# PROPERTIES OF M-PROJECTIVE RECURRENT RIEMANNIAN MANIFOLD

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***Abstract: In this paper, analysis of some properties of the m-projective recurrent curvature tensor in Riemannian manifold has been done.***

***Index Terms— Riemannian manifold, recurrent parameter, m-projective curvature tensor, Einstein manifold.***

## I. INTRODUCTION

An n-dimensional Riemannian manifold with Riemannian metric g be denoted by M. The Riemannian curvature and Riemannian connection is denoted by K and D respectively. A Riemannian manifold is recurrent if

(1.1) $(DUK)X,Y,Z) = \alpha(U)\, K\,(X,Y,Z)$

A non zero 1-form α in (1.1) is known as recurrence parameter . The manifold reduces to symmetric manifold if 1-form α is zero in (1.1)

On contracting with respect to X equation (1.1), we get

(1.2 ) $(DURic)(Y,Z) = \alpha(U)\, Ric(Y,Z).$

Thus from (1.2),we have

(1.3) $(DUQ)(Y) = \alpha(U)Q(Y)$

Ricci operator of type (1,1) is denoted by Q, defined as

(1.4) $Ric(Y,Z) = g(Q(Y),Z).$

On contracting with respect to Y equation (1.3), we get

(1.5) $Ur = \alpha(U)r$

The scalar curvature is denoted by r.

## II. M-PROJECTIVE CURVATURE TENSOR

G.P. Pokhariyal and R.S. Mishra [5] in 1971 defined a tensor field $W^*$ on a Riemannian manifold as

(2.1) $$W^*(X,Y)Z = R(X,Y)Z - \frac{1}{2(n-1)}\,[Ric(Y,Z)X - Ric(X,Z)Y + g(Y,Z)QX - g(X,Z)QY].$$

In Sasakian manifolds R.H.Ojha [3] defined and studied the properties of the m-projective curvature. It bridges the gap between the conformal curvature tensor,Coharmonic curvature tensor and Concircular curvature tensors was shown by him.

In this paper,we have considered a non flat n-dimensional smooth Riemannian manifold in which the M-projective curvature tensor $W^*$ satisfies the following condition

(2.2) $(D_u W^*)(X,Y)Z = \propto(U) W^*(X,Y)Z$

Manifold reduces to the m-projectively symmetric manifold if the 1-form α is zero.

P being the Projective curvature tensor is given by

(2.3) $P(X,Y)Z = R(X,Y)Z - (\frac{1}{n-1})[Ric(Y,Z)X - Ric(X,Z)Y].$

A projectively recurrent manifold a Riemannian manifold which obeys

(2.4) $(DUP)(X,Y)Z = \alpha(U)P(X,Y)Z$

An Einstein manifold is a manifold which obeys

(2.5) $Ric(X,Y) = kg(X,Y)$

where k stands for a constant. From (2.5),we get

(2.6) $Q(X) = kX.$

On contracting the above equation with respect to X,we get

(2.7) $r = nk.$

## III. III. M-PROJECTIVELY RECURRENT MANIFOLD

Theorem 3.1. The constant curvature tensor r in an n-dimensional m-projectively recurrent Riemannian manifold, is given by

(3.1) (n − 3)(Ur) + 2(2 − n)α(U)r − 2nα(QU) = 0.

Proof. Let $M^n$ be an n-dimensional M-projectively recurrent manifold. Then from equations (2.1) and (2.2), it follows that

(3.2) $$(DUR)(X,Y)Z = \alpha(U)R(X,Y)Z + \frac{1}{2(n-1)}[(DURic(Y,Z)X - (DURic)(X,Z)Y + g(Y,Z)(DUQ)X - g(X,Z)(DUQ)Y - \alpha(U)\{Ric(Y,Z)X - Ric(X,Z)Y + g(Y,Z)QX - g(Y,Z)QY\}].$$

