FUZZY PRE GENERALIZED SEMI-CLOSED SETS

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Abstract : In this paper, we introduced a new classes of sets called fuzzy pre-generalized semi-closed sets in fuzzy topological spaces is introduced and investigate their properties. As an application of this set we also introduces the new kinds of separation axioms namely, $F_{pg}T_{1/2}$ -space F_{pgs}-continuity and F_{pgs} irresolute mappings. Fuzzy pre generalized semi $T_{1/2}$ fuzzy pre generalized semi $T_{3/4}$ spaces and fuzzy pre semi $T_{1/3}$ and characterized them.

Key words and phrases :Fuzzy pre generalized semi-closed sets, Fuzzy pre generalized semi-continuity, Fuzzy pre generalized semi-irresoluteness, Fuzzy semi-$T_{1/2}$space.

INTRODUCTION :

We define a new class of fuzzy pre generalized sets namely, fuzzy semi-closed sets and investigate their properties. The fts X denote a fuzzy topological spaces (X,τ). Fuzzy sets in X will be denoted by $(\upsilon,\eta,\lambda,\mu)$. The operators can be denoted by fuzzy closure and fuzzy interior. The concept of fuzzy semi-preopen sets and introduced fuzzy pre generalized $T_{1/2}$ spaces,F_{pgs} continuity and F_{pgs}- irresoluteness. The aim of this paper is to introduce the notion of fuzzy pre generalized semi-closed sets, an alternative generalization of fuzzy semi pre open set in fuzzy topological spaces. We introduce a class of fuzzy topological spaces, called $F_{pg}T_{1/2}$- spaces and obtain some of its characterizations. Further, we also introduce F_{pgs}- continuity and F_{pgs}- irresoluteness.

1.1 Definition : A fuzzy set A of (X,τ) is called

1) Fuzzy semi open (shortly, Fs- open) if $A \leq Cl\big(int(A)\big)$ and a fuzzy semi closed (shortly, Fs-closed) if $Int\big(Cl(A)\big) \leq A$.
2) Fuzzy pre open (shortly, Fp-open) if $A \leq Int\big(Cl(A)\big)$ and a fuzzy pre closed (shortly, Fp- closed) if $Cl\big(Int(A)\big) \leq A$.
3) Fuzzy pre semi open (shortly, Fps - open) if $A \leq Int\, Cl\big(Int(A)\big)$ and a fuzzy pre semi- closed (shortly Fps - closed) if $Cl\, Int\big(Cl(A)\big) \leq A$.

1.2 Definition :

A fuzzy set ν in fuzzt topological space (X,τ) is called

1) Fuzzy generalized closed set if $Cl\nu \leq \eta$ whenever $\nu \leq \eta$ and η is fuzzy open. We shortly denoted it as Fg- closed.
2) Fuzzy pre- generalized closed set if $pCl\nu \leq \eta$ whenever $\nu \leq \eta$ and ν is fuzzy semi open. We shortly denoted it as F_{pg}- closed set.
3) Fuzzy generalized semi- closed set if $sCl\nu \leq \mu$ whenever $\nu \leq \eta$ and η is fuzzy open. We shortly denoted it as F_{gs}- closed set.
4) Fuzzy pre generalized semi- closed set if $psCl\nu \leq \eta$ whenever $\nu \leq \eta$ and η is fuzzy open. We shortly denoted it as F_{pgs}- closed set.

1.3 Definition :

A fuzzy topological space (X,τ) is said to be a

1) Fuzzy - $T_{1/2}$ space if every F_g- closed set is fuzzy closed.
2) Fuzzy semi- $T_{1/2}$ space if every F_{sg}- closed set is fuzzy semi- closed.
3) Fuzzy pre- $T_{1/2}$ space if every F_{pg}- closed set is fuzzy pre closed.

FUZZY SEMI – CLOSED SETS

2.1 Definition :

Let η be a fuzzy set in a fuzzy topological spaces (X,τ)**.** Then ν is called a fuzzy semi- closed set X if $spCl\eta \leq \nu$, whenever $\eta \leq \nu$ and ν is a F_g- open set in X.

2.2 Proposition :

Every fuzzy semi pre-closed set in a fuzzy topological space (X,τ) is fuzzy pre semi-closed.

