ON TYPE-2 FUZZY SOFT TOPOLOGICAL SPACES

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Abstract: In the present paper, a notion of type-2 fuzzy soft topological spaces is introduced and some of its important properties are studied. For this we have defined product of type-2 fuzzy soft sets, type-2 fuzzy soft product spaces, type-2 fuzzy soft continuous mappings etc. and their topological behaviours are examined.

Indexterms: Fuzzy soft sets, Fuzzy soft mappings, Fuzzy soft topologies, Type-2 fuzzy soft sets, Type-2 fuzzy soft mappings, Type-2 fuzzy soft topologies

INTRODUCTION:

In natural occurance of events, uncertainty is the most prevalent aspect and several theories were developed to deal with it. Probability theory deals with the uncertainties caused by the randomness of a situation but it can not handle other type of situations where the uncertainties evolving out of imprecise informations, limitations of computability and perceptibility etc. L.A. Zadeh [18], in 1965 introduced fuzzy set theory (FST) and attempted to formalize this grey area. Afterwards many generalizations, modifications and extensions of FST came up with different perspectives and several research works were published on fuzzy subgroups, fuzzy topological spaces, fuzzy topological spaces, fuzzy topological groups etc. In 1975, Zadeh [19] coined the notion of type-2 fuzzy sets which are characterized by fuzzy membership functions that are themselves type-1 fuzzy in nature. In the theory of fuzzy set as well as type-2 fuzzy sets, membership function play the main role and this function depends, on various factors and because of this many complexities are encountered while dealing with it. In 1999, D. Molodstov [11] proposed soft sets as an alternative way. He took parameterized family of sets instead of membership function. He also proved that Zadeh's fuzzy sets were spacial types of fuzzy soft sets. Therefore, it might be stated that in the field of applications in real life problems the parameterization approach of fuzzy soft set theory is better user friendly as compared to the membership function approach of fuzzy set theory. Since then research activities are going on with all these theories in their pure form as well as in their hybridizations. Currently research on fuzzy soft set theory are progressing at a very fast pace and in areas involving fuzzy soft groups [7, 9], fuzzy soft topology [3, 5, 6, 17], fuzzy soft topological groups [13, 14] etc. which are applied to decision making problems including texture classification, data analysis etc.

Recently in 2015, Rajashi Chatterjee et al. [4] proposed a parameterized structure for type-2 fuzzy sets and named it type-2 fuzzy soft sets. As a continuation and observing a huge potential of fuzzy soft set theory, it is natural to investigate the behavior of the topological structures in type-2 fuzzy soft set settings. In this paper, we have introduced a notion of type-2 fuzzy soft topologies and investigate some of its Important properties.

PRELIMINARIES

In this section, following [7, 8, 11, 15, 16], some definitions and results of fuzzy soft sets, fuzzy soft mappings and type-2 fuzzy soft mappings are given. Unless otherwise stated, X will be assumed to be an initial universal set, E will be taken to be a set of parameters, $P(X)$ denote the power set of X.

1 Definition :[11, 8]

A pair (F, A) where F is a mapping from $A \subseteq E$ to $P(X)$, is called a fuzzy soft set or type-1 fuzzy soft set over X. Let $S_1(X, E)$ denotes the set of all fuzzy soft sets or type-1 fuzzy soft sets over X under the parameter E.

2 Definition :

Let $(F, A), (G, B) \in S_1(X, E)$. Then

(i) (F, A) is said to be fuzzy soft subset of (G, B) if $A \subseteq B$ and $F(\alpha) \leq G(\alpha), \forall\, \alpha \in A$. This relation is denoted by $(F, A) \mathrel{\tilde{\sqsubseteq}} (G, B)$ [11, 8].

(ii) The complement of a fuzzy soft set (F, A) is defined as $(F, A)^C = (F^C, A)$, where $F^C(\alpha) = (F(\alpha))^C = X - F(\alpha), \forall\, \alpha \in A$ [11, 8].

(iii) (F, A) is said to be a null fuzzy soft set (an absolute fuzzy soft set) if $F(\alpha) = \phi (F(\alpha) = X), \forall\, \alpha \in A$. This is denoted by $(\tilde{\phi}, A)((\tilde{X}, A))$[10].

3 Definition : [11, 8]

Let $(F, A), (G, B) \in S_1(X, E)$. Then their

(a) Union, is a fuzzy soft set $(H, A \sqcup B) \in S_1(X, E)$, denoted by $(F, A) \mathbin{\tilde{\sqcup}} (G, B) = (H, A \sqcup B)$, is defined by $\forall\, \alpha \in (A \sqcup B)$

$$H(\alpha) = \begin{cases} F(\alpha) & if \alpha \in (A - B) \\ G(\alpha) & if \alpha \in (B - A) \\ F(\alpha) \sqcup G(\alpha) & if\ \alpha \in (A \sqcap B) \end{cases}$$

(b) Intersection, is a fuzzy soft set $(H, A \sqcap B) \in S_1(X, E)$, denoted by $(F, A) \tilde{\sqcap} (G, B) = (H, A \sqcap B)$, is defined by $H(\alpha) = F(\alpha) \sqcap G(\alpha), \forall \alpha \in (A \sqcap B)$.

(c) AND, is a fuzzy soft set $(H, A \times B) \in S(X, E \times E)$, denoted by $(F, A) \tilde{\wedge} (G, B) = (H, A \times B)$, is defined by $H(\alpha, \beta) = F(\alpha) \sqcap G(\beta), \forall(\alpha, \beta) \in (A \times B)$.

(d) OR, is a fuzzy soft set $(H, A \times B) \in S(X, E \times E)$, denoted by $(F, A) \tilde{\vee} (G, B) = (H, A \times B)$, is defined by $H(\alpha, \beta) = F(\alpha) \sqcup G(\beta), \forall (\alpha, \beta) \in (A \times B)$.

4 Definition : [15]

A fuzzy soft set $(E, A) \neq (\tilde{\phi}, A)$ over X is called a

(i) Generalized fuzzy soft element (or briefly g-fuzzy soft element) of X, if $E(\alpha) = \phi$ or $\exists\, x \in X$ such that $E(\alpha) = \{x\}, \forall \alpha \in A$.

(ii) Constant g-fuzzy soft element of X, if $\exists\, x \in X$ such that
$E(\alpha) = \{x\}, \forall \alpha \in A$.

(iii) Pseudo constant g-fuzzy soft element of X, if $\exists\, x \in X$ such that $E(\alpha) = \phi$ or $\{x\}$, $\forall \alpha \in A$.

We denotes E_x, the constant fuzzy soft element defined by, $E(\alpha) = \{x\}, \forall \alpha \in A$ and fuzzy soft element $E(\alpha) = \{x\}$ and $E(\beta) = \phi$, $\forall \beta (\neq \alpha) \in A$.

5 Definition : [15]

The fuzzy soft mapping $f_\varphi: S_1(X, A) \longrightarrow S_1(Y, B)$ is called fuzzy constant if $\exists$ a fuzzy soft element $(E, B) \in S_1(Y, B)$ such that, $f_\varphi[(F, A)] = (E, B), \forall (F, A) \left[\neq \left(\tilde{\phi}, A\right)\right] \in S_1(X, A)$.

6 Definition : [2]

Let $(F, A) \in S_1(X, A)$ and $(G, B) \in S_1(Y, B)$. The Cartesian product of (F, A) and (G, B) is denoted by $(F, A) \tilde{\times} (G, B) = (F \tilde{\times} G, A \times B) \in S(X \times Y, \times A \times B)$ and defined as $\forall (\alpha, \beta) \in A \times B, (F \tilde{\times} G)(\alpha, \beta) = F(\alpha) \times G(\beta)$.

TYPE-2 FUZZY SOFT SETS:

Following [4], the definition and operations of Type-2 fuzzy soft sets are given first and then we have introduced type-2 fuzzy soft mappings and study some of its properties.

7 Definition : [4]

The pair $[\mathcal{F}, A]$ where $\mathcal{F}$ is a mapping from A to $S_1(X, A)$ is called type-2 fuzzy soft set over (X, A). In this case, corresponding to each parameter $\alpha \in A, \exists$ a type-1 fuzzy soft set (F_α, A) (say) over (X, A) such that $\mathcal{F}(\alpha) = (F_\alpha, A)$. Where $F_\alpha: A \longrightarrow P(X)$. Also $S_2(X, A)$ denotes the set of all type-2 fuzzy soft sets over (X, A).

