MIRACLE OF MATHEMATICAL OPERATIONS OF NUMBERS, DIGITS, ZERO, AND INFINITY

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ABSTRACT: This article reviews and explores the basic conceptions of division by zero. Zero is not a number, but it is one of the digits of decimal system. Zero (0) is the last numerical digit to come into use. Some basic ideas about division and multiplication as well as why those ideas appear to fail when dividing by zero. Zero is neither positive nor negative and appears in the middle of a number line. It is neither a prime number nor a composite number. It cannot be prime because it has an infinite number of factors and cannot be composite because it cannot be expressed by multiplying prime numbers (0 must always be one of the factors). We will discuss in detail some early attempts to resolve those issues and why they were unsuccessful, before providing a modern definition for the process based upon the existence of hidden subspaces, and the indeterminate form, etc.

Keywords: Zero, Infinity, Indeterminate form, Undefined Number, Invention of zero, & Algebraic Operations of zero, etc.

I. INTRODUCTION

The first indubitable appearance of a symbol for zero appears in 876AD, in India on a stone tablet in Gwalior. Documents on copper plates, with the same small o in them, dated back as far as the sixth century C.E. abound. Zero (0) was the last numerical digit to come into use. 0 (zero) is both a number and a numerical digit used to represent that number in numerals. As a number, zero means *nothing*-an absence of other values. It plays a central role in mathematics as the identity element of the integers, real numbers, and many other algebraic structures. As a digit, zero is used as a placeholder in place value systems. Historically, it was the last digit to come into use. In the English language, zero may also be called nil when a number, when a numeral, and nought/naught in either context. In the English language, 0 may be called - zero, oh, null, nought/naught or nil.

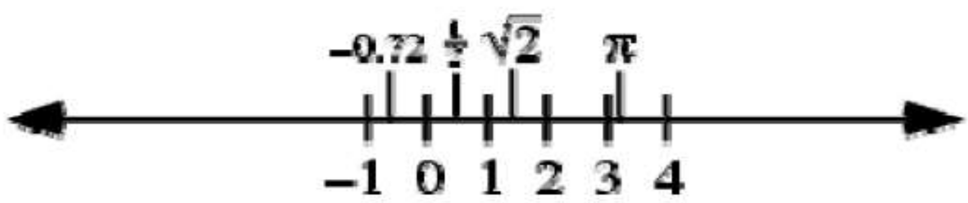
In around 1200, Leonardo Fibonacci wrote *Liber Abaci* where he described the nine Indian symbols together with the sign '0'. However, the concept of *zero* took some time for acceptance. It is only around 1600 that *zero* begain to come into widespread use after encountering a lot of supports and criticisms from mathematicians of the world. That is how *shunyam* given by our forefathers was recognized in the world and made its place permanently as *zero*. Interestingly, the word *zero* probably came from Sanskrit word for *shunyam* or the Hindi equivalent of *shunya*. The word *shunyam* was translated to Arabic as *al-sifer*. Fibonacci mentioned it as *cifra* from which we have obtained our present *cipher*, meaning empty space. From this original Italian word or from alteration of Medieval Latin *zephirum*, the present word *zero* might have originated.

II. THE MIGHTY DIGIT ZERO

There are ten digits as- 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The combination of digits makes a numbers. So, zero is a digit and any number multiplied by zero is always equal to zero. Zero is neither positive nor negative and appears in the middle of a number line. It is neither a prime number nor a composite number. It cannot be prime because it has an infinite number of factors and cannot be composite because it cannot be expressed by multiplying prime numbers (0 must always be one of the factors). Division by zero is a tricky one. The uniqueness of division breaks down when we attempt to divide any number by zero since we cannot recover the original number by the inverting the process of multiplication. Division by zero is undefined. This is the reason that in all computer programs or mathematical calculations, one should take care of this vital operation and there should have appropriate strategy to deal with this situation. Zero is tiny number, but we should never ignore it might. Imagine the world without zero. Not only mathematics, but all branches of sciences would have struggled for more clear definitions in their individual contexts, had zero not exist in our number system. Thanks to the ingenuity of our forefathers. Infinity is the greatest number which could not defined still now. So, it is neither the greatest digit nor a number, but it is undefined.

III. THE UNIQUE DIGIT ZERO

The number 0 is neither positive nor negative but appears in the middle of a number line. It is neither a prime number nor a composite number. It cannot be prime because it has an infinite number of factors and cannot be composite because it cannot be expressed by multiplying prime numbers (0 must always be one of the factors). Real numbers consist all rational (i.e. the numbers which can be express as p/q, like 2) and irrational numbers (which cannot be expressed as fraction, like $\sqrt{2}$). Now all these real numbers can be placed uniquely in a real line towards both positive and negative direction. Hence all positive, negative, even, odd, rational and irrational numbers correspond to only a single point on the line.



