Cordial Labeling Of One Point Union Of Double -Tail (C_3,tP_2) Garphs and their invariance

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1. Abstract: We discuss graphs of type $G^{(k)}$ i.e. one point union of k-copies of G for cordial labeling. We take G as double-tail graph. A double-tail graph is obtained by attaching a path P_m to a pair of adjacent vertices of given graph. It is denoted by double-tail(G,P_m) where G is given graph. We take G as C_3 and restrict our attention to m = 3 in P_m and consider upto t paths P_3 each attached at a pair of adjacent vertices . Further we consider all possible structures of $G^{(k)}$ by changing the common point in one point union and obtain non-isomorphic structures. We show all these structures as cordial graphs. This is called as invariance of different structures of $G^{(k)}$ under cordial labeling.

Key words: cordial, one point union, double-tail graph, tail graph, cycle, labeling,
Subject Classification: 05C78

2. Introduction

The graphs we consider are simple, finite, undirected and connected. For terminology and definitions we depend on Holton [5] Graph Theory by Harary [6], A dynamic survey of graph labeling by J.Gallian [8] and Douglas West.[9].I.Cahit introduced the concept of cordial labeling[5]. f:V(G)→{0,1} be a function. From this label of any edge (uv) is given by |f(u)-f(v)|. Further number of vertices labeled with 0 i.e $v_f(0)$ and the number of vertices labeled with 1 i.e.$v_f(1)$ differ at most by one .Similarly number of edges labeled with 0 i.e.$e_f(0)$ and number of edges labeled with 1 i.e.$e_f(1)$ differ by at most one. Then the function f is called as cordial labeling. Cahit has shown that : every tree is cordial; Kn is cordial if and only if $n \leq 3$; $K_{m,n}$ is cordial for all m and n; the friendship graph $C_3^{(t)}$ (i.e., the one-point union of t copies of C_3) is cordial if and only if t is not congruent to 2 (mod 4); all fans are cordial; the wheel W_n is cordial if and only if n is not congruent to 3 (mod 4).A lot of work has been done in this type of labeling. One may refer dynamic survey by J. Gallian [8].

Our focus of attention is on one point unions on different graphs. For a given graph there are different one point unions (upto isomorphism) structures possible. It depends on which point on G is used to fuse to obtain one point union. We have shown that for G = bull on C_3,bull on C_4,C_3^+,C_4^+-e the different path union $P_m(G)$ are cordial [3].It is called as invariance under cordial labeling. We use the convention that $v_f(0,1)$ = (a,b) to indicate the number of vertices labeled with 0 are a in number and that number of vertices labeled with 1 are b. Further $e_f(0,1)$ = (x,y) we mean the number of edges labeled with o are x and number of edges labeled with 1 are. The graph whose cordial labeling is available is called as cordial graph. In this paper we define double-tail graph and obtain one point union graphs on it. Let G be a (p,q) graph.To one of it's pair of adjacent vertices we fuse t number of paths P_m.We denote this by double –tail(G,tPm).We choose m = 2 and t = 1,2,3, 4.and discuss their one point union graph at different vertices of G and it's invariance under cordial labeling.

3. Preliminaries

3.1 Tail Graph: A (p,q) graph G to which a path P_m is fused at some vertex. This also can be explained as take a copy of graph G and at any vertex of it fuse a path P_m with it's one of the pendent vertex. It's number of vertices are P+m-1 and edges are by q + m-1. It is denoted by tail(G, P_m). In this paper we fix G as C_3 and take P_m for m =2, 3, 4, 5.

3.2 Double tail graph : To any graph G we attach paths of equal length to adjacent pair of vertices .When these paths are gust an edge each then it is referred as bull graph.This graph is denoted by double-tail(G,Pm) when both tails are identical and equal to p_m .if tails are p_m and p_n then the graph is denoted by double-tail(G,p_n,p_m).It has p+m+n-2 vertices and q+m+n-1 edges where G is (p,q) graph.