8 Definition : [4]

Let $[\mathcal{F}, A], [\mathcal{G}, A] \in S_2(X, A)$. Then

(i) $[\mathcal{F}, A]$ is said to be fuzzy soft subset of $[\mathcal{G}, A]$ if $\mathcal{F}(\alpha) \tilde{\sqsubseteq} \mathcal{G}(\alpha), \forall \alpha \in A$. This relation is denoted by $[\mathcal{F}, A] \sqsubseteq [\mathcal{G}, A]$.

(ii) The complement of a type-2 fuzzy soft set $[\mathcal{F}, A]$ is defined as $[\mathcal{F}, A]^C = [\mathcal{F}^C, A]$, Where
$\mathcal{F}^C(\alpha) = [\mathcal{F}(\alpha)]^C, \forall \alpha \in A$.

(iii) $[\mathcal{F}, A]$ is said to be a type-2 null fuzzy soft set (absolute fuzzy soft set) if
$\mathcal{F}(\alpha) = \left(\tilde{\phi}, A\right) \left(\mathcal{F}(\alpha) = \left(\tilde{X}, A\right)\right), \forall \alpha \in A$. This is denoted by $[\tilde{\phi}, A]$ ($[\tilde{x}$, A]).

9 Definition : [4]

Let $[\mathcal{F}, A], [\mathcal{G}, A] \in S_2(X, A)$. Then their

(a) Union (Intersection), is a type-2 fuzzy soft set $[\mathcal{H}, A]$, denoted by $[\mathcal{F}, A] \sqcup [\mathcal{G}, A] = [\mathcal{H}, A]$ ($[\mathcal{F}, A] \sqcap [\mathcal{G}, A] = [\mathcal{H}, A]$), is defined by $\mathcal{H}(\alpha) = \mathcal{F}(\alpha) \tilde{\sqcup} \mathcal{G}(\alpha) \left(\mathcal{H}(\alpha) = \mathcal{F}(\alpha) \tilde{\sqcap} \mathcal{G}(\alpha)\right), \forall \alpha \in A$.

(b) AND (OR), is a type-2 fuzzy soft set $[\mathcal{H}, A \times A]$, denoted by $[\mathcal{F}, A] \wedge [\mathcal{G}, A] = [\mathcal{H}, A \times A]$ ($[\mathcal{F}, A] \vee [\mathcal{G}, A] = [\mathcal{H}, A \times A]$), is defined by $\mathcal{H}(\alpha, \beta) = \mathcal{F}(\alpha) \tilde{\sqcap} \mathcal{G}(\beta)$ ($\mathcal{H}(\alpha, \beta) = \mathcal{F}(\alpha) \tilde{\sqcup} \mathcal{G}(\beta), \forall (\alpha, \beta) \in (A \times A)$.

10 Definition : [16]

Let $S_2(X, A)$ and $S_2(Y, B)$ be the families of all type-2 soft sets over (X, A) and (Y, B) respectively. The mapping $f_{\varphi_\Psi}: S_2(X, A) \longrightarrow S_2(Y, B)$ is called a type-2 fuzzy soft mapping, where $f_\varphi: S_1(X, A) \longrightarrow S_1(Y, B)$ is a type-1 soft mapping and $\Psi: A \longrightarrow B$ is a mapping. Also,

(i) The image of a type-2 fuzzy soft set $[\mathcal{F}, A] \in S_2(X, A)$ under the mapping f_{φ_Ψ} is denoted by $f_{\varphi_\Psi}\big[[\mathcal{F}, A]\big] = \big[f_{\varphi_\Psi}(\mathcal{F}), B\big]$, and is defined by $\forall \beta \in B$,

$$\left[f_{\varphi_\Psi}(\mathcal{F})\right](\beta) = \begin{cases} \tilde{\bigcup}_{\alpha \in \psi^{-1}(\beta)} \left[f_\varphi[\mathcal{F}(\alpha)]\right] & if\, \psi^{-1}(\beta) \neq \phi \\ \left(\tilde{\phi}, B\right) & otherwise \end{cases}$$

(ii) The inverse of a type-2 fuzzy soft set $[\mathcal{G}, B] \in S_2(Y, B)$ under the mapping f_{φ_Ψ} is denoted by $f_{\varphi_\Psi}^{-1}\big[[\mathcal{G}, B]\big] = [f_{\varphi_\Psi}^{-1}(\mathcal{G}), A]$, and is defined by $[f_{\varphi_\Psi}^{-1}(\mathcal{G})](\alpha) = f_\varphi^{-1}\big[\mathcal{G}[\psi(\alpha)]\big]$, $\forall \alpha \in A$.

(iii) The fuzzy soft mapping f_{φ_Ψ} is called injective (surjective) if f_φ and ψ are both injective (surjective).

(iv) The fuzzy soft mapping f_{φ_Ψ} is identity fuzzy soft mapping, if f_φ and ψ are both identity mappings,

1 Proposition : [16]

Let X and Y be two nonempty sets and $f_\varphi: S_2(X, A) \longrightarrow S_2(Y, B)$ be a type-2 fuzzy soft mapping. If $[\mathcal{F}, A], [\mathcal{F}_i, A] \in S_2(X, A)$ and $[\mathcal{G}, B], [\mathcal{G}_i, B] \in S_2(Y, B), i \in \Delta$. Then

(i) $[\mathcal{F}_1, A] \sqsubseteq [\mathcal{F}_2, A] \Longrightarrow f_{\varphi_\Psi}([\mathcal{F}_1, A]) \sqsubseteq f_{\varphi_\Psi}([\mathcal{F}_2, A])$;

(ii) $[\mathcal{G}_1, B] \sqsubseteq [\mathcal{G}_2, B] \Longrightarrow f_{\varphi\Psi}^{-1}([\mathcal{G}_1, B]) \sqsubseteq f_{\varphi\Psi}^{-1}([\mathcal{G}_2, B])$;

(iii) $[\mathcal{F}, A] \sqsubseteq f_{\varphi\Psi}^{-1}\left(f_{\varphi\Psi}([\mathcal{F}, A])\right)$, the equality holds if $f_{\varphi\Psi}$ is injective.

(iv) $f_{\varphi\Psi}\left(f_{\varphi\Psi}^{-1}([\mathcal{G}, B])\right) \sqsubseteq [\mathcal{G}, B]$, the equality holds if $f_{\varphi\Psi}$ is surjective.

(v) $f_{\varphi\Psi}[\sqcup_{i\in\Delta}[\mathcal{F}_i, A]] = \sqcup_{i\in\Delta}[f_{\varphi\Psi}([\mathcal{F}_i, A])]$;

(vi) $f_{\varphi\Psi}[\sqcap_{i\in\Delta}[\mathcal{F}_i, A]] = \sqcap_{i\in\Delta}[f_{\varphi\Psi}([\mathcal{F}_i, A])]$, the equality holds if $f_{\varphi\Psi}$ is injective.

(vii) $f_{\varphi\Psi}^{-1}[\sqcup_{i\in\Delta}[\mathcal{G}_i, B]] = \sqcup_{i\in\Delta}[f_{\varphi\Psi}^{-1}([\mathcal{G}_i, B])]$;

(viii) $f_{\varphi\Psi}^{-1}[\sqcap_{i\in\Delta}[[\mathcal{G}_i, B]] = \sqcap_{i\in\Delta}[f_{\varphi\Psi}^{-1}([\mathcal{G}_i, B])]$.

11 Definition :

Let τ be a family of fuzzy sets over (X, A). Define $\tau(e) = \{F(e): F \in \tau\}$ for $e \in A$. This τ said to be a topology of fuzzy soft subsets over (X, A) if $\tau(e)$ is a crisp topology on $X, \forall\, e \in A$.

12 Definition :

Let (X, A) be the universe. Let $T: A \longrightarrow P(P(X)^A)$ be a fuzzy soft over $(P(X)^A, A)$. Now T is said to be a fuzzy soft topology over (X, A) if for each $e \in A, T(e)$ is a of fuzzy soft subsets over (X, A).

TYPE-2 FUZZY SOFT TOPOLOGIES:

In this section, we have introduced type-2 soft topology and study some of its important properties. Unless otherwise stated. Let X be a non empty set. A be the set of parameters and $S_2(X, A)$ be the family of all type-2 soft sets over (X, A).

13 Definition :

A subcollection $\mathcal{I}$ of $S_2(X, A)$ is said to be a type-2 fuzzy soft topology on X if the following properties are satisfied;

(i) $[\tilde{\phi}, A], [\tilde{\mathfrak{x}}, A] \in \mathcal{I}$, where $\tilde{\phi}(\alpha) = (\tilde{\phi}, A)$ and $\tilde{\mathfrak{x}}(\alpha) = (\tilde{X}, A), \forall\, \alpha \in A$;

(ii) The intersection of any two type-2 fuzzy soft sets in τ belongs to τ.