Proof :

Let η be a fuzzy semi pre-closed set in a fuzzy topological space (X,τ). Suppose that $\eta \leq \nu$ and ν is a fuzzy generalized-open set in X. Since $spCl\eta = \eta$, it follows that $spCl\eta = \eta \leq \nu$. Hence, η is fuzzy pre-semi-closed in X. The reverse implication in the above proposition is not true as seen in the following example.

Example :

Consider the fuzzy topological space (X,τ), where $X = \{a,b,c\}$ and

$$\tau = \left\{0,1,\eta = \frac{0.9}{a} + \frac{0.2}{b} + \frac{1}{c}, \nu = \frac{0.9}{a} + \frac{0}{b} + \frac{0}{c}\right\}.$$

Fuzzy closed sets in X are

$$0,1,\eta' = \frac{0.2}{a} + \frac{0.8}{b} + \frac{0}{c}, \nu' = \frac{0.2}{a} + \frac{1}{b} + \frac{1}{c}.$$

So the family of fuzzy generalized-closed sets is

$$\left\{0,1,\eta',\nu',\frac{\alpha_1}{a} + \frac{\alpha_2}{b} + \frac{\alpha_3}{c}\ either\ \alpha_1 > 0.8\ or\ \alpha_2 > 0.2\right\}.$$

Hence the family of generalized open sets is

$$\left\{0,1,\eta,\nu,\frac{\alpha_1}{a} + \frac{\alpha_2}{b} + \frac{\alpha_3}{c}\ either\ \alpha_1 < 0.2\ or\ \alpha_2 < 0.8\right\}.$$

Now

$$\nu = \frac{1}{a} + \frac{0.2}{b} + \frac{0}{c}$$

is not a fuzzy semi-pre closed set in X, for $Int\ \lambda = \nu$ and so, $Int\ Cl\ Int\ \lambda = Int\ Cl\ \nu = 1 > \lambda$.

Moreover, λ is fuzzy pre semi-closed. Indeed, let $\lambda \leq \mu$ and μ be fuzzy generalized open in X. Then $\mu = 1$ and $spCl\lambda \leq \mu$.

From the succeeding two examples, it can be seen that fuzzy pre-semi-closedness is independent from fuzzy generalized semi-closedness.

Example :

Consider the fuzzy topological space (X,τ), where $X = \{a,b,c\}$ and

$$\tau = \left\{0,1,\nu = \frac{0.7}{a} + \frac{0.3}{b} + \frac{0}{c}\right\}.$$

Fuzzy closed sets are :

$$0,1,\nu' = \frac{0.2}{a} + \frac{0.9}{b} + \frac{1}{c}$$

So the family of fuzzy generalized closed sets in X is

$$\left\{0,1,\nu', and\ \frac{\alpha_1}{a} + \frac{\alpha_2}{b} + \frac{\alpha_3}{c}\ either\ 0.7 < \alpha_1\ or\ 0.3 < \alpha_2\right\}.$$

Hence the family of fuzzy generalized-open sets is

$$\left\{0,1,\nu,\frac{\alpha_1}{a} + \frac{\alpha_2}{b} + \frac{\alpha_3}{c}\ either\ \alpha_1 < 0.2\ or\ \alpha_2 < 0.9\right\}$$

Now,

$$\lambda = \frac{0.7}{a} + \frac{0.2}{b} + \frac{0}{c}$$

is a fuzzy pre semi-closed set in X, for if $\lambda \leq \mu$ and μ is fuzzy generalized open set in X. Then $\mu = 1$. Hence, $spCl\lambda \leq \mu$. But, $Int\ Cl\ \lambda = 1, \lambda \leq \nu$ and $Int\ Cl\ \lambda = 1 > \nu$, so λ is not a fuzzy generalized closed set in X.