Among these real numbers, zero has the most important and unique position. It is in the intersection between positive and negative numbers. If one goes to the right side from zero, it is positive numbers and if one goes towards the left side of zero, it is all negative numbers. So essentially zero is neither positive nor negative number, it is the borderline for positive and negative numbers, or it is neutral in that sense. In fact this is the only number in the real number world, which is neither positive nor negative. Zero as single entity has no power of its own. Even if one puts zero to the left side of any number still it is powerless. But if one starts adding it to the right side of a number, then zero starts showing its power and the number increases by ten times for each additional zero.

IV. THE SECRET MEANING OVAL AND CIRCULAR-SHAPED OF DIGIT ZERO

The number and the letter O are both round, but there is great difference in modern science. The difference is important on a computer. For one thing, a computer will not do arithmetic with the letter O, because it does not know that it should have been a zero. The oval-shaped zero and circular letter O came into use together on modern character displays. It is shown in fig. [1]: (a) and (b) as:

Oval-shaped zero Circular-shaped zero

Fig. [1]: (a) and (b)

The zero with a dot in the centre seems to have begun as a choice on IBM 3270 controllers (this has the problem that it looks like the Greek letter-Theta). The slashed zero, looking like the letter O with a diagonal line drawn inside it, is used in old-style ASCII graphic sets that came from the default type-wheel on the well-known ASR-33 teletype. This format causes problems because it looks like the symbol, representing the empty set, as well as for certain Scandinavian languages which use Ø as a letter.

V. BASIC ALGEBRAIC OPERATIONS OF DIGIT ZERO

Zero is neither positive nor negative, neither a prime number nor a composite number, nor is it a unit. The following are some basic rules for dealing with the number zero. These rules apply for any complex number x, unless otherwise stated.

- **Addition: $x + 0 = 0 + x = x$. (That is, 0 is an identity element with respect to addition.)**
- **Subtraction: $x - 0 = x$ and $0 - x = - x$.**
- **Multiplication: $x \cdot 0 = 0 \cdot x = 0$.**
- **Division: $0 / x = 0$, for nonzero x. But $x / 0$ is undefined, because 0 has no multiplicative inverse, a consequence of the previous rule. For positive x, as y in x / y approaches zero from positive values, its quotient increases toward positive infinity, but as y approaches zero from negative values, the quotient increases toward negative infinity. The different quotients confirms that division by zero is undefined.**
- **Exponentiation: $x = 1$, except that the case $x = 0$ may be left undefined in some contexts. For all positive real x, $0 = 0$.**
- **The sum of 0 numbers is 0, and the product of 0 numbers is also 0.**
- **The expression "0/0" is an "indeterminate form." That does not simply mean that it is undefined; rather, it means that if $f(x)$ and $g(x)$ both approach 0 as x approaches some number, then $f(x)/g(x)$ could approach any finite number or ∞ or- ∞; it depends on which functions f and g. (L'Hopital's rule).**

VI. DIVISION OF NUMBERS BY DIGIT ZERO

If zero is added with a positive or a negative number, then one will remain in the same number point in the real line scale, i.e. no change in the value. And if one multiplies any positive and negative real number with zero, result is zero. However, division by zero is a tricky one. Brahmagupta himself could not describe the operation properly and later Bhaskara also mentioned it incorrectly.

Bhaskara said: if any number is divided by zero, it is infinity. Therefore, as one keep dividing by a smaller number and go towards zero, the result increases. But still the smaller number is not equal to zero. Therefore, one is not actually doing any division by zero, rather predicting a trend, which might be possible if divisor reaches a value, closer to zero or very small numbers. But whatever the smallest number one can think, another number smaller than that exists. Moreover, it is to be understood that infinity is a concept, an abstract thing, not a number as defined in our number system and all rules of mathematics are invalid while one consider operation with infinity. For example, if infinity is added with infinity result is not twice the value of infinity. It is still infinity! It is therefore wrong to say that a number divided by zero is infinity. In fact, in the very first place it is wrong to attempt to divide a number by zero. The uniqueness of division breaks down when we attempt to divide any number by zero since we cannot recover the original number by the inverting the process of multiplication. And zero is the only number with this property and so division by zero is *undefined* for real numbers. So we should *never* attempt to do a division with zero. In fact, it is meaningless to attempt to do this operation. This is the reason that in all computer programs or mathematical calculations, one should take care of this vital operation and there should have appropriate strategy to deal with this situation. Imagine, a remotely controlled rocket is going towards a distant star and the computer installed in it, is doing millions of vital calculation every second. But the scientists who programmed the computer just inadvertently forgot to tell the computer what it should do if something like division by zero occurs. And unfortunately if it occurs, the computer will stop working and it will wonder what to do with this undefined operation. So all the efforts of the scientists will be a waste! Zero is so powerful.

