E-cordial labeling of path union on mixed graph.

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Abstract: On alternate vertex of a path P_m graphs G_1 and G_2 are fused resulting in mixed path union of G_1 and G_2 denoted by $P_m(G_1,G_2)$.We chose G_1 andG_2 from C_3, C_4 and C_5 and show that $P_m(G_1,G_2)$ are e-cordial.

In general $P_m(G_1,G_2)$ is not same as $P_m(G_2,G_1)$ (upto isomorphism)

Key words: E-cordial, labeling, bull, one point union, cycle.

Subject Classification: 05C78.

2. Introduction:

The graphs we consider are finite, connected, un directed and simple.For terminology and definitions we refer Harary [3] E.J.of G.Theory, Dynamic survey of graph labeling [2]. In 1997 Yilmaz and Cahit [4] introduced a weaker version of edge graceful labeling called E-cordial. The word cordial was used first time in this paper. Let G be a graph with vertex set V and edge set E. Let f be a function that maps E into {0,1} .Define f on V by f(v) = $\sum\{f(uv)/(uv)\in E\}$(mod 2).The function f is called as E cordial labeling if $|e_f(0)-e_f(1)|\leq1$ and $|v_f(0)-v_f(1)|\leq1$. Where $e_f(i)$ is the number of edges labeled with i = 0,1 and $v_f(i)$ is the number of vertices labeled with i = 0,1. We also use $v_f(0,1)$ = (a,b) to denote the number of vertices labeled with 0 are a and that with 1 are b in number. Similarly $e_f(0,1)$=(x, y) to denote number of edges labeled with 0 are x and that labeled with 1 are y in number respectively. A lot of work has been done in this type of labeling and the above mentioned paper gave rise to number of papers on cordial labeling. A graph that admits E-cordial labeling is called as E- cordial graph. Yilmaz and Cahit has shown that Trees T_n with n vertices and Complete graphs K_n on n vertices are E – cordial iff n is not congruent to 2 (modulo 4). Friendship graph $C_3^{(n)}$ for all n and fans F_n for n not congruent to1 (mod 4).They have observed that the graph with n vertices where n≡ 2 (mod 4) is not e-cordial. One may refer A Dynamic survey of graph labeling [3]for more details on completed work

3. Preliminaries:

3.1 Fusion of vertex. Let G be a (p,q) graph. let u≠v be two vertices of G. We replace them with single vertex w and all edges incident with u and that with v are made incident with w. If a loop is formed is deleted. The new graph has p-1vertices and at least q-1 edges.[5]

3.2 **Path union of G** i.e. $P_m(G)$ is obtained by taking a path P_m and m copies of graph G. Fuse a copy each of G at every vertex of path at given fixed point on G. It has mp vertices and mq +m-1 edges, where G is a (p, q) graph. If we change the vertex on G that is fused with vertex of Pm then we generally get a path union non isomorphic to earlier structure. In this paper we define a e-cordial function f that does not depends on which vertex of given graph G is used to obtain path union. This allows us to obtain path union in which the same graph G is fused with vertices of Pm at different vertices of G, as our choice and the same function f is applicable to all such structures that are possible on $P_m(G)$.

4. Main results:

Theorem.4.1 $P_m(G_1,G_2)$ where $G_1 = C_3$ and $G_2 = C_4$ is e-cordial for all m not congruent to 3,4 (mod 8)

Proof. Define a function f:E(G)→{0,1}.That gives us labeled structures as follows. In all diagrams below numbers shown are edge labels.

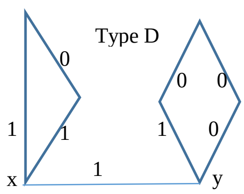


Fig 4.1: $v_f(0,1)$ = (3,4) $e_f(0,1)$ = (4,4)

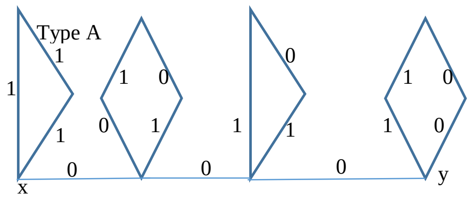


Fig 4.2: $v_f(0,1)$ = (6,8) $e_f(0,1)$ = (8,9)