(iii) The union of any number of type-2 fuzzy soft sets in τ belongs to τ.

The triple $[X, A, \mathcal{I}]$ is called a type-2 fuzzy soft topological space over X.

Example 1:

Let X be a non-empty set and A be a set of parameters. Then $\mathcal{I}_1 = \{[\tilde{\phi}, A], [\tilde{\mathfrak{x}}, A]\}$, $\mathcal{I}_2 = S_2(X, A)$ and $\mathcal{I}_3 = \{[\tilde{\phi}, A], [\mathcal{F}, A], [\tilde{\mathfrak{x}}, A]\}$, where $[\mathcal{F}, A](\neq [\tilde{\phi}, A])$ be any type-2 fuzzy soft set are the examples of type-2 fuzzy soft topologies. $\mathcal{I}_1, \mathcal{I}_2$ are called type-2 indiscrete fuzzy soft topology and type-2 discrete fuzzy soft topology on X respectively.

Remark:

The union of any two type-2 fuzzy soft topologies on X may not be a type-2 fuzzy soft topology on X in general, which shows the following example.

Example 2:

Let X be a nonempty set, Y is a proper subset of X and $A = \{\alpha, \beta\}$. Let $[\mathcal{F}_1, A] = \{(\tilde{Y}, A), (\tilde{\phi}, A)\}$ and $[\mathcal{F}_2, A] = \{(\tilde{\phi}, A), (\tilde{Y}, A)\}$ be two type-2 fuzzy soft set. Then $\mathcal{I}_1 = \{[\tilde{\phi}, A], [\mathcal{F}_1, A], [\tilde{\mathfrak{x}}, A]\}, \tau_2\{[\tilde{\phi}, A], [\mathcal{F}_2, A], [\tilde{\mathfrak{x}}, A]\}$ are type-2 fuzzy soft topologies on X but their union $\mathcal{I}_1 \sqcup \mathcal{I}_2 = \{[\tilde{\phi}, A], [\mathcal{F}_1, A], [\mathcal{F}_2, A], [\tilde{\mathfrak{x}}, A]\}$ is not a type-2 fuzzy soft topology on X.

2 Proposition :

Let $[X, A, \mathcal{I}]$ be a type-2 fuzzy soft topological space over X. Then for each $\alpha \in A$, the collection $\mathcal{J}^\alpha = \{\mathcal{F}(\alpha): [\mathcal{F}, A] \in \mathcal{J}\}$ defines a fuzzy soft topology of Shabir and Naz [13] on X.

Proof:

Let $[X, A, \mathcal{I}]$ be a type-2 fuzzy soft topological space over X and α be any member of A. Since $[\tilde{\phi}, A], [\tilde{\mathfrak{x}}, A] \in \mathcal{I}$, it follows that $[\tilde{\phi}, A], [\tilde{\mathfrak{x}}, A] \in \mathcal{J}^\alpha$.

Again let $(\mathcal{F}_1, A), (\mathcal{F}_2, A)$ be any two members of $\in \mathcal{I}^\alpha$. Then $\exists$ two type-2 fuzzy soft sets $[\mathcal{F}_1, A], [(\mathcal{F}_1, A]$ in $\mathcal{J}$ such that $\mathcal{F}_1(\alpha) = (F_1, A)$ and $\mathcal{F}_2(\alpha) = (F_2, A)$. Thus $[\mathcal{F}_1, A] \sqcap [\mathcal{F}_2, A] = [\mathcal{F}_1 \sqcap \mathcal{F}_2, A]$ is a member of $\mathcal{J}$ and hence, $(\mathcal{F}_1 \sqcap \mathcal{F}_2)(\alpha) = \mathcal{F}_1(\alpha) \,\tilde{\cap}\, \mathcal{F}_2(\alpha) = (F_1, A) \,\tilde{\cap}\, (F_2, A)$ is a member of $\mathcal{I}^\alpha$.

Next, let $(\mathcal{F}_i, A)$, $i \in \Delta$ be any collection of members of $\mathcal{I}^\alpha$. Then $\exists$ type-2 fuzzy soft sets $[\mathcal{F}_i, A], i \in \Delta$ in $\mathcal{I}$ such that $\mathcal{F}_i(\alpha) = (\mathcal{F}_i, A)$, $i \in \Delta$. Thus $\sqcup_{i\in\Delta} [\mathcal{F}_i, A] = [\sqcup_{i\in\Delta} \mathcal{F}_1, A]$ is a member of $\mathcal{I}$ and hence,

$(\sqcup_{i\in\Delta} \mathcal{F}_i)(\alpha) = \tilde{\cup}_{i\in\Delta} \mathcal{F}_1(\alpha) = \tilde{\cup}_{i\in\Delta}(F_i, A)$ is a member of $\mathcal{I}^\alpha$. Therefore, $\mathcal{I}^\alpha$ is a fuzzy soft topology of Shabir and Naz over $X, \forall\, \alpha \in A$.

3 Proposition :

Let X be a non empty set and A be a set of parmeters. If $\mathcal{I}: A \longrightarrow P(S_1(X, A))$ be a mapping defined by $\mathcal{J}(\alpha)$ is a fuzzy soft topology over $X, \forall\, \alpha \in A$, then $\exists$ a type-2 fuzzy soft topology $\mathcal{I}^*$ over X such that $[\mathcal{I}^*]^\alpha = \mathcal{I}(\alpha), \forall\, \alpha \in A$.

Proof:

Define $\mathcal{I}^* \in S_2(X, A): \mathcal{I}(\alpha) \in \mathcal{I}(\alpha),\ \forall\, \alpha \in A\}$. Since $\tilde{\Phi}(\alpha) = (\tilde{\phi}, A) \in \mathcal{I}(\alpha), \forall\, \alpha \in A \Rightarrow [\tilde{\Phi}, A] \in \mathcal{I}^*$ and $\tilde{\mathfrak{x}}(\alpha) = (\tilde{X}, A) \in \mathcal{I}(\alpha), \forall\, \alpha \in A \Rightarrow [\tilde{\mathfrak{x}}, A] \in \mathcal{I}^*$.

Again let $(\mathcal{F}_1, A), (\mathcal{F}_2, A) \in \mathcal{I}^*$. Then $\mathcal{F}_1(\alpha), \mathcal{F}_2(\alpha) \in \mathcal{J}(\alpha), \forall\, \alpha \in A$.

Thus $\mathcal{F}_1(\alpha) \,\tilde{\cap}\, \mathcal{F}_2(\alpha) = [\mathcal{F}_1 \sqcap \mathcal{F}_2](\alpha) \in \mathcal{I}(\alpha), \forall\, \alpha \in A$.

Therefore $[\mathcal{F}_1, A] \sqcap [\mathcal{F}_2, A] = [\mathcal{F}_1 \sqcap \mathcal{F}_2, A] \in \mathcal{J}^*$.

Also let $[\mathcal{F}_i, A] \in \mathcal{J}^*, \forall\, i \in \Delta$. Then $\mathcal{F}_i(\alpha) \in \mathcal{J}(\alpha), \forall\, \alpha \in A, \forall\, i \in \Delta$. Thus, $\tilde{\cup}_{i\in\Delta} \mathcal{F}_i(\alpha) = [\sqcup_{i\in\Delta} \mathcal{F}_i](\alpha) \in \mathcal{J}(\alpha), \forall\, \alpha \in A$. Therefore $[\sqcup_{i\in\Delta} \mathcal{F}_i, A] = [\sqcup_{i\in\Delta} \mathcal{F}_i, A] \in \mathcal{J}^*$. Therefore $\mathcal{J}^*$ is a type-2 fuzzy soft topology on X.

Next let $(U, A) \in \tau(\alpha)$. Now we can define a type-2 fuzzy soft set $[\mathcal{F}, A]$ by $\mathcal{F}(\alpha) = (U, A)$ and $\mathcal{F}(\beta) = (\phi, A), \forall\, \beta \neq \alpha$. Then $[\mathcal{F}, A] \in \mathcal{J}^*$ and $(U, A) = \mathcal{F}(\alpha) \in [\mathcal{J}^*]^\alpha$. Therefore, $\mathcal{J}(\alpha) \subseteq [\mathcal{J}^*]^\alpha$ ……..(1). Also let $(V, A) = [\mathcal{J}^*]^\alpha$. Then $\exists\, [\mathcal{F}, A] \in \mathcal{J}^*$ such that $(V, A) = \mathcal{J}(\alpha) \in \mathcal{J}(\alpha)$. Therefore, $[\mathcal{J}^*]^\alpha \subseteq \mathcal{J}(\alpha)$ ……..(2). Therefore, from (1) and (2) we get $\mathcal{J}(\alpha) = [\mathcal{J}^*]^\alpha, \forall\, \alpha \in A$.