Permuting equation (3.1) twice with respect to U,X, Y and adding the three equations and using Bianchi's second identity, we have

(3.3) $$\alpha(U)R(X,Y)Z + \alpha(X)R(Y,U)Z + \alpha(Y)R(U,X)Z + \frac{1}{2(\text{n-1})}[(DURic)(Y,Z)X - (DURic)(X,Z)Y + g(Y,Z)(DUQ)(X) - g(X,Z)(DUQ)(Y) + (DXRic)(U,Z)Y - (DXRic)(Y,Z)U + g(U,Z)(DXQ)(Y) - g(Y,Z)(DXQ)(U)$$
$$+ (\text{DY Ric})(X,Z)U - (\text{DY Ric})(U,Z)X + g(X,Z)(DYQ)(U) - g(U,Z)(DYQ)(X) - \alpha(U)\{Ric(Y,Z)X - Ric(X,Z)Y + g(Y,Z)QX - g(X,Z)QY\} - \alpha(X)\{Ric(U,Z)Y - Ric(Y,Z)U + g(U,Z)QY - g(Y,Z)QU\} - \alpha(Y)\{Ric(X,Z)U - Ric(U,Z)X + g(X,Z)QU - g(U,Z)QX\}] = 0.$$

On contracting the above equation with respect to X, we obtain

(3.4) $$\alpha(U)Ric(Y,Z) - \alpha(Y)Ric(U,Z) + R'(Y,U,Z,\rho) + 1/(2(n-1))[(n-1)(DURic(Y,Z) + g(Y,Z)(Ur) - g((DUQ)Y,Z) + (DY\ Ric)(U,Z) - (DURic)(Y,Z + 1/2\, g(U,Z)(Y r) - 1/2\, g(Y,Z)(Ur) + (1-n)(DY\ \ Ric)(U,Z) + g((DYQ)U,Z) - g(U,Z)(Y r) + (1-n)\alpha(U)Ric(Y,Z) + \alpha(U)g(Y,Z)r + \alpha(U)g(Q(Y),Z) - \alpha(Y)Ric(U,Z) + \alpha(U)Ric(Y,Z) - \alpha(Q(Y)g(U,Z) + \alpha(Q(U)g(Y,Z) - \alpha(Y)g(R(U),Z) + (n-1)\alpha(Y)Ric(U,Z) + \alpha(Y)g(U,Z)r] = 0$$

Where vector field ρ is defined as

(3.5) $g(X,\rho) = \alpha(X)$

Factoring off Z in above, we have

(3.6) $$\alpha(U)QY - \alpha(Y)QU - R(Y,U,\rho) + 1/(2(n-1))[(DYQ)U + (n-1)(DUQ)Y + Y(Ur) - (DUQ)Y + 1/2U(Y r) - (DUQ)Y - 1/2\,Y(Ur) + (1-n)(DYQ)U + (DY\,Q)U - U(Y r) - (n-1)\alpha(U)QY - \alpha(U)(Y r) + \alpha(U)QY - \alpha(Y)QU + \alpha(U)QU - \alpha(Q(Y))U + \alpha(Y)Ur + \alpha(Q(U))Y + (n-1)\alpha(Y)QU] = 0.$$

Or

$$R(Y,U,\rho) = \frac{1}{2(n-1)}[(n-3)(DUQ)Y + \alpha(U)QY - (n-3)(DYQ)U - \alpha(Y)QU + \alpha(Q(U))Y - \alpha(Q(Y))U - \alpha(U)Y r + \alpha(Y)Ur].$$

(3.6)

On contracting (3.6) with respect to Y, we get

(3.7) $$Ric(Y,\rho) = \frac{1}{2(n-1)}[(n-3)(Ur) - \frac{(n-1)}{2}(Ur) + (2-n)\alpha(U)r + (n-2)\alpha(QU)].$$

Using (1.4) and (3.4) in equation (3.7), we get

$$(n-3)(Ur)+2(2-n)\alpha(U)r-2n\alpha(QU)=0.$$

This completes the proof of the theorem.

