PAIR SUM LABELING OF TREES IN ZERO DIVISOR GRAPHS

J.Periaswamy[1], N.Selvi[2]
[1] Part-Time Research Scholar, Bharathidasan University
Tiruchirapalli, Tamilnadu, India-620 024.
A.V.C.College(Autonomous), Mayiladuthurai, Tamilnadu, India-606 305.
A.D.M.College for Women(Autonomous)
Nagappattinam, Tamilnadu, India-611 001.

ABSTRACT: The first simplification of Beck's [2] zero divisor graph was introduced by Anderson and Livingston[1]. Their motivation was to give a better illustration of the zero divisor structure of the ring. In this paper, we investigate the pair sum labeling behavior of several trees which are obtained from stars and Bi-stars in $\Gamma(Z_n)$. Finally we show that all the trees $\Gamma(Z_{2p})$ of order less than 9 are pair sum graph. Here, we generate pair sum trees from stars in $\Gamma(Z_n)$. Clearly, we denote the vertex and edge sets of the star $\Gamma(Z_{2p})$ by, $V(\Gamma(Z_{2p})) = \{u, u_i : 1 \le i \le p-1\}$ and $E(\Gamma(Z_{2p})) = \{uu_i : 1 \le i \le p-1\}$.

Keywords: Labeling, pair Sum Labeling, Zero divisor graph.
AMS Subject classification: 05C25, 05C69.

1. INTRODUCTION

Let G be a (r, s) graph. An one to one map $f : V(G) \to \{\pm 1, \pm 2, \ldots\ldots\ldots, \pm r\}$ is called a pair sum labeling if the induced edge mapping, $f_e : E(G) \to Z - \{0\}$ defined by $f_e(uv) = f(u) + f(v)$ is one-one and $f_e(E(G))$ is either of the form $\{\pm l_1, \pm l_2, \ldots, \pm l_{s/2}\}$ or $\{\pm l_1, \pm l_2, \ldots, \pm l_{(s-1)/2}\} \cup \{l_{(s+1)/2}\}$ according as S is even or odd. A graph with a pair sum labeling defined on it is called pair sum graph. Pair sum labeling satisfies the following observations.

(i) If f is a pair sum labeling defined on $\Gamma(Z_n)$ then $\sum_{u \in V(G)} d(u) f(u) = 0$ iff G is a even size.

(ii) If f is a pair sum labeling then x and $-x$ are not labels of two adjacent vertices. [otherwise, zero appears as an edge label].

(iii) If $\Gamma(Z_n)$ is an even size pair sum graph then $\Gamma(Z_n) - e$ is also a pair sum graph for every edge e.

(iv) Let $\Gamma(Z_n)$ be an odd size pair sum graph with $-f_e(e) \notin f_e(E)$. Then $\Gamma(Z_n) = e$ is a pair sum graph.

(v) Let $\Gamma(Z_n)$ be a pair sum graph with even size and let f be a pair sum labeling of G with $f(n) = M$. Then the graph $\Gamma(Z_n)^*$ with $V(\Gamma(Z_n)^*) = V(\Gamma(Z_n)) \cup \{v\}$ and $E(\Gamma(Z_n)^*) = E(\Gamma(Z_n)) \cup \{uv\}$ is also a pair sum graph.

(vi) Let $\Gamma(Z_n)$ be an even order and size graph. If G is super vertex graceful then G is a pairsum graph.

(vii) Every graph is a subgraph of a connected pair sum graph.

The pair sum labeling is introduced in [3] by Ponraj and et al. In [3], [4], [5] and [6] they study the pair sum labeling of cycle, path, star and some of their related graphs. Let R be a commutative ring and let Z(R) be its set of zero-divisors. We associate a graph $\Gamma(R)$ to R with vertices $\Gamma(R)^* = Z(R) - \{0\}$, the set of non-zero zero divisors of R and for distinct $u, v \in Z(R)^*$, the vertices u and v are adjacent if and only if $uv = 0$. The zero divisor graph is very useful to find the algebraic structures and properties of rings. The idea of a zero divisor graph of a commutative ring was introduced by Beck in [2]. The first simplication of Beck's zero divisor graph was introduced by Anderson and Livingston [1]. Their motivation was to give a better illustration of the zero divisor structure of the ring. Anderson and Livinston, and others e.g., [7, 8, 9], investigate the interplay between the graph theoretic properties of $\Gamma(R)$ and the ring theoretic properties of R. Throught this paper, we consider the commutative ring R by Z_n and zero divisor graph $\Gamma(R)$ by $\Gamma(Z_n)$.

2. PAIR SUM LABELING OF SOME TREES IN ZERO DIVISOR GRAPH

Theorem 2.1. Let G be the tree with $V(G)=V(\Gamma(Z_{2p}))\cup\{v_i:1\le i\le p\}$ and $E(G)=E(\Gamma(Z_{2p}))\cup\{u_iv_i:1\le i\le p\}\cup\{uv_p\}$. Then, G is a pair sum graph.