Example 3:

Let X be a non-empty set and $A = \{\alpha, \beta\}$. Let $\mathcal{J} = \{[\widetilde{\Phi}, A], [\widetilde{\mathrm{X}}, A]\}$ be al type-2 fuzzy soft topology on X. Then $\mathcal{J}^\alpha = \mathcal{J}^\beta = \{(\tilde{\phi}, A), (\widetilde{\mathrm{X}}, A)\}$.

Proof:

Let $[\mathcal{F}_1, A] = \{(\widetilde{\mathrm{X}}, A)/\alpha, (\tilde{\phi}, A)/\beta\}$ and $[\mathcal{F}_2, A] = \{(\tilde{\phi}, A)/\alpha, (\widetilde{\mathrm{X}}, A)/\beta\}$. Thus $\mathcal{J}^* = \{[\widetilde{\Phi}, A], [\mathcal{F}_1, A], [\mathcal{F}_2, A], [\tilde{\mathfrak{x}}, A]\}$. So, $\mathcal{J}^*$ is different from $\mathcal{J}$ but $[\mathcal{J}^*]^\alpha = [\mathcal{J}]^\alpha, \forall\, \alpha \in A$.

Example 4:

Let X be a non-empty set and $A = \{\alpha, \beta\}$. Define a soft set $T: A \longrightarrow P(P(X)^A)$ by $T(\alpha) = \{(F, A), (G, A)\}$, $T(\beta) = \{(F, A), (G, A)\}$ Where $F(\alpha) = \phi, F(\beta) = X, G(\beta) = \phi$. Then T is a Hazra et al. type fuzzy soft topology on X, but T is not a type-2 fuzzy soft topology on X since $T(\alpha) = \{(F, A), (G, A)\}$ is not a Naz type fuzzy soft topology on X.

Again we define a fuzzy soft set $T: A \longrightarrow P(P(X)^A)$ by $T^*(\alpha) = \{(F, A): F(\beta) \in [T(\alpha)](\beta), \forall\, \beta \in A$. of Hazra et al., a type-2 fuzzy soft topology and $[T^*(\alpha)](\beta) = [T(\alpha)](\beta), \forall\, \alpha, \beta \in A$.

14 Definition :

Let X and Y be two non-empty sets, $\mathcal{J}, \mathcal{U}$ be two type-2 fuzzy soft topologies on X, Y respectively and $f_{\varphi\psi}: [X, A, \mathcal{J}] \longrightarrow [Y, B, \mathcal{U}]$ be a type-2 fuzzy soft mapping. The image of $\mathcal{J}$ and the preimage of $\mathcal{U}$ under $f_{\varphi\psi}$ are denoted by $f_{\varphi\psi}(\mathcal{J})$ and $f_{\varphi\psi}^{-1}(\mathcal{U})$ respectively, defined by

(i) $f_{\varphi\psi}(\mathcal{J}) = \{[\mathcal{G}, B] \in S_2(Y, B): f_{\varphi\psi}^{-1}([\mathcal{G}, B]) = [f_{\varphi\psi}^{-1}(\mathcal{G}), A] \in \mathcal{J}\}$ and

(ii) $f_{\varphi\psi}^{-1}(\mathcal{U}) = \{f_{\varphi\psi}^{-1}([\mathcal{G}, B]) = [f_{\varphi\psi}^{-1}(\mathcal{G}), A]: [\mathcal{G}, B] \in \mathcal{U}\}$

4 Proposition :

Let $[X, A, \tau]$ be a type-2 fuzzy soft topological space and $G \subseteq X$. Then $\mathcal{J}_G = \{[F, A] \sqcap [\tilde{\mathcal{G}}, A]: [F, A] \in \mathcal{J}\}$, where $\tilde{\mathcal{G}}(\alpha) = (\tilde{G}, A), \forall \alpha \in A$ is type-2 fuzzy soft topology on G.

Proof:

(i) Since $[\widetilde{\Phi}, A]$ and $[\tilde{\mathfrak{x}}, A] \in \mathcal{J} \Rightarrow [\widetilde{\Phi}, A]$ and $[\tilde{\mathcal{G}}, A] \in \mathcal{J}_G$.

(ii) Let $[\mathcal{H}_1, A], [\mathcal{H}_2, A] \in \mathcal{J}_G$. Then $\exists [\mathcal{F}_1, A], [\mathcal{F}_2, A] \in \mathcal{J}$ such that $[\mathcal{H}_1, A] = [\mathcal{F}_2, A] \sqcap [\tilde{\mathcal{G}}, A]$ and $[\mathcal{H}_2, A] = [\mathcal{F}_2, A] \sqcap [\tilde{\mathcal{G}}, A]$. Since $\mathcal{J}$ is a type-2 fuzzy soft topology, it follows that $[\mathcal{F}_1, A] \sqcap [\mathcal{F}_2, A] \in \mathcal{J}$ and $[\mathcal{H}_1, A] \sqcap [\mathcal{H}_2, A] = [\mathcal{F}_1, A] \sqcap [\mathcal{F}_2, A] \sqcap [\tilde{\mathcal{G}}, A] \in \mathcal{J}_G$.

(iii) Next let $[\mathcal{H}_i, A] \in \mathcal{J}_G, i \in \Delta$. Then $\exists [\mathcal{F}_i, A] \in \mathcal{J}$ such that $[\mathcal{H}_i, A] = [\mathcal{F}_i, A] \sqcap [\tilde{\mathcal{G}}, A]$ $i \in \Delta$. Since ,$\mathcal{J}$ is a type-2 fuzzy soft topology, it implies that $\sqcup_{i\in\Delta} [\mathcal{H}_i, A] = \sqcup_{i\in\Delta} ([\mathcal{F}_i, A] \sqcap [\tilde{\mathcal{G}}, A]) = [\sqcup_{i\in\Delta} [\mathcal{F}_i, A]] \sqcap [\tilde{\mathcal{G}}, A] \in \mathcal{J}_G$. Therefore, $\mathcal{J}_G$ is a type-2 fuzzy soft topology on G.

15 Definition :

Let $[X, A, \mathcal{J}]$ and $[Y, B, \delta]$ be two type-2 fuzzy soft topological spaces. The type-2 fuzzy soft mapping $f_{\varphi_\Psi}: [X, A, \mathcal{J}] \longrightarrow [Y, B, \delta]$ is said to be

(i) Type-2 fuzzy soft continuous if $f_{\varphi_\Psi}^{-1}([\mathcal{G}, B]) \in \mathcal{J}, \forall\, [\mathcal{G}, B] \in \mathcal{U}$.

(ii) Type-2 fuzzy soft open if $f_{\varphi_\Psi}([\mathcal{F}, A]) \in \delta, \forall\, [\mathcal{F}, A] \in \mathcal{J}$.

5 Proposition :

Let $[X, A, \mathcal{J}]$ and $[Y, B, \delta]$ be two type-2 fuzzy soft topological spaces. A type-2 fuzzy soft mapping $f_{\varphi_\Psi}: [X, A, \mathcal{J}] \longrightarrow [Y, B, \delta]$ is type-2 fuzzy soft continuous iff $\forall$ type-2 fuzzy soft element $[\varepsilon, A]$ of X and

$\forall\, [\mathcal{G}, B] \in \mathcal{U}$ such that $f_{\varphi_\Psi}([\varepsilon, A]) \sqsubseteq [\mathcal{G}, B], \exists [\mathcal{F}, A] \in \mathcal{J}$ such that $[\varepsilon, A] \sqsubseteq [\mathcal{F}, A]$ and $f_{\varphi_\Psi}([\mathcal{F}, A]) \sqsubseteq [\mathcal{G}, B]$.

Proof:

Let $f_{\varphi_\Psi}: [X, A, \mathcal{J}] \longrightarrow [Y, B, \delta]$ be type-2 fuzzy soft continuous. Let n $[\varepsilon, A]$ be any type-2 fuzzy soft element in X and $[\mathcal{G}, B] \in \mathcal{U}$ such that $f_{\varphi_\Psi}([\mathcal{G}, B]) \sqsubseteq [\mathcal{G}, B]$. Then $[f_{\varphi_\Psi}([\varepsilon, A])(\beta) \mathrel{\tilde{\sqsubseteq}} \mathcal{G}(\beta), \forall\, \beta \in B$.