VII. INDETERMINATE-FORM / UNDEFINED / MEANINGLESS OF DIGIT ZERO

Another interesting case is zero divided by zero. Mathematically speaking, an expression like zero divided by zero is called *indeterminate*. To put it simply, this is a sort of expression, which cannot be determined accurately. If we see the expression properly, you can't assign any value to it. That means (0 / 0) can be equal to 10, 100 or anything else and interestingly the rule of multiplication also holds true here since 10 or 100 multiplied by zero will give the product as zero. So the basic problem is that we

cannot determine the exact or precise value for this expression. That's why mathematically (0 / 0) is said to be *indeterminate*. Zero to the power zero is also *indeterminate.* Mathematically, this situation is similar to zero divided by zero. Using limit theorem, it can be found that as *x and a tend to zero*, the function *ax* takes values between 0 and 1, inclusive. So zero to the power zero is also termed as *indeterminate*. But modern day mathematicians are giving many new theories and insights regarding proper explanation of zero to the power zero. Some mathematicians say that accepting $0^0 = 1$, allows some formulae to be expressed simply while some others point out that $0^0 = 0$ makes the life easier.

VIII. THE UNIQUE FEATURES OF DIGIT ZERO

Zero is a special number. If there are zero things, there are no things at all. There are none. For example, if DR. VINAY has zero hats, that means he does not have a hat at all. Some peculiar properties of zero are:

- The zero is not a number but it is one of the digit.
- The zero is a whole number.
- The zero is not a positive number.
- The zero is not a negative number.
- The zero is neither negative nor positive.
- The zero is a neutral number.
- Any number divided by itself equals one, except if that number is zero. In operations:
  0 ÷ 0 = not a number, but it is indeterminate form.

IX. THE MATHEMATICAL STORY OF DIGIT ZERO

An Engineer, a Physicist and a Mathematician are all put in a room with a burning fire in the middle and one bucket of water next to it. This is how they respond:

THE ENGINEER, practical as he is, takes the bucket and throws the water over the fire to put it off instantly.

THE PHYSICIST, curious as he is, takes the bucket and pours out water all around the fire and watches the fire to die slowly.

THE MATHEMATICIAN, walks about in the room, observes the fire and the bucket of water, thinks for a moment, determines there is a solution and leaves the room again.

X. MATHEMATICAL OPERATIONS OF DIGIT ZERO, INFINITY AND NUMBERS

[a]. ADDITION WITH INFINITY

Infinity plus a number, $\infty \pm K = \infty$

Infinity plus infinity, $\infty + \infty = \infty$

Infinity minus infinity, $\infty - \infty =$ Indeterminate form

[b]. MULTIPLICATION WITH INFINITY,

Infinity by a number, $\infty \times (\pm k) = \pm\infty$, if $k \neq 0$

Infinity by infinity, $\infty \times \infty =$ Indeterminate form

Infinity by zero, $\infty \times 0 =$ Indeterminate form

[c]. DIVISION WITH INFINITY AND ZERO

Zero over a number, $\frac{0}{K} \neq 0$

A number over zero, $\frac{k}{0} = \pm\infty, k \neq 0, \infty$

A number over infinity, $\frac{K}{\infty} = 0$

A number over infinity, $\frac{\infty}{k} = \infty$

Zero over infinity, $\frac{0}{\infty} = 0$

Infinity over zero, $\frac{\infty}{0} = \infty$ or Undefined

Zero over zero, $\frac{0}{0} =$ Indeterminate form

Infinity over infinity $\frac{\infty}{\infty} =$ Indeterminate form

[d]. POWERS WITH INFINITY AND ZERO

A number to the zero power, $K^0 = 1, \quad k \neq 0$

Zero to the power zero, $0^0 =$ Indeterminate form

Infinity to the power zero, ∞^0 = Indeterminate form

Zero to the power of a number, $0^K = \{0,\ \text{if } k > 0$
$\{\infty,\ \text{if } k < 0$

[e]. A Number to the Power of Infinity,

$$k^\infty = \{\infty, \quad k > 1$$
$$k^\infty = \{0, \quad 0 < k > 1$$

Zero to the power of infinity, $0^\infty = 0$
Infinity to the power of infinity, $\infty^\infty = \infty$
One to the power of infinity, 1^∞ = Indeterminate form

XI. CONCLUSIONS

Zero (0) is both a number and a numerical digit used to represent that number in numerals. In the English language, 0 may be called- *zero, oh, null, nought or nil.* Every aspect of zero presented in this paper supports the notion that zero is exceptional. It not only took man centuries to invent the digit zero, it took added centuries to discover the number zero and still more centuries to accept and use it. Yet, after all this, evidence indicates that we still do not recognize its significance and importance. The uniqueness of zero is more general and deeper than that of any other number and yet pedagogically zero is treated superficially as a trivial and obvious notion.

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