown that for G = bull on C_3,bull on C_4,C_3^+,C_4^+-e the different path union $P_m(G)$ are cordial [4].It is called as invariance under cordial labeling. We use the convention that $v_f(0,1) = (a,b)$ to indicate the number of vertices labeled with 0 are a in number and that number of vertices labeled with 1 are b. Further $e_f(0,1) = (x,y)$ we mean the number of edges labeled with o are x and number of edges labeled with 1 are. The graph whose cordial labeling is available is called as cordial graph. In this paper we define double-tail graph and obtain one point union graphs on it. Let G be a (p,q) graph.To one of it's pair of adjacent vertices we fuse t number of paths P_m.We denote this by double –tail(G,tPm).We choose m = 2 and t = 1,2,3.and discuss their one point union graph at different vertices of G and it's invariance under cordial labeling.

3. Preliminaries

3.1 Tail Graph: A (p,q) graph G to which a path P_m is fused at some vertex. This also can be explained as take a copy of graph G and at any vertex of it fuse a path P_m with it's one of the pendent vertex. It's number of vertices are P+m-1 and edges are by q + m-1. It is denoted by tail(G, P_m). In this paper we fix G as C_3 and take P_m for m =2, 3, 4, 5.

3.2 Double tail graph : To any graph G we attach paths of equal length to adjacent pair of vertices .When these paths are gust an edge each then it is referred as bull graph.This graph is denoted by double-tail(G,Pm) when both tails are identical and equal to p_m .if tails are p_m and p_n then the graph is denoted by double-tail(G,p_n,p_m).It has p+m+n-2 vertices and q+m+n-1 edges where G is (p,q) graph. 3.3 Fusion of vertices. Let $u \neq v$ be any two vertices of G. We replace these two vertices by a single vertex say x and all edges incident to u and v are now incident to x. If loop is formed then it is deleted.[6] 3.4 $G^{(K)}$ it is One point union of k copies of G is obtained by taking k copies of G and fusing a fixed vertex of each copy with same fixed vertex of other copies to create a single vertex common to all copies. If G is a (p, q) graph then $|V(G_{(k)}| = k(p-1)+1$and $|E(G)|= k.q$

-Results Proved:

Theorem4.1All non- isomorphic one point union on k-copies of graph obtained on $G =$ double-tail(C_5,P_2) given by $G^{(k)}$ are cordial graphs.

Proof:

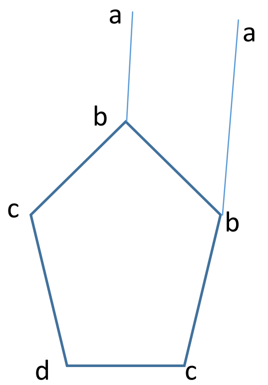

Fig 4.1 Four non-isomorphic one point union are possible at points ‘a’, ‘b’,’c’ and ‘d’