Since f_{φ_Ψ} is type-2 fuzzy soft continuous, it follows that $f_{\varphi_\Psi}^{-1}([\mathcal{G}, B]) = [f_{\varphi_\Psi}^{-1}(\mathcal{G}), A] \in \mathcal{J}$.

Let $[\mathcal{F}, A] = [f_{\varphi_\Psi}^{-1}(\mathcal{G}), A]$. Now for each $\alpha \in A$.

$$\mathcal{F}(\alpha) = [f_{\varphi_\Psi}^{-1}(\mathcal{G})](\alpha) = f_\varphi^{-1}[\mathcal{G}(\Psi(\alpha))] \mathrel{\tilde{\sqsupseteq}} f_\varphi^{-1}[[f_{\varphi_\Psi}([\varepsilon, A])](\Psi(\alpha))]$$
$$= f_{\varphi_\Psi}^{-1}\left[[f_{\varphi_\Psi}([\varepsilon, A])](\alpha)\right] = \left[f_{\varphi_\Psi}^{-1}[f_{\varphi_\Psi}([\varepsilon, A])]\right](\alpha) \mathrel{\tilde{\sqsupseteq}} [\varepsilon, A](\alpha)$$

Thus $[\varepsilon, A] \sqsubseteq [\mathcal{F}, A]$ and $f_{\varphi_\Psi}\left(f_{\varphi_\Psi}^{-1}([\mathcal{G}, B])\right) \sqsubseteq [\mathcal{G}, B]$.

Conversely, let the given condition be satisfied.

Let $[\mathcal{G}, B] \in \mathcal{U}$. We are to show that $f_{\varphi_\Psi}^{-1}([\mathcal{G}, B]) \in \mathcal{J}$. Let $[\varepsilon, A]$ be any type-2 fuzzy soft element of X such that $[\varepsilon, A] \sqsubseteq f_{\varphi_\Psi}^{-1}([\mathcal{G}, B])$. Then $f_{\varphi_\Psi}([\varepsilon, A]) \sqsubseteq f_{\varphi_\Psi}\left(f_{\varphi_\Psi}^{-1}([\mathcal{G}, B])\right) \sqsubseteq [\mathcal{G}, B]$. So by the given condition $\exists\, [\mathcal{F}_\epsilon. A] \in \mathcal{J}$ such that $[\varepsilon, A] \sqsubseteq [\mathcal{F}_\epsilon. A]$ and $f_{\varphi_\Psi}([\mathcal{F}_\epsilon. A]) \sqsubseteq [\mathcal{G}, B]$

i.e., $[\mathcal{F}_\epsilon. A] \sqsubseteq f_{\varphi_\Psi}^{-1}[f_{\varphi_\Psi}([\mathcal{F}_\epsilon. A])] \sqsubseteq f_{\varphi_\Psi}^{-1}([\mathcal{G}, B])$. Now,

$f_{\varphi_\Psi}^{-1}([\mathcal{G}, B]) = \sqcup \{[\mathcal{G}, A]: [\mathcal{G}, A]$ be any type-2 fuzzy soft element of X such that $[\varepsilon, A] \sqsubseteq f_{\varphi_\Psi}^{-1}([\mathcal{G}, B])\}$.

$\sqsubseteq \sqcup \{[\mathcal{F}_\epsilon. A]: [\varepsilon, A]$ be any type-2 fuzzy soft element of X such that $[\varepsilon, A] \sqsubseteq f_{\varphi_\Psi}^{-1}([\mathcal{G}, B])\}$.

$\sqsubseteq f_{\varphi_\Psi}^{-1}([\mathcal{G}, B])$.

Thus $f_{\varphi_\Psi}^{-1}([\mathcal{G}, B]) = \sqcup \{[\mathcal{F}_\epsilon. A]\}$ where each $[\mathcal{F}_\epsilon. A] \in \mathcal{J}$ and hence, $f_{\varphi_\Psi}^{-1}([\mathcal{G}, B]) \in \mathcal{J}$. Therefore f_{φ_Ψ} is type-2 fuzzy soft continuous.

6 Proposition :

Let $[X, A, \mathcal{J}]$ be a type-2 fuzzy soft topological space. Then the identity type-2 fuzzy soft mapping $f_{\varphi_\Psi}: [X, A, \mathcal{J}] \longrightarrow [X, A, \mathcal{J}]$ is type-2 fuzzy soft continuous.

Proof:

Since f_{φ_Ψ} is an identity type-2 fuzzy soft mapping, it follows that f_φ and Ψ are identity mappings. Let $[\mathcal{F}, A] \in \mathcal{J}$. Then, $[f_{\varphi_\Psi}^{-1}([\mathcal{F}, A])(\alpha) = f_\varphi^{-1}[\mathcal{F}(\Psi(\alpha))] = f_\varphi^{-1}[\mathcal{F}(\alpha)] = \mathcal{F}(\alpha), \forall\, \alpha \in A$. Thus $f_{\varphi_\Psi}^{-1}([\mathcal{F}, A]) = [\mathcal{F}, A]$. Therefore, the identity type-2 fuzzy soft mapping $f_{\varphi_\Psi}: [X, A, \mathcal{J}] \longrightarrow [X, A, \mathcal{J}]$ is type-2 fuzzy soft continuous.

7 Proposition :

Let $[X, A, \mathcal{J}]$ and $[Y, B, \delta]$ be two type-2 fuzzy soft topological spaces. If $\mathcal{J}$ contains all those type-2 fuzzy soft sets of the form $[(\widetilde{X_0, A}), A]$ where X_0 be any subset of X, then the constant type-2 fuzzy soft mapping $f_{\varphi_\Psi}: [X, A, \mathcal{J}] \longrightarrow [Y, B, \delta]$ is type-2 fuzzy soft continuous.

Proof:

Since $f_{\varphi_\Psi}: [X, A, \mathcal{J}] \longrightarrow [Y, B, \delta]$ is a constant type-2 fuzzy soft mapping, it follows that the mappings $f: X \longrightarrow Y, \Psi: \mathrm{A} \longrightarrow B$ and $\varphi: A \longrightarrow B$ are constant. Let $\Psi(\alpha) = \beta'$ and $\varphi(\alpha) = \beta''\ \forall\, \alpha \in A$. Let $[\mathcal{G}, B]$ be any member of δ. Then for any $\alpha \in A$

$$[f_{\varphi_\Psi}^{-1}([\mathcal{G}, B])(\alpha) = f_\varphi^{-1}[\mathcal{G}(\Psi(\alpha))] = f_\varphi^{-1}[\mathcal{G}(\beta')] = \begin{cases} (\tilde{\phi}, A), \forall\, \alpha \in A\ if\, [\mathcal{G}(\beta')](\beta'') = \phi \\ (\widetilde{X_0}, A), where\ X_0 \subseteq X, \forall\, \alpha \in A\ if\ [\mathcal{G}(\beta')](\beta'') \\ \neq \phi \end{cases}$$

Hence,

$f_{\varphi_\Psi}^{-1}([\mathcal{G}, B]) = [\tilde{\Phi}, A]$ or $[(\widetilde{X_0, A})\ A]$. Since $[\tilde{\Phi}, A], [(\widetilde{X_0, A}), A]\mathcal{J}$, it follows that the constant type-2 fuzzy soft mapping $f_{\varphi_\Psi}: [X, A, \mathcal{J}] \longrightarrow [Y, B, \delta]$ is type-2 fuzzy soft continuous.

8 Proposition :

Let $[X, A, \mathcal{J}], [Y, B, \mathcal{U}]$ and $[Z, C, \mathcal{W}]$ be type -2 fuzzy soft topological spaces. If $f_{\varphi_\Psi}: [X, A, \mathcal{J}\,] \longrightarrow [Y, B, \mathcal{U}]$ and $g_{\varepsilon_\eta:}\ [Y, B, \mathcal{U}] \longrightarrow [Z, C, \mathcal{W}]$ are type-2 fuzzy soft continuous, then the fuzzy soft mapping $g_{\varepsilon_\eta} \circ f_{\varphi_\Psi} = (gf)_{\varepsilon\varphi_{\eta\psi}}$: $[X, A, \mathcal{J}] \longrightarrow [Z, C, \mathcal{W}]$ is type-2 fuzzy soft continuous.

Proof:

Proof is straightforward.

16 Definition :

Let $[X, A, \mathcal{J}]$ be a type-2 fuzzy soft topological spaces. A subcollection $\mathbb{B}$ of $\mathcal{J}$ is said to be an open base for $\mathcal{J}$ if every member of $\mathcal{J}$ can be expressed as the union of some members of $\mathbb{B}$.