3.3 Fusion of vertices. Let u $\neq$v be any two vertices of G. We replace these two vertices by a single vertex say x and all edges incident to u and v are now incident to x. If loop is formed then it is deleted.[6]

3.4 $G^{(K)}$ it is One point union of k copies of G is obtained by taking k copies of G and fusing a fixed vertex of each copy with same fixed vertex of other copies to create a single vertex common to all copies. If G is a (p, q) graph then $|V(G_{(k)}| = k(p-1)+1$and $|E(G)|= k.q$

4. Results Proved:

Theorem 4.1All non- isomorphic one point union on k-copies of graph obtained on G = double-tail(C_3,P_2) given by $G^{(k)}$ are cordial graphs for all k.

Proof: From figure 4.1 it follows that there are three different structures on one point union of k copies of G are possible depending on if we use vertex a, b, or c as common point.

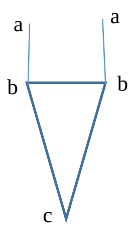


Fig. 4.1 One point union may be taken at vertex a, or b or c

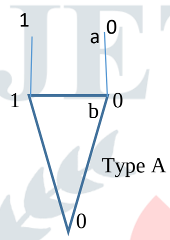


Fig. 4.2 $v_f(0,1)$ = (3,2) $e_f(0,1)$= (3,2)

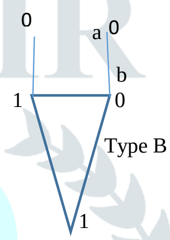
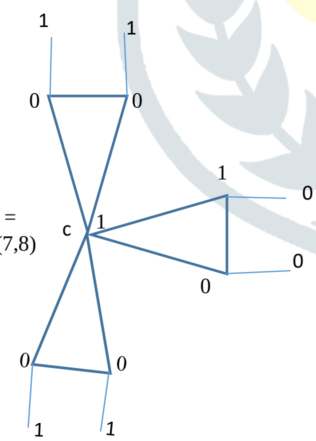


Fig. 4.3 $v_f(0,1)$ = (3,2) $e_f(0,1)$= (2,3)

Type C

Fig. 4.4 $v_f(0,1)$ = (7,6) $e_f(0,1)$= (7,8)



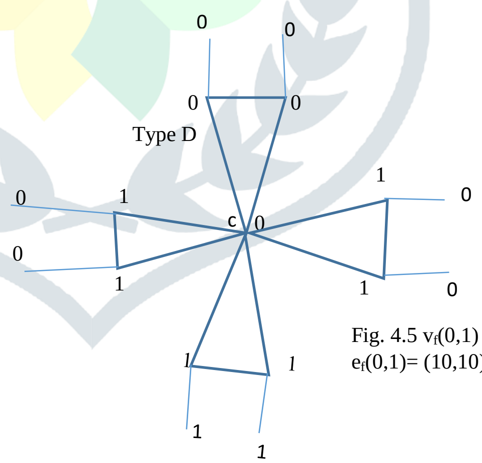


Fig. 4.5 $v_f(0,1)$ = (9,8) $e_f(0,1)$= (10,10)

To take one point union at vertex a or b the type A or Type B label is used alternately. In $G^{(k)}$ where k = 2x the type A label and type b label each is used for x times. If k = 2x+1 (x = 0,1,2,..) then type A label is used for x+1 times and type B label is used for x times. The label number distribution is $v_f(0,1)$ = (3+4x,2+4x) $e_f(0,1)$= (3+5x,2+5x) for k =2x+1 , x=0, 1, 2, …If k= 2x , x= 1, 2, 3, ..then $v_f(0,1)$ = (5+4(x-1),4+4(x-1)) $e_f(0,1)$= (5x,5x).When one point union is taken at point c then the graph $G^{(k)}$ has no cordial labeling when k ≡ 2(mod 4).

For all other values of k we proceed as follows.