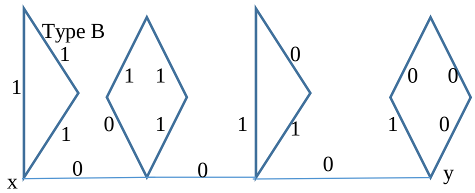


Fig 4.3 : $v_f(0,1) = (8,6)$ $e_f(0,1) = (8,9)$

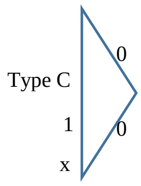


Fig 4.4 : $v_f(0,1) = (1,2)$ $e_f(0,1) = (2,1)$

Obtain labeled copy of $P_8(C_3,C_4)$ by Joining point x on Type A with point y on Type B by an edge with label '0'.We have $v_f(0,1) = (14,14)$ $e_f(0,1) = (17,18)$.We repeat this procedure on $P_8(C_3,C_4)$ each time adding a new edge with label '0' to obtain path union on 8x vertices where we have $v_f(0,1) = (14x,14x)$ $e_f(0,1) = (18x-1,18x)$.To obtain a path on 8x+t length x = 0,1,2,3,.. and t = 1,2,3,4,5,6,7 we first obtain $P_{8x}(C_3,C_4)$ and with an edge join it with $P_t(C_3,C_4)$ obtained below. The edge label of new edge is '0'.

For t=1 use Type C label at vertex x. The new edge is (yx) with label '0'..Resultant label distribution for P_{8x+1} is $v_f(0,1) = (14x+1,14x+2)$, $e_f(0,1) = (18x+2,18x+1)$.(If x =0 just Type C label will work)

For t = 2 use Type D label at vertex x. The new edge is (yx).Resultant label distribution is for P_{8x+2} is $v_f(0,1) = (14x+3,14x+4)$, $e_f(0,1) = (18x+4,18x+4)$.

For t = 3 and t = 4. The desired labeling does not exist.

For t=5 use Type B join with Type C label at vertex y on Type B and vertex x on type C. The new edge is (yx). Resultant label distribution for P_{8x+5} is $v_f(0,1) = (14x+9,14x+8)$, $e_f(0,1) = (18x+10,18x+10)$.

For t = 6 use Type B and Type D label at vertex y on Type B and x on Type D with the new edge (yx).Resultant label distribution for P_{8x+6} is $v_f(0,1) = (14x+11,14x+10)$, $e_f(0,1) = (18x+12,18x+13)$. For t =7 use Type B join with Type C label join with type D label at vertex y on Type B and vertex x on type C , further x on C with y on D. The new two new edges are (yx). Resultant label distribution for P_{8x+5} is $v_f(0,1) = (14x+12,14x+12)$, $e_f(0,1) = (18x+14,18x+15)$.

Theorem.4.2 $P_m(G_1,G_2)$, $G_1 = C_4$ and $G_2 = C_3$ is e-cordial for all m not congruent to 4,5 (mod 8) Proof. Define a function $f:E(G)\rightarrow\{0,1\}$.We obtain a labeled copy of $P_{8x}(C_4,C_3)$ by joining Type B and type A with vertex x on type B and vertex y on type A with new edge (xy) with label '0' given in above theorem 4.1. We have $v_f(0,1) = (14x,14x)$ $e_f(0,1) = (18x-1,18x)$.To obtain a path on 8x+t length x = 0,1,2,3,.. and t = 1,2,3,4,5,6,7 we first obtain $P_{8x}(C_3,C_4)$ and with an edge join it with $P_t(C_3,C_4)$ obtained below. The edge label of new edge is '0'. For t=1 use Type X label at vertex x. The new edge is (yx) with label '0'.Resultant label distribution for P_{8x+1} is $v_f(0,1) = (14x+2,14x+2)$, $e_f(0,1) = (18x+3,18x+2)$. (If x =0 just Type X label will work)

For t = 2 use Type D label at vertex y. The new edge is (yy) with label '0'.Resultant label distribution is for P_{8x+2} is $v_f(0,1) = (14x+3,14x+4)$, $e_f(0,1) = (18x+4,18x+4)$.