Example :

Consider the fuzzy topological space (X,τ) : $X = \{a,b,c\}$ and

$$\tau = \left\{0,1,\nu = \frac{0}{a} + \frac{1}{b} + \frac{1}{c}\right\}.$$

Fuzzy closed sets are :

$$0,1,\nu' = \frac{1}{a} + \frac{0}{b} + \frac{0}{c}.$$

So the family of fuzzy generalized closed sets in X is

$$\{0,1,\frac{\alpha_1}{a} + \frac{\alpha_2}{b} + \frac{\alpha_3}{c}\ either\ \alpha_1 > 0\ or\ \alpha_2 > 0\}.$$

Hence the family of fuzzy generalized open sets in X is

$$\left\{0,1,\nu,\frac{\alpha_1}{a} + \frac{\alpha_2}{b} + \frac{\alpha_3}{c}\ either\ \alpha_1 < 1\ or\ \alpha_2 < 1\right\}.$$

Now

$$\lambda = \frac{1}{a} + \frac{1}{b} + \frac{0}{c}$$

is not a fuzzy pre-semi-closed set in X; if $\lambda \leq \nu$ and λ is fuzzy generalized open set, but, $Int\ Cl\ Int\ \lambda = 1$, and hence, $spCl\lambda = 1 > \lambda$. But, λ is fuzzy generalized closed set in X and hence it is fuzzy generalized semi-closed.

2.3 Proposition :

Let η be a fuzzy set in a fuzzy topological spaces (X,τ). Then the following are equivalent :

i. η is fuzzy open and fuzzy pre semi- closed.
ii. η is fuzzy open and F_{pgs}- closed.

Proof :

Let η be fuzzy open and fuzzy pre semi- closed. Then, by known proposition,

"Every fuzzy pre semi- closed set in a fuzzy topological space (X,τ) is F_{pgs}- closed."

Proof :

Let λ be a fuzzy pre semi- closed set in a fuzzy topological space (X,τ). Suppose that $\lambda \leq \nu$ and ν is a fuzzy open set in X. Then, $psCl\lambda \leq \nu$ and ν is F_g- open in X and hence λ is fuzzy pre generalized semi- closed in X.

Hence, it is fuzzy pre generalized semi- closed.

2.3 Proposition :

Let η be a fuzzy pre-semi-closed set in a fuzzy topological space (X,τ). If η is a fuzzy set in X such that $\eta \leq \nu \leq spCl\eta$, then ν is also fuzzy pre-semi-closed.

Proof :

Let $\nu \leq \mu$ and μ be fuzzy generalized-open in X. Then $\eta \leq \lambda$ and since μ is fuzzy pre semi-closed. It is clear that, $spCl\nu \leq sp\ Cl(sp\ Cl\eta) = sp\ Cl\eta \leq \upsilon$. Hence, ν is fuzzy pre-semi-closed.

FUZZY PRE GENERALIZED SEMI - CLOSED SETS

3.1 Definition :

A fuzzy set A of (X,τ) is called fuzzy pre generalized semi- closed (shortly, F_{pgs}-closed) if $psCl(\nu) \leq \eta$, whenever $\nu \leq \eta$ and η is F_s- open in X.

By fuzzy pre generalized semi- closed (X,τ), we denote the family of all fuzzy pre generalized semi- closed sets of fuzzy topological space X.

Example :

Let $X = \{a,b\}$ and $Y = \{x,y,z\}$ and fuzzy sets A, B, E, H, K and M be defined by :

$A(a) = 0.3,\ A(b) = 0.4,\ B(a) = 0.4,\ B(b) = 0.5$;
$E(a) = 0.3,\ E(b) = 0.7,\ H(a) = 0.7,\ H(b) = 0.6$;
$K(x) = 0.1,\quad K(y) = 0.2,\quad K(z) = 0.7;$
$M(x) = 0.9,\quad M(y) = 0.2,\quad M(z) = 0.5.$

Let $\tau = \{0, A, 1\}$, $\sigma = \{0, E, 1\}$ and $\gamma = \{0, K, 1\}$. Then B is F_{pgs}- closed in (X,τ) but not F_s- closed ; M is F_{pgs}- closed in (Y,γ) but not F_{gs}- closed because, If we consider a fuzzy set $T(x) = 0.9, T(y) = 0.2, T(z) = 0.7$, then clearly $sCl(M) \nleq T$ where as $M \leq T$ and T is fuzzy semi open in (Y,γ) and H is F_{pgs}- closed in (X,σ) but neither F_{pgs}-closed because, If we consider a fuzzy set $L(a) = 0.8, L(b) = 0.7$, then clearly $spCl(H) \nleq L$ where as $H \leq L$ and L is fuzzy semiopen in (X,σ) nor F_{sp}- closed because $Int\left(Cl\big(Int(H)\big)\right) \nleq H$.