Case k= 4x. We use type D label repeatedly. The label number distribution is $v_f(0,1)$ = (1+8x, 8x) $e_f(0,1)$= (10x,10x).

Case k = 4x+1.

We first obtain labeling for k = 4x as shown above. With this labeled copy we fuse type A label at vertex c. The label number distribution is $v_f(0,1) = (3+8x, 2+8x)$ $e_f(0,1)= (3+10x,2+10x)$.

Case k = 4x+3. We first obtain labeling for k = 4x as shown above. With this labeled copy we fuse type C label at vertex c. The label number distribution is $v_f(0,1) = (7+8x, 6+8x)$ $e_f(0,1)= (7+10x,8+10x)$.

Thus the graph is cordial. #

Theorem 4.2 All non- isomorphic one point union on k-copies of graph obtained on G =double -tail(C_3,$2P_2$) given by $G^{(k)}$ are cordial graphs.

Proof: From Fig 4.6 it follows that we can take one point union at four vertices ‘a’, ‘b’, ‘c’. For the one point union at any of these vertices we fuse the type A and Type B label at respective vertex. In $G^{(k)}$ where k = 2x the type A label and type B label each is used for x times. If k = 2x+1 (x = 0,1,2,..) then type A label is used for x+1 times and type B label is used for x times.

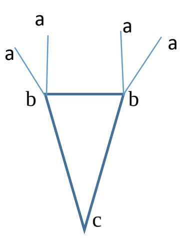


Fig. 4.6 One point union may be taken at vertex a, or b or c.

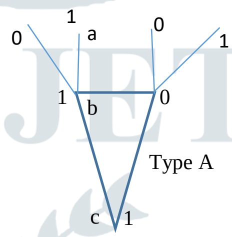


Fig. 4.7 $v_f(0,1)$ = (3,2) $e_f(0,1)$= (3,2)

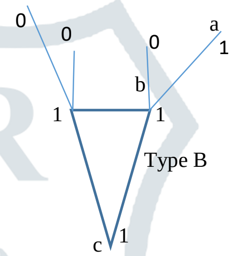


Fig. 4.8 $v_f(0,1)$ = (3,2) $e_f(0,1)$= (2,3)

The label number distribution is $v_f(0,1) = (3+6x,4+6x)$ $e_f(0,1)= (3+7x,4+7x)$ for k =2x+1 , x=0, 1, 2, …If k= 2x , x= 1, 2, 3, ..then $v_f(0,1) = (6+6(x-1),7+6(x-1))$ $e_f(0,1)= (7x,7x)$.Thus the graph is product cordial even if we change the common point in $G^{(k)}$

Theorem 4.3 All non- isomorphic one point union on k-copies of graph obtained on G = double-tail(C_3,$3P_2$) given by $G^{(k)}$ are cordial graphs.

Proof: From Fig 4.9 it follows that we can take one point union at three vertices ‘a’, ‘b’, or ‘c’. For the one point union at vertex a or vertice b we use Type A label and type B label alternately in $G^{(k)}$. In $G^{(k)}$ where k = 2x the type A label and type b label each is used for x times. If k = 2x+1 (x = 0,1,2,..) then type A label is used for x+1 times and type B label is used for x times. The label number distribution is given by $v_f(0,1) =(5+8x,4+8x)$, $e_f(0,1)=(4+9x,5+9x)$ where k = 2x+1, x=0,1,2… If k = 2x; x= 1,2,.. then we have, $v_f(0,1) = (9+8(x-1),8+8(x-1))$, $e_f(0,1) = (9k,9k)$. Thus the graph is cordial.

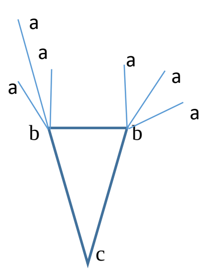


Fig. 4.9 One point union may be taken at vertex a, or b or c.