Proof. Define a map $f:V(G)\to\{\pm1,\pm2,\dots,\pm2p\}$ by

$f(u)=1$

$f(u_i)=-i-1,\ 1\le i\le p-1$

$f(v_i)=2i+1,\ 1\le i\le p-1$

$f(v_p)=2p$

Here, $f_e(E(G))=\{\pm1,\pm2,\dots,\pm(p-1)\}\cup\{2p+1\}$. Then, G is a pair sum graph.

Theorem 2.2. If G is a tree with $V(G)=V(\Gamma(Z_{2p}))\cup\{v_1,v_2,v_3,v_4\}$ and $E(G)=E(\Gamma(Z_{2p}))\cup\{uv_1,v_1v_2,v_2v_3,v_3v_4\}$, then G is a pair sum graph.

Proof. Define a map $f:V(G)\to\{\pm1,\pm2,\dots,\pm2p\}$ by

$f(u)=-1$

$f(v_1)=-4$

$f(v_2)=1$

$f(v_3)=2$

$f(v_4)=3$

$f(u_i)=2i+3,\ 1\le i\le \frac{p-1}{2}$

$f\left(u_{\left\lfloor\frac{p}{2}\right\rfloor+i}\right)=-2i-1,\ 1\le i\le\frac{p-1}{2}$

Here, $f_e(E(G))=\{\pm3,\pm4\}\cup\{\pm4,\pm6,\pm8\dots,\pm(p+1)\}$. Here $p+1$ is always even, because p is any prime number which is greater than 2. Then, G is a pair sum graph.

Theorem 2.3. Let G be the tree with $V(G)=V(\Gamma(Z_{2p}))\cup\{v_i:1\le i\le5\}$ and $E(G)=E(\Gamma(Z_{2p}))\cup\{uv_1,v_1v_2,v_2v_3,v_3v_4,v_4v_5\}$. Then G is a pair sum graph.

Proof. Define a map $f:V(G)\to\{\pm1,\pm2,\dots,\pm(p+5)\}$ by

$f(u)=-1$

$f(v_1)=4$

$f(v_2)=-3$

$f(v_3)=2$

$f(v_4)=-2$

$f(v_5) = -4$

$f(u_i) = 6 + i,\ 1 \le i \le \frac{p-1}{2}$

$f\left(u_{\left\lfloor \frac{p}{2} \right\rfloor + i}\right) = -i - 5,\ 1 \le i \le \frac{p-1}{2}$

Here, $f_e(E(G)) = \{\pm 1, \pm 3, -6\} \cup \{6, \pm 4, \pm 7, \pm 8, \pm 9, ..., \pm (p-1)\} \cup \{-p\}$. Then, the mapping f is pair sum labeling.

Theorem 2.4. If G is a tree with $V(G) = V(\Gamma(Z_{2p})) \cup \{v_1, v_2\}$ and $E(G) = E(\Gamma(Z_{2p})) \cup \{u_1v_1, u_2v_2\}$. Then G is a pair sum graph.

Proof. Define $f : V(G) \to \{\pm 1, \pm 2, ..., \pm (p+2)\}$ by

$f(u) = 1$

$f(v_1) = 3$

$f(v_2) = 5$

$f(u_1) = -2$

$f(u_2) = -3$

$f(u_{2+i}) = 2i + 2,\ 1 \le i \le \left\lceil \frac{p-3}{2} \right\rceil$

$f\left(u_{\left\lceil \frac{p-3}{2} \right\rceil + 2 + i}\right) = -(2i + 4),\ 1 \le i \le \left\lfloor \frac{p-3}{2} \right\rfloor$

Here, $f_e(E(G)) = \{\pm 1, \pm 2\} \cup \{\pm 5, \pm 7, ..., \pm p\}$. Hence, f is a pair sum labeling.

Theorem 2.5. The tree with vertex set $V(G) = V(\Gamma(Z_{2p})) \cup \{v_i : 1 \le i \le 5\}$ and $E(G) = E(\Gamma(Z_{2p})) \cup \{uv_1, v_1v_2, v_2v_3, v_3v_4, v_4v_5\}$ is a pair sum graph.

Proof. Define a map $f : V(G) \to \{\pm 1, \pm 2, ..., \pm (p+5)\}$ by

$f(u) = -1$

$f(v_1) = -4$

$f(v_{1+i}) = i,\ 1 \le i \le 4$

$f(u_1) = -6$

$f(u_{2i}) = -6 - i, 1 \le i \le \frac{p-1}{2}$

Here, $f_e(E(G)) = \{\pm 1, \pm 5, -7\} \cup \left\{-7, \pm 8, \pm 9, ..., \pm \frac{p+1}{2}\right\} \cup \left\{\frac{-(p+13)}{2}\right\}$.