3.2 Theorem :

If A is fuzzy semi open and F_{pgs}- closed in (X,τ), then A is a F_{ps}- closed in (X,τ).

Proof :

Since $A \leq A$ and A is fuzzy semi open and F_{pgs}- closed, then $psCl(A) \leq A$. Since $A \leq psCl(A)$, we have $A = psCl(A)$ and thus A is a F_{ps}- closed set in X.

3.3 Theorem :

If A is a F_{pgs}- closed set of (X,τ) and $A \leq B \leq psCl(A)$, then B is a F_{pgs}- closed set of (X,τ).

Proof :

Let H be a F_s- open set of (X,τ) such that $B \leq H$. Then $A \leq H$. Since A is F_{pgs}- closed, it follows that $spCl(A) \leq H$. Now, $B \leq psCl(A)$ implies $psCl(B) \leq psCl\big(psCl(A)\big) = psCl(A)$. Thus, $psCl(B) \leq H$. This proves that B is also a F_{pgs}- closed set of (X,τ).

3.4 Definition :

A fuzzy set A of (X,τ) is called fuzzy pre generalized semi-open (shortly, F_{pgs}- open) iff $(1 - A)$ is F_{pgs}- closed set in X. That is , A is F_{pgs}- open iff $E \leq sp\ Int(A)$ whenever $E \leq A$ and E is a F_s- closed set in X.

3.5 Theorem :

Fuzzy pre semi- open $(X,\tau) \leq$ fuzzy pre generalized semi- open (X,τ).

Proof :

Let A any fuzzy pre semi- open set in X. Then, $1 - A$ is fuzzy pre semi- closed and hence, fuzzy pre generalized semi- closed. This implies that A is fuzzy pre generalized semi- open. Hence, B is fuzzy pre semi- open $(X,\tau) \leq$ fuzzy pre generalized semi open(X,τ).

3.6 Theorem :

Let A be fuzzy pre generalized semi- open in X and $ps\ Int(A) \leq B \leq A$, then B is fuzzy pre generalized semi- open.

Proof :

Suppose A is F_{pgs}- open in X and $ps\ Int\ A(A) \leq B \leq A$. Then $1 - A$ is F_{pgs}- closed and $1 - A \leq 1 - B \leq psCl(1 - A)$. Then $1 - A$ is F_{pgs}- closed and $1 - A \leq 1 - B \leq psCl(1 - A)$. Then $1 - B$ is F_{pgs}- closed set. Hence, B is F_{pgs}- open set in X.

FUZZY PRE GENERALIZED SEMI- IRRESOLUTE MAPPINGS :

4.1 Theorem :

Let $f: (X,\tau) \rightarrow (Y,\sigma)$ be F_{pgs}- irresolute, then f is F_{pgs}- continuous.

Proof :

Proof is immediate as every fuzzy closed set is F_{pgs}- closed and f is F_{pgs}- irresolute map.

4.2 Theorem :

Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be fuzzy irresolute and fuzzy semi- closed. Then for every F_{pgs}- closed set A of X, $f(A)$ is a F_{pgs}- closed in Y.

Proof :

Let A be a F_{pgs}- closed set of X. Let V be a fuzzy semi open set of Y containing $f(A)$. Since f is fuzzy irresolute, $f^{-1}(V)$ is a fuzzy semi open set of X. As $A \leq f^{-1}(V)$ and A is a F_{pgs}- closed in X, then $psCl(A) \leq f^{-1}(V)$ implies that $f\big(psCl(A)\big) \leq V$. Since f is fuzzy semi pre-closed, then $f\big(psCl(A)\big) = psCl\left(f\big(spCl(A)\big)\right)$.

Then, $spCl\big(f(A)\big) \leq spCl\left(f\big(spCl(A)\big)\right) = f\big(psCl(A)\big) \leq V$.

Therefore, $f(A)$ is a F_{pgs}- closed set in Y.