Theorem 3.2. The necessary and sufficient condition for an n-dimensional Ricci recurrent Riemannian manifold to be a recurrent manifold is that it is m-projectively recurrent manifold for the same recurrence parameter.

Proof. Taking the covariant derivative of (2.1) with respect to U, we get

(3.8) $$(DUW^{*})(X,Y)Z = (DUR)(X,Y)Z - 1/(2(n-1))\,[(DURic)(Y,Z)X - (DURic)(X,Z)Y + g(Y,Z)(DUQ)(X) - g(X,Z)(DUQ)(Y)].$$

Let $M^n$ be a Ricci recurrent Riemannian manifold, then from (1.2),(1.3) and (3.8),we have

(3.9) $(DUW^{*})(X,Y)Z = (DUR)(X,Y)Z - \alpha(U)2(n-1)[Ric(Y,Z)X - (Ric(X,Z)Y + g(Y,Z)QX - g(X,Z)QY].$
From (3.5) it is evident that if any one of the equations (1.1) and (2.5) hold then the second also hold.

Theorem 3.3. In an Einstein manifold $M^n$ the m-projective curvature tensor satisfies the following identity
(3.10) $(DUW^{*})(X,Y)Z + (DXW^{*})Y,U)Z + (DY)(U,X)Z = 0.$

Proof. Using (2.7) in (2.1) , ,it follows that

(3.11) $W^{*}(X,Y)Z = R(X,Y)Z - [\frac{k}{(n-1)}]\,[g(Y,Z)X - g(X,Z)Y].$

Taking covariant derivative of the above with respect to U , we get

(3.12) $(DUW^{*})(X,Y)Z = (DUR)(X,Y)Z.$

permuting equation(3.10) twice with respect to U,X, Y ;adding the three equations
and using Bianchi's second identity, we get the required result.

Theorem 3.4. Let $M^n$ be an n-dimensional Ricci recurrent Riemannian manifold.Then $M^n$ is m-projective recurrent if and only if it is a projectively recurrent manifold of the same reccurence parameter.

Proof. We have the following relation in projective curvature tensor and m-Projective curvature tensor

(3.13) $W^{*}(X,Y)Z = P(X,Y)Z + \frac{1}{2(n-1)}\,[Ric(Y,Z)X - Ric(X,Z)Y - g(Y,Z)QX + g(X,Z)QY].$

Taking the covariant derivative of (3.10) with respect to U ,we get

(3.14) $$(DUW^{*})(X,Y)Z = (DUP)(X,Y)Z + \frac{1}{2(n-1)}\,[(DURic)(Y,Z)X - (DURic)(X,Z)Y - g(Y,Z)(DUQX) + g(X,Z)(DUQY)].$$

Let $M^n$ be a Ricci-recurrent Riemannian manifold,then from(1.2),(1.3) and

(3.12) it follows that

(3.15) $$(DUW^{*}(X,Y)Z = (DUP)(X,Y)Z + [\alpha(U)/2(n-1)][Ric(Y,Z)X - Ric(X,Z)Y - g(Y,Z)QX + g(X,Z)QY].$$

With the help of equations (3.12) and (3.13) it follows that if any one of the equations (2.6) and (2.2) hold then the second equation also holds.

## IV. CONCLUSIONS

The constant curvature tensor r in an n-dimensional m-projectively recurrent Riemannian manifold, is given by $(n-3)(Ur)+2(2-n)\alpha(U)r-2n\alpha(QU)=0$. The necessary and sufficient condition for an n-dimensional Ricci recurrent Riemannian manifold to be a recurrent manifold is that it is m-projectively recurrent manifold for the same recurrence parameter. $M^n$ being an n-dimensional Ricci recurrent Riemannian manifold and is m-projective recurrent iff it is a projectively recurrent manifold of the same recurrence parameter.

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