9 Proposition :

Let $[X, A, \mathcal{J}]$ and $[Y, B, \mathcal{U}]$ be type-2 fuzzy soft topological spaces. Then $\mathbb{F} = \{[\mathcal{F}, A] \times [\mathcal{G}, B]: [\mathcal{F}, A] \in \mathcal{J}, [\mathcal{G}, B] \in \mathcal{U}\}$ forms an open base of a type- fuzzy soft topology of $\mathbb{B}$.

We note that $[\tilde{\Phi}, A \times B] = [\tilde{\mathfrak{x}}, A] \times [\tilde{\Phi}, B] = [\tilde{\Phi}, A] \times [\tilde{\mathcal{Y}}, B]$. Since $[\tilde{\Phi}, A], [\tilde{\mathfrak{x}}, A] \in \mathcal{J}$ and $[\tilde{\Phi}, B], [\tilde{\mathcal{Y}}, B] \in \mathcal{U}$, it follows that $[\tilde{\Phi}, A \times B], [\mathcal{X} \times \mathcal{Y}, A \times B] \in \mathbb{F}$. Again let $[\mathcal{F}_1, A] \times [\mathcal{G}_1, B], [\mathcal{F}_2, A] \times [\mathcal{G}_2, B]$be any two member of $\mathbb{F}$, where $[\mathcal{F}_1, A], [\mathcal{F}_2, A] \in \mathcal{J}$ and $[\mathcal{G}_1, B], [\mathcal{G}_2, B] \in \mathcal{U}$ .So $[\mathcal{F}_1, A] \sqcap [\mathcal{F}_2, A] \in \mathcal{J}$ and $[\mathcal{G}_1, B] \sqcap [\mathcal{G}_2, B] \in \mathcal{U}$. Thus $[[\mathcal{F}_1, A] \times [\mathcal{G}_1, B]] \sqcap [[\mathcal{F}_2, A] \times [\mathcal{G}_2, B]] = [[\mathcal{F}_1, A] \sqcap [\mathcal{F}_2, A]] \times [[\mathcal{G}_1, B] \sqcap [\mathcal{G}_2, B]] \in \mathbb{F}$. Therefore $\mathbb{F}$ forms an open base of the type-2 fuzzy soft topology on $X \times Y$.

17 Definition :

The type-2 fuzzy soft topology in $X \times Y$ induced by the open base $\mathbb{F}$ is said to be the product type-2 fuzzy soft topology of the type-2 fuzzy soft topology $\mathcal{J}$ and $\mathcal{U}$. It is denoted by $\mathcal{J} \times \mathcal{U}$. The type-2 fuzzy soft topological space $[X \times Y, A \times B, \mathcal{J} \times \mathcal{U}]$ is said to be the type-2 fuzzy soft topological product of the type-2 fuzzy soft topological spaces $[X, A, \mathcal{J}]$ and $[Y, B, \mathcal{U}]$.

10 Proposition :

Let $[X, A, \mathcal{J}]$ and $[Y, B, \mathcal{U}]$ be two type-2 fuzzy soft topological spaces. Then the projection fuzzy soft mappings $[\pi^X]_{\varphi^A_{\psi^A}}: [X \times Y, A \times B, \mathcal{J} \times \mathcal{U}] \longrightarrow [X, A, \mathcal{J}]$ and $[\pi^Y]_{\varphi^B_{\psi^B}}: [X \times Y, A \times B, \mathcal{J} \times \mathcal{U}] \longrightarrow [Y, B, \delta]$ are type-2 fuzzy soft continuous and type-2 fuzzy soft open where $\pi^X: X \times Y \longrightarrow X, \varphi^A: A \times B \longrightarrow B$ and $\varphi^A: A \times B \longrightarrow A$ are mappings defined by $\pi^X(x, y) = x, \varphi^A(\alpha, \beta) = \alpha$ and $\varphi^A(\alpha, \beta) = \alpha$. Also $\mathcal{J} \times \mathcal{U}$ is the smallest type-2 fuzzy soft topology in $X \times Y$ for which the projection fuzzy soft mappings are type-2 fuzzy soft continuous.

Proof:

$[\mathcal{F}, A] \in \mathcal{J} \Rightarrow [\pi^X]^{-1}{}_{\varphi^A_{\psi^A}}([\mathcal{F}, A]) = [\mathcal{F}, A] \times [\widetilde{\mathcal{Y}}, B]$ is a basic open set in $\mathcal{J} \times \mathcal{U}$. So $\pi^X{}_{\varphi^A_{\psi^A}}$ is type-2 fuzzy soft continuous.

Again let $[\mathcal{F}, A \times B] \in \mathcal{J} \times \mathcal{U}$. Then there exists a subfamily $\mathbb{F}'$ of $\mathbb{F}$ such that $[\mathcal{F}, A \times B]$ is the union of the members of $\mathbb{F}'$. So,

$$\pi^X{}_{\varphi^A_{\psi^A}}([\mathcal{F}, A \times B]) = \sqcup \left\{\pi^X{}_{\varphi^A_{\psi^A}}[[\mathcal{G}, A] \times [\mathcal{H}, B]]: [\mathcal{G}, A] \times [\mathcal{H}, B] \in \mathbb{F}'\right\}$$
$$= \sqcup \{[\mathcal{G}, A]: [\mathcal{G}, A] \times [\mathcal{H}, B] \in \mathbb{F}'\} \in \mathcal{J}\ \ (\text{since } [\mathcal{G}, A] \in \mathcal{J}\ ).$$

Therefore, $\pi^X{}_{\varphi^A_{\psi^A}}$ is type-2 fuzzy soft open.

Similarly, it can be shown that $\pi^Y{}_{\varphi^B_{\psi^B}}$ is also type-2 fuzzy soft continuous and type-2 fuzzy soft open.

Next let $\mathcal{w}$ be any type-2 fuzzy soft topology on $X \times Y$ such that fuzzy soft mappings $[\pi^X]_{\varphi^A_{\psi^A}}: [X \times Y, A \times B, \mathcal{J} \times \mathcal{u}] \longrightarrow [X, A, \mathcal{J}]$ and $[\pi^Y]_{\varphi^B_{\psi^B}}: [X \times Y, A \times B, \mathcal{J} \times \mathcal{u}] \longrightarrow [Y, B, \mathcal{u}]$ are type-2 fuzzy soft continuous.

Let $[[\mathcal{F}, A] \times [\mathcal{G}, B]]$ be any basic open type-2 fuzzy set in $\mathcal{J} \times \mathcal{u}$. Now,

$$[[\mathcal{F}, A] \times [\mathcal{G}, B]] = [[\mathcal{F}, A] \sqcap [\tilde{x}, A]] \times [[\tilde{y}, B] \sqcap [\mathcal{G}, B]$$
$$= [[\mathcal{F}, A] \times [\tilde{y}, B]] \sqcap [[\tilde{x}, A] \times [\mathcal{G}, B]]$$
$$= \left(\prod\nolimits^X_{\phi^A_{\Psi^A}}\right)^{-1} [[\mathcal{F}, A]] \sqcap \left(\prod\nolimits^X_{\phi^A_{\Psi^A}}\right)^{-1} [\mathcal{G}, B] \in \mathcal{w}$$

(since $\left(\prod^X_{\phi^A_{\Psi^A}}\right)^{-1} [[\mathcal{F}, A]], \left(\prod^X_{\phi^A_{\Psi^A}}\right)^{-1} [\, [\mathcal{G}, B]\,] \in \mathcal{w}$)

Thus $\mathcal{J} \times \mathcal{u}$ is a subcollection of $\mathcal{w}$. Therefore $\mathcal{J} \times \mathcal{u}$ is the smallest type-2 fuzzy soft topology in $X \times Y$ for which the projection fuzzy soft mappings are type-2 fuzzy soft continuous.

11 Proposition :

Let $[X \times Y, A \times B, \mathcal{J} \times \mathcal{u}]$ be the product space of two type-2 fuzzy soft topological spaces $[X, A, \mathcal{J}]$ and $[Y, B, \mathcal{u}]$ and $[\pi^X]_{\varphi^A_{\psi^A}}: [X \times Y, A \times B, \mathcal{J} \times \mathcal{u}] \longrightarrow [X, A, \mathcal{J}]$ and $[\pi^Y]_{\varphi^B_{\psi^B}}: [X \times Y, A \times B, \mathcal{J} \times \mathcal{u}] \longrightarrow [Y, B, \mathcal{u}]$ be the fuzzy soft projection mappings. If $[Z, C, \mathcal{w}]$ be any type-2 fuzzy soft topological space, then the fuzzy soft mapping $f_{\varphi\Psi}: [Z, C, \mathcal{w}] \longrightarrow [X \times Y, A \times B, \mathcal{J} \times \mathcal{u}]$ is fuzzy soft type-2 continuous iff the fuzzy soft mappings $[\pi^X]_{\varphi^A_{\psi^A}} \circ f_{\varphi\Psi}: [Z, C, \mathcal{w}] \longrightarrow [X, A, \mathcal{J}]$ and $[\pi^Y]_{\varphi^B_{\psi^B}} \circ f_{\varphi\Psi} : [Z, C, \mathcal{w}] \longrightarrow [Y, B, \mathcal{u}]$ are type-2 fuzzy soft continuous.