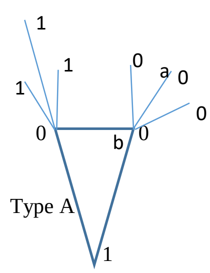


Fig. 4.10 $v_f(0,1) = (5,4)$ $e_f(0,1)= (4,5)$

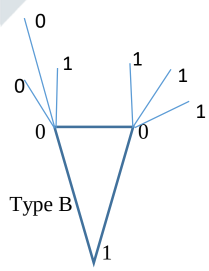


Fig. 4.11 $v_f(0,1) = (5,4)$ $e_f(0,1)= (4,5)$

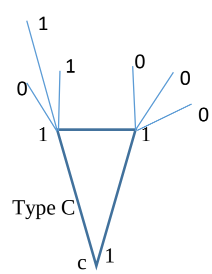


Fig. 4.12 $v_f(0,1) = (4,5)$ $e_f(0,1)= (5,4)$

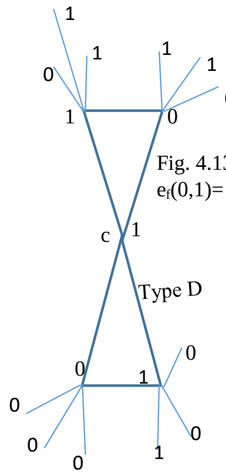


Fig. 4.13 $v_f(0,1) = (9,12)$
$e_f(0,1)= (11,11)$

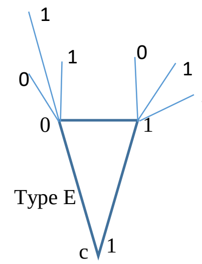


Fig. 4.14 $v_f(0,1) = (3,6)$
$e_f(0,1)= (4,5)$

The table shows scheme to obtain labeled copy $G^{(K)}$ for given k with point common to all copies being c.

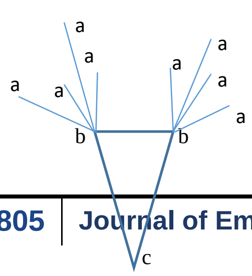
k	Type of labels fused at vertex c	$V_f(0,1)$	$e_f(0,1)$	Label of common vertex	Remarks
1	C	(4,5)	(5,4)	1	cordial
2	D	(9,8)	(9,9)	1	cordial
3	D and C	(13,12)	(14,13)	1	cordial
4	C and D and E	(16,17)	(18,18)	1	cordial
4x	Copy of $G^{(4)}$ fuse at vertex 'c' for x times	(16x,16x+1)	(18x,18x)	1	cordial
4x+1	Copy of $G^{(4x)}$ fuse at vertex 'c' with type C	(16x+4,16x+5)	(18x+5,18x+4)	1	cordial
4x+2	Copy of $G^{(4x)}$ fuse at vertex 'c' with type D	(16x+9,16x+8)	(18x+9,18x+9)	1	Cordial
4x+3	Copy of $G^{(4x)}$ fuse at vertex 'c' with type D and typeC	(16x+13,16x+12)	(18x+14,18x+13)	1	cordial

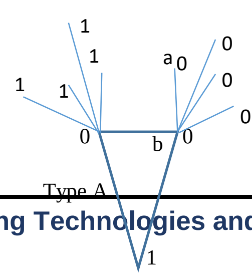
Table 4.1 : Elobarates the scheme to obtain labeled copy of $G^{(k)}$ with common vertex as two degree vertex c on double-tail(C_3,$3p_2$)

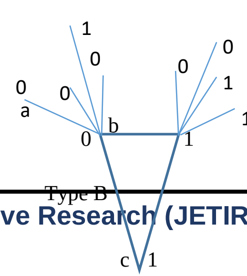
Thus the graphs are cordial.

Theorem 4.4 All non- isomorphic one point union on k-copies of graph obtained on G = double-tail(C_3,$4P_2$) given by $G^{(k)}$ are cordial graphs.