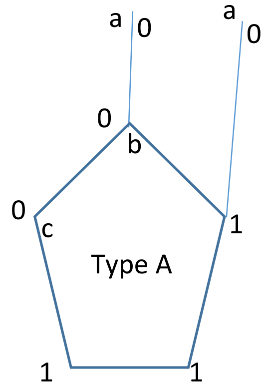

Fig 4.2 $v_f(0,1) = (4,3)$ $e_f(0,1)= (4,3)$

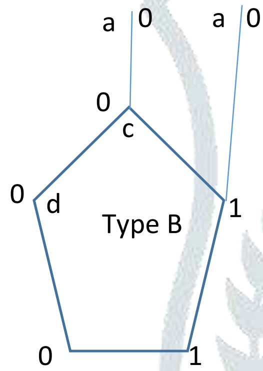

Fig 4.3 $v_f(0,1) = (4,3)$ $e_f(0,1)= (3,4)$

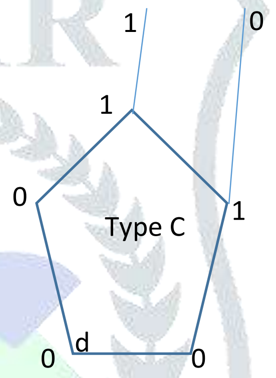

Fig 4.4 $v_f(0,1) = (4,3)$ $e_f(0,1)= (4,3)$

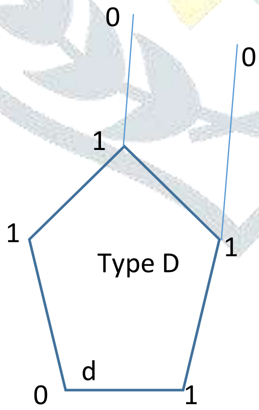

Fig 4.5 $v_f(0,1) = (4,3)$ $e_f(0,1)= (3,4)$

From Fig 4.1 it follows that we can take one point union at four vertices ‘a’, ‘b’, ‘c’and ‘d’. For the one point union at vertices a, b or c we fuse the type A and Type B label at vertices a, b or c respectively. For the one point union at vertex ‘d’ we use type C and type D label and fuse it at vertex d. For given k , if $k = 2x$ then x copies of type A (type C) and x copies of type B (type D) are fused at desired point .If $K = 2x+1$ then one more copy of Type A (type C) is used than the copies of type B (type D) used.

In both case the label number distribution is given by $v_f(0,1) =(4+6x, 3+6x)$, $e_f(0,1)= (4+7k,3+7k)$ where $k = 2x+1$, $x=0,1,2\ldots$ If $k = 2x$; $x= 1,2,..$ then we have, $v_f(0,1) = (7+5(x-1),6+5(x-1))$, $e_f(0,1) = (7k,7k)$. Thus the graph is cordial. #

Theorem 4.2 All non- isomorphic one point union on k-copies of graph obtained on G =double - tail(C_5,$2P_2$) given by $G^{(k)}$ are cordial graphs. Proof:

From Fig 4.6 it follows that we can take one point union at four vertices ‘a’, ‘b’, ‘c’and ‘d’. For the one point union at vertices a, or b we fuse the type A and Type B label at vertices a, b respectively. For the one point union at vertex ‘c’ and ‘d’ we use type C and type D label and fuse it at vertex c or d as required. For given k , if k = 2x then x copies of type A (type C) and x copies of type B (type D) are fused at desired point .If K = 2x+1 then one more copy of Type A (type C) is used than the copies of type B (type D) used.

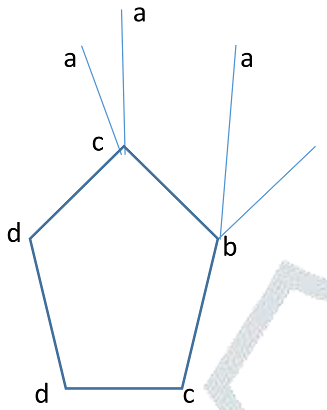

Fig 4.6 Four non-isomorphic one point union are possible at points ‘a’, ‘b’,’c’ and ‘d’

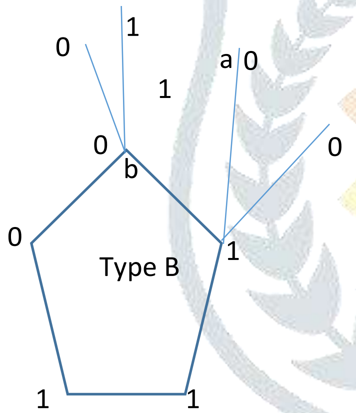

Fig 4.8 $v_f(0,1)$ = (5,4) $e_f(0,1)$= (4,5)

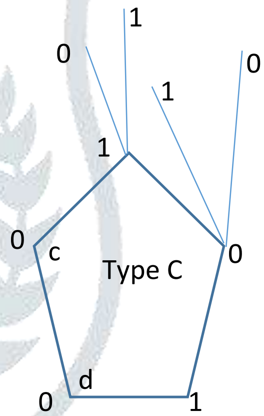

Fig 4.9 $v_f(0,1)$ = (5,4) $e_f(0,1)$= (5,4)

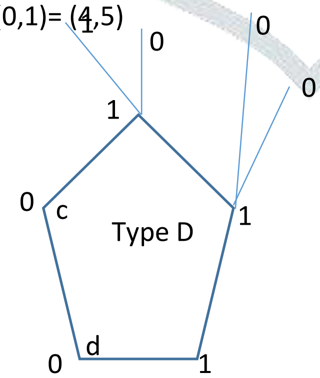

Fig 4.10 $v_f(0,1)$ = (5,4) $e_f(0,1)$= (4,5)

In both cases the label number distribution is given by $v_f(0,1)$ =(5+8x, 4+8x), $e_f(0,1)$= (5+9x,4+9x) where k = 2x+1, x=0,1,2… If k = 2x; x= 1,2,.. then we have, $v_f(0,1)$ = (9+8(x-1),8+8(x-1)), $e_f(0,1)$ = (9(k-1),9(k-1)). Thus the graph is cordial. Theorem 4.3 All non- isomorphic one point union on k-copies of graph obtained on G = double- tail(C_5,3P_2) given by $G^{(k)}$ are cordial graphs.