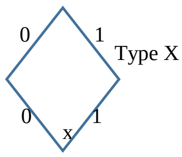


Fig 4.5: $v_f(0,1) = (2,2)$ $e_f(0,1) = (2,2)$

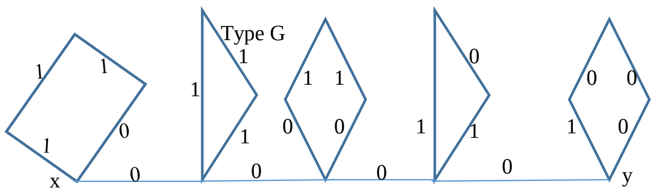


Fig 4.6 : $v_f(0,1) = (10,8)$ $e_f(0,1) = (11,11)$

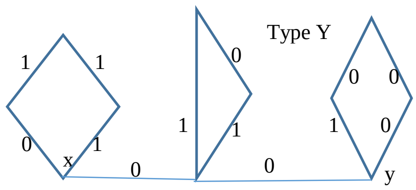


Fig 4.7: $v_f(0,1) = (5,6)$ $e_f(0,1) = (7,6)$

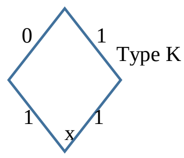


Fig 4.8: $v_f(0,1) = (2,2)$ $e_f(0,1) = (1,3)$

For t=3 use Type Y label at vertex x The new edge is (xx) has label '0'. Resultant label distribution for P_{8x+3} is $v_f(0,1) = (14x+5,14x+6)$, $e_f(0,1) = (18x+7,18x+6)$.

For t = 4 and t = 5, the desired labeling does not exists.

For t=6 join Type Y and type C label with an edge with label number '0'.

Resultant label distribution for P_{8x+3} is $v_f(0,1) = (14x+11,14x+10)$, $e_f(0,1) = (18x+13,18x+13)$.

For t=7 join Type Y and type C label and the Type C end with type K each by an edge with label '0'. Resultant label distribution for P_{8x+3} is $v_f(0,1) = (14x+13,14x+12)$, $e_f(0,1) = (18x+16,18x+15)$.To complete the case t =t we start with the case t = 7 and join it with Type K label by an edge with label '0'.The resultant label distribution is $v_f(0,1) = (14,14)$ $e_f(0,1) = (17,18)$.Thus the graph is e-cordial.

Theorem.4.3 $P_m(G_1,G_2)$ is e-cordial where $G_1 = C_4$ and $G_2 = C_5$ for all m not congruent to 4, 5 (mod 8).

Proof: Define a function $f:E(G) \rightarrow \{0,1\}$.That gives us labeled structures as follows. In all diagrams below numbers shown are edge labels. Except Type C all are e-cordial.

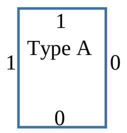


Fig 4.5 : $v_f(0,1) = (2,2)$ $e_f(0,1) = (2,2)$

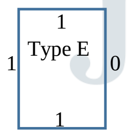


Fig 4.6 : $v_f(0,1) = (2,2)$ $e_f(0,1) = (2,2)$

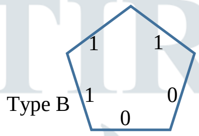


Fig 4.7 : $v_f(0,1) = (3,2)$ $e_f(0,1) = (2,3)$

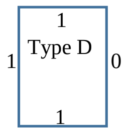


Fig 4.8 : $v_f(0,1) = (2,2)$ $e_f(0,1) = (1,3)$

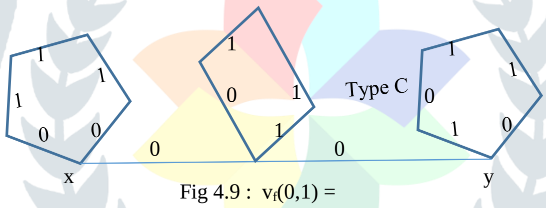


Fig 4.9 : $v_f(0,1) = (6,8)$ $e_f(0,1) = (7,9)$

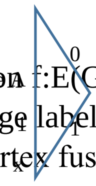
We consider following cases on t = 1,2,3,4,5,6,7,8 and obtain labeled copy of $Pt(G_1,G_2)$.The path under consideration will be pt = (v_1, v_2, $v_3 \ldots v_t$).Having obtained $P_{t-1}(G_1,G_2)$ we just fuse a labeled copy from above at vertex v_t to to obtain $Pt(G_1,G_2)$. case t =1 : Type A label. Case t = 2 : at v1 fuse a copy of type A label and at V_2 fuse a copy of Type B label. The label number distribution will be $v_f(0,1) = (5,4)$, $e_f(0,1) = (5,5)$. Case t = 3 at vertex V_3 fuse Type A label. The label number distribution will be $v_f(0,1) = (7,6)$, $e_f(0,1) = (8,7)$. For t ≡ 4, 5 (mod 8) the labeling does not exists. Case t = 6.We have already obtained $P_3(G_1,G_2)$.Add a new edge with label '0' and extend $P_3(G_1,G_2)$ by Type C label. The label number distribution will be $v_f(0, 1) = (13,14)$, $e_f(0,1) = (16, 16)$. Case t = 7.We have already obtained $P_6(G_1,G_2)$.Add a new edge with label '0' and extend $P_6(G_1,G_2)$ by Type E label. The label number distribution will be $v_f(0, 1) = (15,16)$, $e_f(0,1) = (18, 19)$. Case t = 8.We have already obtained $P_7(G_1,G_2)$.Add a new edge with label '0' and extend $P_7(C_4,C_5)$ by Type C label. The label number distribution will be $v_f(0, 1) = (18,18)$, $e_f(0,1) = (21, 22)$.To obtain A pathunion of length P_{8x+t} (x=0,1,2,..7) we first obtain a path union on P_{8x} (take x copies of c and concanate it each time by adding an edge with label '0' and fuse with other copy of $P_8(C_4,C_5)$. This procedure repeated for x times will produce labeled copy of $P_{8x}(C_4,C_5)$) The label number distribution will be $v_f(0, 1) = (18x,18x)$, $e_f(0,1) = (22x-1, 22x)$.The labeled copy of $P_{8x+t}(C_4,C_5)$ is obtained by joining a labeled copy of $P_t(C_4,C_5)$ with $P_{8x}(C_4,C_5)$ for t= 1, 2, 3, 6,7. The label number distribution is given below.