Proof: From Fig 4.15 it follows that we can take one point union at three vertices 'a', 'b', or 'c'. For the one point union at vertex a or vertice b we use Type A label and type B label alternately in $G^{(k)}$. In $G^{(k)}$ where k = 2x the type A label and type b label each is used for x times. If k = 2x+1 (x = 0,1,2,..) then type A label is used for x+1 times and type B label is used for x times. The label number distribution is given by $v_f(0,1)$ =(6+10x,5+10x) , $e_f(0,1)$=(5+11x,6+11x) where k = 2x+1, x=0,1,2… If k = 2x; x= 1,2,.. then we have, $v_f(0,1)$ = (11+10(x-1),10+10(x-1)), $e_f(0,1)$ = (11k,11k). Thus the graph is cordial.







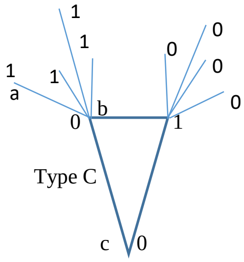


Fig. 4.18 $v_f(0,1) = (3,6)$
$e_f(0,1)= (4,5)$

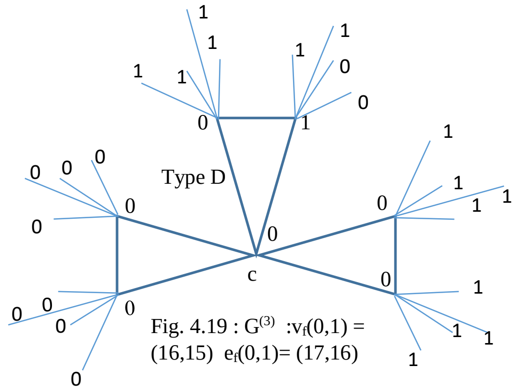


Fig. 4.19 : $G^{(3)}$:$v_f(0,1) = (16,15)$ $e_f(0,1)= (17,16)$

k	Type of labels fused at vertex c	$V_f(0,1)$	$e_f(0,1)$	Label of common vertex (c)	Remarks
1	B	(6,5)	(6,5)	1	cordial
2	C and B	(11,10)	(11,11)	1	cordial
3	D	(16,15)	(17,16)	0	cordial
4	D and C	(21,20)	(22,22)	0	cordial
4x	Copy of $G^{(4)}$ fuse at vertex 'c' for x times	(20x+1,20x)	(22x,22x)	0	cordial
4x+1	Copy of $G^{(4x)}$ fuse at vertex 'c' with type C	(20x+6,20x+5)	(22x+5,22x+6)	0	cordial
4x+2	Copy of $G^{(4x)}$ fuse at vertex 'c' with C and B	(20x+11,20x+10	(22x+11,22x+11)	0	Cordial
4x+3	Copy of $G^{(4x)}$ fuse at vertex 'c' with D	(20x+16,20x+15)	(22x+17,22x+16)	0	cordial

Table 4.2 : Elobarates the scheme to obtain labeled copy of G($^{k)}$ with common vertex as two degree vertex c on double-tail(C_3,4p_2)

Thus the graph is cordial.

Conclusions: In this paper we define some new families obtained from C_3 and fusing to two adjacent vertices with pendent edges upto four. We show that

1) All non- isomorphic one point union on k-copies of graph obtained on G =double - tail(C_3,P_2) also called $G^{(k)}$ are cordial graphs.

2) All non- isomorphic one point union on k-copies of graph obtained on G =double- tail(C_3,2P_2) given by $G^{(k)}$ are cordial graphs.

3) All non- isomorphic one point union on k-copies of graph obtained on G =double- tail(C_3,3P_2) given by $G^{(k)}$ are cordial graphs.

4) All non- isomorphic one point union on k-copies of graph obtained on G =double- tail(C_3,4P_2) given by $G^{(k)}$ are cordial graphs.

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