Then G is a pair sum graph.

Theorem 2.6. Let G be the tree with $V(G) = V(\Gamma(Z_{2p})) \cup \{v_i : 1 \le i \le 6\}$ and $E(G) = E(\Gamma(Z_{2p})) \cup \{uv_1, v_1v_2, v_2v_3, v_3v_4, uv_5, v_5v_6\}$. Then, G is a pair sum graph.

Proof. Define a map $f : V(G) \to \{\pm 1, \pm 2, ..., \pm (p+6)\}$ by

$f(u) = -1$

$f(v_1) = -4$

$f(v_2) = 1$

$f(v_3) = 2$

$f(v_4) = 3$

$f(v_5) = -3$

$f(v_6) = 7$

$f(u_i) = -4 - 2i, 1 \le i \le \frac{p-1}{2}$

$f(u_{\frac{p-1}{2}+i}) = 6 + 2i, 1 \le i \le \frac{p-1}{2}$

Here, $f_e(E(G)) = \{\pm 3, \pm 4, \pm 5\} \cup \{\pm 7, \pm 8, ..., \pm (p+3)\}$. Then, G is a pair sum graph.

Theorem 2.7. Let G be the tree with $V(G) = V(\Gamma(Z_{2p})) \cup \{v_i : 1 \le i \le 6\}$ and $E(G) = E(\Gamma(Z_{2p})) \cup \{uv_1, v_1v_2, v_2v_3, v_5v_6, uv_4\}$. Then, G is a pair sum graph.

Proof. Define a map $f : V(G) \to \{\pm 1, \pm 2, ..., \pm (p+6)\}$ by

$f(u) = 1$

$f(v_1) = 2$

$f(v_2) = 3$

$f(v_3) = 4$

$f(v_4) = -4$

$f(v_5) = -1$

$f(v_6) = -6$

$f(u_i) = 6 + i, 1 \le i \le \frac{p-1}{2}$

$f(u_{\frac{p-1}{2}+i}) = -8 - i, 1 \le i \le \frac{p-1}{2}$

Here, $f_e(E(G)) = \{\pm 3, \pm 4, \pm 7\} \cup \left\{\pm 8, \pm 9, ..., \pm \frac{(p+13)}{2}\right\}$. Then, G is a pair sum graph.

Theorem 2.8. The trees G_i $(1 \le i \le 6)$ with vertex set and edge set given below are pair sum,

(i) $V(G_1) = V(\Gamma(Z_{2p})) \cup \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and $E(G_1) = E(\Gamma(Z_{2p})) \cup \{uv_1, v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6\}$.

(ii) $V(G_2) = V(\Gamma(Z_{2p})) \cup \{v_1, v_2, ..., v_7\}$ and $E(G_2) = E(\Gamma(Z_{2p})) \cup \{v_1v_2, v_2v_3, v_3v_4, v_4v_5, uv_5, uv_6, v_6v_7\}$.

(iii) $V(G_3) = V(\Gamma(Z_{2p})) \cup \{v_1, v_2, ..., v_7\}$ and $E(G_3) = E(\Gamma(Z_{2p})) \cup \{v_1v_2, v_2v_3, v_3v_4, v_4u, uv_5, v_5v_6, v_6v_7\}$.

(iv) $V(G_4) = V(\Gamma(Z_{2p})) \cup \{v_iw_i : 1 \le i \le 4\}$ and $E(G_4) = E(\Gamma(Z_{2p})) \cup \{uw_1, w_1w_2, uw_3, w_3w_4, v_1v_2, v_2v_3, v_3v_4, v_4u\}$.

(v) $V(G_5) = V(\Gamma(Z_{2p})) \cup \{v_iw_i : 1 \le i \le 3\}$ and $E(G_5) = E(\Gamma(Z_{2p})) \cup \{uv_1, uv_2, uv_3, v_1w_1, v_2w_2, v_3w_3\}$.

Proof. (i) Define a map $f: V(G_1) \to \{\pm 1, \pm 2, ..., \pm (p+6)\}$ by

$f(u) = -1$

$f(v_1) = -7$

$f(v_2) = -5$

$f(v_3) = 1$

$f(v_4) = 3$

$f(v_5) = 5$

$f(v_6) = 7$

$f(u_i) = -2i - 4,\ 1 \le i \le \frac{p-1}{2}$

Here, $f_e(E(G_1)) = \{\pm 4, \pm 8, \pm 12\} \cup \{\pm 7, \pm 9, \pm 9 ..., \pm (p+4)\}$. Then, G_1 is a pair sum graph.