Proof:

First let the fuzzy soft mapping $f_{\varphi\Psi}: [Z, C, \mathcal{w}] \longrightarrow [X \times Y, A \times B, \mathcal{J} \times \mathcal{u}]$ be type-2 fuzzy soft continuous. Also the fuzzy soft mappings $[\pi^X]_{\varphi^A_{\psi^A}}: [X \times Y, A \times B, \mathcal{J} \times \mathcal{u}] \longrightarrow [X, A, \mathcal{J}]$ and $[\pi^Y]_{\varphi^B_{\psi^B}}: [X \times Y, A \times B, \mathcal{J} \times \mathcal{u}] \longrightarrow [Y, B, \mathcal{u}]$ are type-2 fuzzy soft continuous. Then by proposition 17, we have the composition fuzzy soft mappings $[\pi^X]_{\varphi^A_{\psi^A}} \circ f_{\varphi\Psi}: [Z, C, \mathcal{w}] \longrightarrow [X, A, \mathcal{J}]$ and $[\pi^Y]_{\varphi^B_{\psi^B}} \circ f_{\varphi\Psi} : [Z, C, \mathcal{w}] \longrightarrow [Y, B, \mathcal{u}]$ are type-2 fuzzy soft continuous.

Conversely, let the fuzzy soft mappings $[\pi^X]_{\varphi^A_{\psi^A}}: [X \times Y, A \times B, \mathcal{J} \times \mathcal{u}] \longrightarrow [X, A, \mathcal{J}]$ and $[\pi^Y]_{\varphi^B_{\psi^B}}: [X \times Y , A \times B, \mathcal{J} \times \mathcal{u}] \longrightarrow [Y, B, \mathcal{u}]$ are type-2 fuzzy soft continuous. Let $[\mathcal{F}, A \times B] \in (\mathcal{J} \times \mathcal{u})$ and $\mathbb{F} = \{[\mathcal{G}, A] \times [\mathcal{H}, B]: [\mathcal{G}, A] \in \mathcal{J}, [\mathcal{H}, B] \in \mathcal{u}\}$.
Then $\exists$ a subfamily $\mathbb{F}' = \{[\mathcal{G}_k, A] \times [\mathcal{H}_k, B]: k \in \Delta$ of $\mathbb{F}$ such that $[\mathcal{F}, A \times B] = \sqcup_{k \in \Delta} [[\mathcal{G}_k, A] \times [\mathcal{H}_k, B]]$. Thus, $\pi^X{}_{\varphi^A_{\psi^A}}$

$$[f_{\varphi\Psi}]^{-1}[[\mathcal{F}, A \times B]] = f^{-1}_{\varphi\Psi}[\sqcup_{k \in \Delta} [[\mathcal{G}_k, A] \times [\mathcal{H}_k, B]]]$$
$$= f^{-1}_{\varphi\Psi}[\sqcup_{k \in \Delta} [(\pi^X{}_{\varphi^A_{\psi^A}})^{-1}[\mathcal{G}_k, A] \sqcap (\pi^Y{}_{\varphi^B_{\psi^B}})^{-1}[\mathcal{H}_k, B]$$
$$= \sqcup_{k \in \Delta} (\pi^X{}_{\varphi^A_{\psi^A}} \circ f_{\varphi\Psi})^{-1}([\mathcal{G}_k, A]) \sqcap (\pi^Y{}_{\varphi^B_{\psi^B}} \circ f_{\varphi\Psi})^{-1}([\mathcal{H}_k, B]) \in \mathcal{w}.$$

Therefore $f_{\varphi\Psi}: [Z, C, \mathcal{w}] \longrightarrow [X \times Y , A \times B, \mathcal{J} \times \mathcal{u}]$ is type-2 fuzzy soft continuous.

12 Proposition :

Let $[X \times Y, A \times B, \mathcal{J} \times \mathcal{u}]$ be the product space of two type-2 fuzzy soft topological spaces $[X, A, \mathcal{J}]$ and $[Y, B, \mathcal{u}]$, where $\mathcal{u}$ contains all those type-2 fuzzy soft sets of the form $[\widetilde{(Y_0, B)}, B]$ where Y_0 be any subset of Y.
Then for fixed $x_0 \in X, \alpha_0, \alpha_1 \in A$, the type-2 fuzzy soft mapping $f_{\varphi\Psi}: [Y, B, \mathcal{u}] \longrightarrow [X \times Y, A \times B, \mathcal{J} \times \mathcal{u}]$, defined by $f(y) = (x_0, y), \varphi(\beta) = (\alpha_0, \beta)$ and $\psi(\beta) = (\alpha_1, \beta), \forall\, y \in Y, \forall \beta \in B$ is type-2 fuzzy soft continuous.

Proof:

Let $\pi^X_{\varphi^A_{\psi^A}}: [X \times Y, A \times B, \mathcal{J} \times \mathcal{u}] \longrightarrow [X, A, \tau]$ and $\pi^B_{\varphi^B_{\psi^B}}: [X \times Y,\ A \times B, \mathcal{J} \times \mathcal{u}] \longrightarrow [Y, B, \mathcal{u}]$ be the soft projection mappings. Then $\pi^X_{\varphi^A_{\psi^A}} \circ f_{\varphi\Psi}: [Y, B, \mathcal{u}] \longrightarrow [X, A, \mathcal{J}]$ is a type-2 fuzzy soft mapping such that $[\pi^A \circ f](y) = x_0, \forall\, y \in Y [\varphi^{A} \circ \varphi](\beta) = \alpha_0$ and $[\psi^{A} \circ \psi] = \alpha_1, \forall\, \beta \in B$. So, $\pi^X_{\varphi^A_{\psi^A}} \circ f_{\varphi\Psi}$ is a constant type-2 fuzzy soft mapping and $\mathcal{u}$ contains all those type-2 fuzzy soft sets of the form $\left[\widetilde{(Y_0, B)}, B\right]$, Y_0 be any subset of Y. Hence the fuzzy soft mapping $\pi^X_{\varphi^A_{\psi^A}} \circ f_{\varphi\Psi}$ is type-2 fuzzy soft continuous.

Again $\pi^X_{\varphi^A_{\psi^A}} \circ f_{\varphi\Psi}$: $[Y, B, \mathcal{u}] \longrightarrow [Y, B, \mathcal{u}]$ is a type-2 fuzzy soft mapping such that $[\pi^B \circ f](y) = y, \forall\, y \in Y, [\varphi^{A} \circ \varphi](\beta) = \beta$ and $[\psi^{B} \circ \psi] = \beta, \forall\, \beta \in B$. so $\pi^B_{\varphi^B_{\psi^B}} \circ f_{\varphi\Psi}$ is an identity type-2 fuzzy soft mapping. Hence the fuzzy soft mapping $\pi^B_{\varphi^B_{\psi^B}} \circ f_{\varphi\Psi}$ is type-2 fuzzy soft continuous.

Therefore, for fixed $x_0 \in X, \alpha_0, \alpha_1 \in A$, the type-2 fuzzy soft mapping $f_{\varphi\Psi}: [Y, B, \mathcal{u}] \longrightarrow [X \times Y, A \times B, \mathcal{J} \times \mathcal{u}]$, defined by $f(y) = (x_0, y), \varphi(\beta) = (\alpha_0, \beta)$ and $\psi(\beta) = (\alpha_1, \beta), \forall\, y \in Y, \forall\, \beta \in B$ is type-2 fuzzy soft continuous.

13 Proposition :

Let $[X \times Y, A \times B, \mathcal{J} \times \mathcal{u}]$ be the product space of two type-2 fuzzy soft topological spaces $[X, A, \mathcal{J}]$ and $[Y, B, \mathcal{u}]$, where $\mathcal{J}$ contains all those type-2 fuzzy soft sets of the form $\left[\widetilde{(X_0, A)}, A\right]$ where X_0 be any subset of X. Then for fixed $y_0 \in Y, \beta_0, \beta_1 \in B$, the type-2 fuzzy soft mapping $f_{\varphi\Psi}: [X, A, \mathcal{J}] \longrightarrow [X \times Y, A \times B, \mathcal{J} \times \mathcal{u}]$, defined by $f(x) = (x, y_0), \varphi(\alpha) = (\alpha, \beta_0)$ and $\psi(\alpha) = (\alpha, \beta_1), \forall\, x \in X,\ \forall\, \alpha \in A$ is type-2 fuzzy soft continuous.