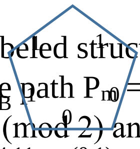
The table below gives label number distribution for vertices and edges

Sr. Number	mx (x = 0,1 ,2,..)	$v_f(0, 1)$	$e_f(0,1)$	
1	8x	(18x,18x)	(22x-1, 22x)	

2	8x+1	(18x+2,18x+2)	(22x+12, 22x+2)	For m = 8x+4, 8x+5 Desired labeling does not exists.
3	8x+2	(18x+5,18x+4)	(22x+5, 22x+5)	
4	8x+3	(18x+7,18x+6)	(22x+8 22x+7)	
5	8x+6	(18x+13,18x+14)	(22x+16,22x+16)	
6	8x+7	(18x+15,18x+16)	(22x+18, 22x+19)	

Theorem.4.4 $P_m(G_1,G_2)$ is e-cordial where $G_1 = C_3$ and $G_2 = C_5$ for all m .





Proof: Define a function f:E(G)→{0,1}.That gives us labeled structures as follows. In all diagrams above numbers shown are edge labels. All are e-cordial. On the path $P_m = (v_1, v_2, v_3 \ldots v_m)$.All the labels on path are labeled as '0'.at vertex fuse a copy of Type A if i≡1 (mod 2) and copy of type B if i≡ 0(mod 2) The resultant label number distribution for vertices and edges is given by $v_f(0, 1) = (4x,4x)$, $e_f(0,1) = (5x-1, 5x)$ when m = 2x and when m = 2x-1 we have, $v_f(0, 1) = (4x+1,4x+2)$, $e_f(0,1) = (5x+1, 5x+2)$.Thus the graph is e-cordial for all m.

Fig 4.10 : $v_f(0,1) = (1,2)$ $e_f(0,1) = (1,2)$

Fig 4.11 : $v_f(0,1) = (3,2)$ $e_f(0,1) = (2,3)$

Theorem.4.5 $P_m(G_1,G_2)$ is e-cordial where $G_1 = C_5$ and $G_2 = C_3$ for all m .

Proof: Define a function f:E(G)→{0,1}.That gives us labeled structures same as Type A and Type B in fig 4.10 and 4.11 in theorem 4.3. On the path $P_m = (v_1, v_2, v_3 \ldots v_m)$ all the edges on path are labeled as '0'.At vertex v_i of P_m fuse a copy of Type B if i≡1 (mod 2) and copy of type A if i≡ 0(mod 2) The resultant label number distribution for vertices and edges is given by $v_f(0, 1) = (4x,4x)$, $e_f(0,1) = (5x-1, 5x)$ when m = 2x-1 and when m = 2x we have, $v_f(0, 1) = (4x+1,4x+2)$, $e_f(0,1) = (5x+1, 5x+2)$.Thus the graph is e-cordial for all m.(x =0,1,2..). #

Theorem.4.6

Proof: we define a function f : E(G)→{0,1} as follows. It gives us following different types of labels which we combine to obtain $P_m(G_1,G_2)$.