(ii) Define a map $f: V(G_2) \to \{\pm 1, \pm 2, ..., \pm (p+7)\}$ by

$f(v_1) = -3$

$f(v_2) = -6$

$f(v_3) = -1$

$f(v_4) = -4$

$f(v_5) = 1$

$f(v_6) = 3$

$f(u) = 2$

$f(u_{1+i}) = 4 + 2i,\ 1 \le i \le \frac{p-1}{2}$

$f\left(u_{\left\lceil \frac{p}{2} \right\rceil + i}\right) = -2i - 8,\ 1 \le i \le \frac{p-3}{2}$

Here, $f_e(E(G_2)) = \{\pm 3, \pm 5, \pm 7\} \cup \{\pm 8, \pm 10, ..., \pm (p+3)\} \cup \{p+5\}$. Then, G_2 is a pair sum graph.

(iii) Define a map $f: V(G_3) \to \{\pm 1, \pm 2, ..., \pm (p+7)\}$ by

$f(v_1) = -3$

$f(v_2) = -6$

$f(v_3) = -1$

$f(v_4) = -4$

$f(v_5) = 2$

$f(v_6) = 3$

$f(v_7) = 4$

$f(u) = 2$

$f(u_{1+i}) = 7 + i,\ 1 \le i \le \frac{p-1}{2}$

$f\left(u_{\frac{p-1}{2} + i}\right) = -i - 10,\ 1 \le i \le \frac{p-1}{2}$

Here, $f_e(E(G_3)) = \{\pm 3, \pm 5, \pm 7, \pm 9\} \cup \left\{\pm 10, \pm 11, ..., \pm \frac{(p+15)}{2}\right\} \cup \left\{\frac{-(p+17)}{2}\right\}$. Then, G3 is a pair sum graph

(iv) Define a map $f : V(G_4) \to \{\pm 1, \pm 2, ..., \pm (p+8)\}$ by

$f(u) = -1$

$f(v_1) = 3$

$f(v_2) = 2$

$f(v_3) = 1$

$f(w_1) = -5$

$f(w_2) = -6$

$f(w_3) = 7$

$f(w_4) = 4$

$f(u_1) = 7$

For other vertices we define,

$f(u_i) = -5 - 2i,\ 1 \le i \le \frac{p-1}{2}$

$f\left(u_{\left\lfloor \frac{p}{2} \right\rfloor + i}\right) = 7 + 2i,\ 1 \le i \le \frac{p-1}{2}$

Here, $f_e(E(G_4)) = \{\pm 3, \pm 5, \pm 6, \pm 11\} \cup \{\pm 8, \pm 10, ..., \pm (p+5)\}$. Then, G4 is a pair sum labeling.

(v) Define a map $f : V(G_5) \to \{\pm 1, \pm 2, ..., \pm (p+6)\}$ by

$f(u) = -1$

$f(v_1) = 2$

$f(v_2) = 3$

$f(v_3) = 4$

$f(w_1) = -3$

$f(w_2) = -5$

$f(w_3) = -7$

$f(u_i) = 2i + 4,\ 1 \le i \le \frac{p-1}{2}$

Here, $f_e(E(G_5)) = \{\pm 1, \pm 2, \pm 3, ..., \pm (p+2)\}$. Then, G5 is a pair sum labeling.

REFERENCES

[1] D.F.Anderson, P.S.Livingston, *The zero- divisor graph of a commutative ring*, J.Algebra, 217, No.2, (1999), 434-447.

[2] I.Beck, *Colouring of Commutative Rings*, J.Algebra, 116, (1988), 208-226.

[3] R.Ponraj, J.V.X.Parthipan, *Pair Sum Labeling of Graphs*, The Journal of Indian Academy of Mathematics, 32, No.2, (2010), 587-595.

[4] R.Ponraj,J.V.X.Parthipan and R.Kala, *Some Results on Pair Sum Labeling*, International Journal of Mathematical Combinatorics, 4,(2010), 55-61.

[5] R.Ponraj, J.V.X.Parthipan and R.Kala, *A Note on Pair Sum Graphs*, Journal of Scientific Research, 3, No.2, (2011), 321-329.

[6] R.Ponraj, J.V.X.Parthipan, *Further Results on Pair Sum Labeling of Trees*, Applied Mathematics, 32, No.2, (2011), 1270-1278.

[7] J.RaviSankar and S.Meena, *Changing and Unchanging the Domination Number of a Commutative ring* ,International Journal of Algebra, 6,No.27, (2012), 1343-1352.

[8] J.RaviSankar and S.Meena, *Connected Domination number of a commutative ring*, International Journal of Mathematical Research, 5, No.1, (2012), 5-11.

[9] J.RaviSankar and S.Meena, *On Weak Domination in a Zero Divisor Graph*, International Journal of Applied Mathematics, 26, No.1, (2013), 83-91.