FUZZY PRE- SEMI- $T_{1/2}$ SPACES :

5.1 Proposition :

Every fuzzy pre semi- $T_{1/2}$ space is a fuzzy pre semi- $T_{1/3}$ space.

Proof :

Let (X,τ) be a fuzzy pre semi- $T_{1/2}$ space and μ be a F_{pgs}- closed set in (X,τ). Then μ is fuzzy pre semi- closed. Then, (X,τ) is a fuzzy semi- $T_{1/3}$ space.

5.2 Proposition :

Every fuzzy pre semi- $T_{3/4}$ space is a fuzzy pre semi- $T_{1/2}$ space.

Proof :

Let (X,τ) be a fuzzy pre semi- $T_{3/4}$ space and ν be a fuzzy pre semi- closed set in (X,τ). Then, ν is fuzzy pre closed. Then, (X,τ) is a fuzzy pre semi- $T_{1/2}$ space. Converse of the above proposition is not true as seen in the following example.

Example :

Let (X,τ) be a fuzzy topological space, where $X=\{a,b,c\}$ and

$$\tau=\left\{0,1,\nu_1=\frac{1}{a}+\frac{0}{b}+\frac{0}{c},\nu_2=\frac{0}{a}+\frac{1}{b}+\frac{0}{c},\nu_3=\frac{1}{a}+\frac{1}{b}+\frac{0}{c}\right\}.$$

Fuzzy closed sets in (X,τ) are

$$0,1,\nu'_1=\frac{0}{a}+\frac{1}{b}+\frac{1}{c},\nu'_2=\frac{1}{a}+\frac{0}{b}+\frac{1}{c},\nu'_3=\frac{0}{a}+\frac{0}{b}+\frac{1}{c}.$$

If η is fuzzy generalized-closed then $\eta\le\nu$ implies $Cl\eta\le\nu$ whenever ν is fuzzy open. Thus, fuzzy generalized-closed sets in (X,τ) are :

$$0,1,\nu'_1=\frac{0}{a}+\frac{1}{b}+\frac{1}{c},\nu'_2=\frac{1}{a}+\frac{0}{b}+\frac{1}{c},\nu'_3=\frac{0}{a}+\frac{0}{b}+\frac{1}{c}\ and\ \frac{\alpha_1}{a}+\frac{\alpha_2}{b}+\frac{\alpha_3}{c}$$

Where $\alpha_3\ne 0$. So, the family of fuzzy generalized-open sets in (X,τ) is

$$\left\{0,1,\frac{\alpha_1}{a}+\frac{\alpha_2}{b}+\frac{\alpha_3}{c}\ where\ \alpha_3\ne 1\right\}.$$

It is enough to prove that, if η is not fuzzy semi-pre-closed then it is not fuzzy pre-semi-closed and there is a fuzzy pre-semi-closed set which is not pre-closed. Let $\eta\ne 0$ be a fuzzy set in X. Then,

$$Int\ \eta=\begin{cases}0\\ \nu_1\\ \nu_2\\ \nu_3\\ 1\end{cases},\ Cl\ Int\ \eta=\begin{cases}0\\ \nu'_2\\ \nu'_3\\ 1\\ 1\end{cases}\ and\ Int\ Cl\ Int\ \eta=\begin{cases}0\\ \nu_1\\ \nu_2\\ 1\\ 1\end{cases}$$

So, η is not fuzzy semi-pre-closed if $\nu_3\le\eta$. In that case, η is also not fuzzy pre-semi-closed. For η is fuzzy generalized-open and $\eta\le\eta$. But $sp\ Cl\eta\ge\eta\vee Int\ Cl\ Int\ \nu=1>\eta$. Thus X is a pre-semi-$T_{1/2}$ space.

$$\nu=\frac{1}{a}+\frac{0.3}{b}+\frac{0}{c}$$

Is a fuzzy semi-pre-closed. Hence it is fuzzy pre-semi-closed but it is not fuzzy pre-closed, as $Cl\ Int\ \nu\ne\nu$.

CONCLUSION :

In this paper, by the introduction of fuzzy pre semi- closed sets, we have equivalences of fuzzy spaces namely, fuzzy pre semi- $T_{1/3}$ space, pre semi- $T_{3/4}$ space and fuzzy pre semi- $T_{1/2}$ space.

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