Proof:

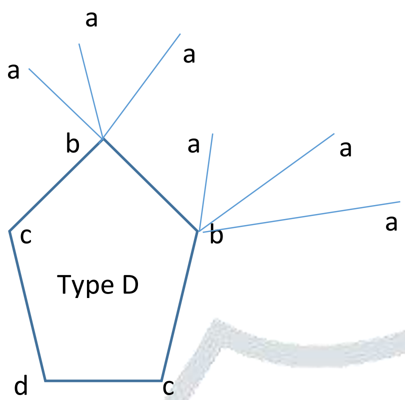

Fig 4.11 One point union may be taken at vertices 'a', 'b' 'c', 'd'

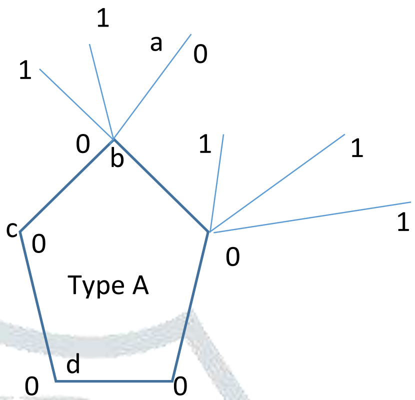

Fig 4.12 $v_f(0,1) = (6,5)$ $e_f(0,1)= (6,5)$

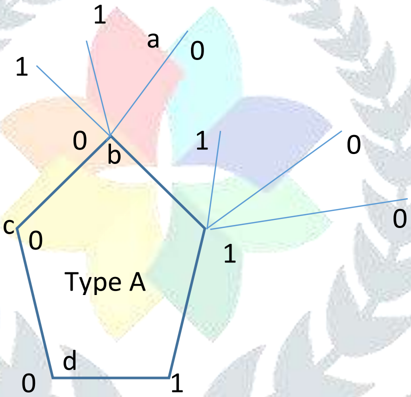

Fig 4.12 $v_f(0,1) = (6,5)$ $e_f(0,1)= (5,6)$

From Fig 4.11 it follows that we can take one point union at five vertices 'a', 'b', 'c', 'd'or 'e'. For the one point union at any of these vertices we use Type A label and type B label alternately in $G^{(k)}$ .In all cases the label number distribution is given by $v_f(0,1)$ =(6+10x,5+10x) , $e_f(0,1)$=(6+11x,5+11x) where k = 2x+1, x=0,1,2… If k = 2x; x= 1,2,.. then we have, $v_f(0,1)$ = (11+10(x-1),10+10(x-1)), $e_f(0,1)$ = (11k,11k). Thus the graph is cordial.

Conclusions: In this paper we define some new families obtained from C_5 and fusing totwo adjacent vertices with pendent edges upto three. We show that 1) 1All non- isomorphic one point union on k-copies of graph obtained on G =double tail(C_5,P_2) also called $G^{(k)}$ are cordial graphs. 2) All non- isomorphic one point union on k-copies of graph obtained on G =double- tail(C_5,2P_2) given by $G^{(k)}$ are cordial graphs. 3) All non-isomorphic one point union on k-copies of graph obtained on G =double- tail(C_5,3P_2) given by $G^{(k)}$ are cordial graphs.

References:

[1] M. Andar, S. Boxwala, and N. Limye, New families of cordial graphs, J. Combin. Math. Combin. Comput., 53 (2005) 117-154. [134]

[2] M. Andar, S. Boxwala, and N. Limye, On the cordiality of the t-ply Pt(u,v), Ars Combin., 77 (2005) 245-259. [135]

[3] Bapat Mukund, Ph.D. thesis submitted to university of Mumbai. India 2004.

[4] Bapat Mukund V. Some Path Unions Invariance Under Cordial labeling, IJSAM feb.2018 issue.

[5] I.Cahit, Cordial graphs: a weaker version of graceful and harmonious graphs, Ars Combin., 23 (1987) 201-207.
[6] J. Clark and D. A. Holton, A first look at graph theory; world scientific.
[7] Harary, Graph Theory, Narosa publishing ,New Delhi
[8] Yilmaz, Cahit, E-cordial graphs, Ars combina, 46,251-256.
[9] J.Gallian, Dynamic survey of graph labeling, E.J.C 2017
[10] D. WEST, Introduction to Graph Theory, Pearson Education Asia.

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