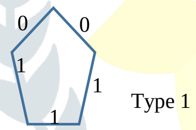


Fig 4.12 : $v_f(0,1) = (3,2)$ $e_f(0,1) = (2,3)$

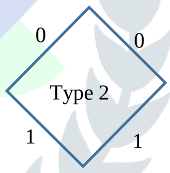


Fig 4.13 : $v_f(0,1) = (2,2)$ $e_f(0,1) = (2,2)$

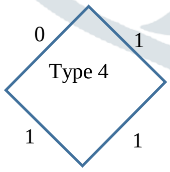


Fig 4.14 : $v_f(0,1) = (2,2)$ $e_f(0,1) = (1,3)$

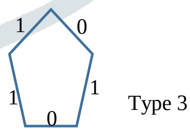


Fig 4.15 : $v_f(0,1) = (1,4)$ $e_f(0,1) = (2,3)$

Take a path on m vertices say $P_m = (v_1, v_2, v_3, .. v_m)$.To obtain $P_m(G_1,G_2)$ label all the edges on it with '0'.At each vertex v_i fuse a copy of type 1 if i≡ 1,3,7 (mod 8) . At each vertex v_i fuse a copy of type 2 if i≡ 2,4 (mod 8). At each vertex v_i fuse a copy of type 3 if i≡ 5 (mod 8). At each vertex vi fuse a copy of type 4 if i≡ 6,8 (mod 8). The label number distribution is given by $v_f(0, 1) = (3,2)$, $e_f(0,1) = (2,3)$ when m =1.If x ≠ 0 we have $v_f(0, 1) = (18x+3,18x+2)$, $e_f(0,1) = (22x+2, 22x+3)$ when m = 8x+1, x= 0,1,2... The label number distribution is given by $v_f(0, 1) = (18x+5,18x+4)$, $e_f(0,1) = (22x+5, 22x+5)$ when m = 8x+2, x= 0, 1, 2, .. The label number distribution is given by $v_f(0, 1) = (18x+8,18x+6)$, $e_f(0,1) = (22x+8, 22x+8)$ when m = 8x+3, x= 0,1 ,2... In this case the graph is not e-cordial.

The label number distribution is given by $v_f(0, 1) = (18x+10,18x+8)$, $e_f(0,1) = (22x+11, 22x+10)$ when m = 8x+4, x= 0, 1, 2, .. In this case the graph is not e-cordial.

The label number distribution is given by $v_f(0, 1) = (18x+11,18x+12)$, $e_f(0,1) = (22x+14, 22x+13)$ when $m = 8x+5$, $x= 0, 1, 2, ..$

The label number distribution is given by $v_f(0, 1) = (18x+13,18x+14)$, $e_f(0,1) = (22x+16, 22x+16)$ when $m = 8x+6$, $x= 0, 1, 2, ..$

The label number distribution is given by $v_f(0, 1) = (18x+16,18x+16)$, $e_f(0,1) = (22x+19, 22x+19)$ when $m = 8x+7$, $x= 0, 1, 2, ..$

The label number distribution is given by $v_f(0, 1) = (18x+18,18x+18)$, $e_f(0,1) = (22x+21, 22x+22)$ when $m = 8x+8$, $x= 0, 1, 2, ..$

Conclusions: In this paper we have discussed mixed path unions for e-cordial labeling. These path unions are obtained by fusing two graphs G_1 and G_2 at alternate nodes of path P_m. It is denoted by $P_m(G_1,G_2)$.If $G_1 \neq G_2$ (equality upto isomorphism.) $P_m(G_1,G_2) \neq P_m(G_2,G_1)$We have shown this actually by taking G_1 and G_2 from C_3, C_4, C_5.

We prove that 1) $P_m(G_1,G_2)$ where $G_1 = C_3$ and $G_2 = C_4$ is e-cordial for all m not congruent to 3,4 (mod 8).

2) $P_m(G_1,G_2)$, $G_1 = C_4$ and $G_2 = C_3$ is e-cordial for all m not congruent to 4,5 (mod 8).

3) $P_m(G_1,G_2)$ is e-cordial where $G_1 = C_4$ and $G_2 = C_5$ for all m not congruent to 4, 5. (mod 8).

4) $P_m(G_1,G_2)$ is e-cordial where $G_1 = C_3$ and $G_2 = C_5$ for all m.

5) $P_m(G_1,G_2)$ is e-cordial where $G_1 = C_5$ and $G_2 = C_3$ for all m

6) $P_m(G_1,G_2)$ is e-cordial where $G_1 = C_5$ and $G_2 = C_4$ for all m not congruent to 3,4 (mod 8). Another observation is that $P_m(C_3)$is not e-cordial when $m \equiv 2 \pmod 4$.And $P_m(C_4)$ is e-cordial for all m. But in mixed path union of C_3 and C_4 we get that mixed path union of C_3 and C_4 is e-cordial for all m not congruent to 3,4 (mod 8).In this path union if we interchange position of C_3 and C_4 we get $P_m(C_4,C_3)$ which is e-cordial iff m not congruent to 4,5 (mod 8).

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