Proof:

Proof is similar to that of Proposition 12.

14 Proposition :

Let $f_{\varphi_\Psi}: [X, A, \mathcal{J}] \longrightarrow [X, A, \mathcal{J}]$ and $g_{\xi_\eta}: [Y, B, \mathcal{u}] \longrightarrow [Y, B, \mathcal{u}]$ be two type-2 fuzzy soft continuous mappings. Then the type-2 fuzzy soft mapping $h_{\rho_\zeta}: [X \times Y, A \times B, \mathcal{J} \times \mathcal{u}] \longrightarrow [X \times Y, A \times B, \mathcal{J} \times \mathcal{u}]$, where $h(x, y) = \big(f(x), g(y)\big), \rho(\alpha, \beta) = (\varphi(\alpha), \xi(\beta))$ and $\zeta(\alpha, \beta) = (\psi(\alpha), \eta(\beta))$ is type-2 fuzzy soft continuous.

Proof:

Let $[\mathcal{F}, A \times B] \in (\mathcal{J} \times \mathcal{u})$. Then there exist $[\mathcal{G}_i, A] \in \mathcal{J}$ and $[\mathcal{H}_i, B] \in \mathcal{u}, i \in \Delta$ such that $[\mathcal{F}, A \times B] = \sqcup_{i \in \Delta} [[\mathcal{G}_i, A] \times [\mathcal{H}_i, B]]$. Since f_{φ_Ψ} and $g_{\xi_\eta}$are type-2 fuzzy soft continuous, it follows that $f_{\varphi_\Psi}^{-1}[[\mathcal{G}_i, A]] \in \mathcal{J}$ and $g_{\xi_\eta}^{-1}[[\mathcal{H}_i, B]] \in \mathcal{u}$. Now $\forall\ (\alpha, \beta) \in (A \times B)$, we have,

$$
\begin{aligned}
h_\rho^{-1}[(G_i, A) \tilde{\times} (H_i, B)](\alpha, \beta) &= h^{-1}[[G_i \tilde{\times} H_i]\big(\rho(\alpha, \beta)\big)] \\
&= h^{-1}[[G_i \tilde{\times} H_i]\big(\varphi(\alpha), \xi(\beta)\big)] \\
&= h^{-1}[G_i\big(\varphi(\alpha)\big) \times H_i\big(\xi(\beta)\big)] \\
&= f^{-1}[G_i\big(\varphi(\alpha)\big) \times g^{-1}[H_i\big(\xi(\beta)\big)] \\
&= [f_\varphi^{-1}(G_i) \tilde{\times} g_\xi^{-1}(H_i)(\alpha, \beta).
\end{aligned}
$$

Therefore, $h_\rho^{-1}[(G_i, A) \tilde{\times} (H_i, B)] = (f_\varphi^{-1}(G_i), A) \tilde{\times} g_\xi^{-1}(H_i), B)$. Thus

$$
\begin{aligned}
[h_{\rho_\xi}^{-1}\big[[[\mathcal{G}_i, A] \times [[\mathcal{H}_i, B]\big](\alpha, \beta) &= h_\rho^{-1}[[\mathcal{G}_i \times \mathcal{H}_i](\zeta(\alpha, \beta))] \\
&= h_\rho^{-1}[[\mathcal{G}_i \times \mathcal{H}_i](\psi(\alpha), \eta(\beta))] \\
&= h_\rho^{-1}[\mathcal{G}_i\big(\psi(\alpha)\big) \tilde{\times} \mathcal{H}_i(\eta(\beta))] \\
&= f_\varphi^{-1}[\mathcal{G}_i\big(\psi(\alpha)\big)] \tilde{\times} g_\xi^{-1}\mathcal{H}_i(\eta(\beta))] \\
&= [f_{\varphi_\Psi}^{-1}[[\mathcal{G}_i, A](\alpha)] \tilde{\times} [g_{\xi_\eta}^{-1}[[\mathcal{H}_i, B]](\beta)] \\
&= [f_{\varphi_\Psi}^{-1}[[\mathcal{G}_i, A]]\ \times g_{\xi_\eta}^{-1}[[\mathcal{H}_i, B]]](\alpha, \beta).
\end{aligned}
$$

Therefore,

$$h_{\rho_\xi}^{-1}\big[[[\mathcal{G}_i, A] \times [[\mathcal{H}_i, B]\big] = f_{\varphi_\Psi}^{-1}[[\mathcal{G}_i, A]]\ \times g_{\xi_\eta}^{-1}[[\mathcal{H}_i, B]] \in (\mathcal{J} \times \mathcal{u})$$

and hence,

$$h_{\rho_\xi}^{-1}\big[[\mathcal{F}, A \times B]\big] = h_{\rho_\xi}^{-1}[\sqcup_{i \in \Delta} [[\mathcal{G}_i, A]] \times [[\mathcal{H}_i, B]]] = \sqcup_{i \in \Delta} [[\mathcal{G}_i, A]] \times [[\mathcal{H}_i, B]] \in (\mathcal{J} \times \mathcal{u}).$$

Therefore, the type-2 fuzzy soft mapping $h\rho_\zeta: [X \times Y, A \times B, \mathcal{J} \times \mathcal{u}] \longrightarrow [X \times Y, A \times B, \mathcal{J} \times \mathcal{u}]$, where $h(x, y) = \big(f(x), g(y)\big), \rho(\alpha, \beta) = (\varphi(\alpha), \xi(\beta))$ and $\zeta(\alpha, \beta) = (\psi(\alpha), \eta(\beta))$ is type-2 fuzzy soft continuous.

CONCLUSION:

In this paper, concepts of type-2 fuzzy soft topology is introduced and studied of its important properties. There is an wide scope for further research to extend it in topological group theory which have may applications in abstract integration theory viz. Haar measure, Haar integral etc. and also in manifold theory through the development of Lie groups.

REFERENCES:

1. H. Aktas and N. Cagman, Soft Sets and Soft Groups, Inform Sci.177 (13) (2007), 2726-2735.
2. B. Ahmad and A. Kharal, On Fuzzy Soft Sets, Adv Fuzzy syst, 2009 (2009).
3. Athar Kharal and B. Ahmad., Mappings on fuzzy soft classes, Advances in Fuzzy Systems, 2009.
4. Chakraborty Rp, Mukherjee P(2015), On generalized fuzzy soft topological spaces, Afr J. Math. Comput. Sci. Res 8:1-11.
5. F. Feng, Y.B. Jun, X.Y. Liu and L.F. Li, An adjustable approach to fuzzy soft set based decision making, J Comput. Appl. Math. 234(2010) 10-20.
6. P.H. Khedr, S.A Abd El-Baki and M.s. Malfi, Results on generalized fuzzy soft topological spaces, Vol.11(3), PP: 35-45, April 2018.
7. Maji P.k, Biswas R. Roy A.R: fuzzy soft sets, Journal of fuzzy Mathematics, 9(3), (2001), PP: 589-602.
8. Maji P.K, Biswas R. Roy A.R: soft set theory, Computers and Mathematics with Applications. 45(2003), PP: 19-31.
9. Maji P.K and Roy, A.R, An application of soft sets in a decision making problem, Compu. Math. Appl., 44(2002), 1077-1083.
10. Maji P.K, Biswas R. Roy AR (2001a). Fuzzy soft sets, Journal of fuzzy Mathematics, Vol 9, PP: 589-602, 2001.
11. Majumdar P, Samanta SK (2010), Generalized fuzzy soft sets, Comp. Math. Appl 59:1425-1432.
12. Nazmul, SK and Samanta, S.K., Some properties of soft groups and fuzzy soft groups under soft mappings, accepted at PJM, on Dec, 2016.,
13. Shabir. M and Naz. M., On fuzzy soft topological spaces, Comput.Math. Appl., 61(7) (2011).
14. TanayB, Burc Kandemir M (2011). Topological structure of fuzzy soft sets, Compt. Math. Appl. 61: 2952-2957.
15. B.K. Tripathy, Sooraj. T.R, R.K Mohanty, A New Approach to fuzzy soft set theory and its application in decision making, Dec (2015).
16. Yang HL(2011). Notes on Generalized fuzzy soft sets, J. Math Res. Expo. 31: 567-570.
17. L.A Zadeh: Fuzzy sets, Information and Control.8(1965), PP: 338-353.