g*-Binary Regular Closed and Open Sets in g-Binary Topological Spaces

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Abstract: In this paper we introduce g*-binary regular closed and open sets and investigate some of their basic properties in g-binary topological space

Index Terms - g-binary regular open, g-binary regular closed, g*-binary regular open, g*-binary regular closed.

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1. INTRODUCTION

The concept of binary topology is recently introduced and studied some of its basic properties, where a binary topology from X to Y is a binary structure satisfying certain axioms that are analogous to the axioms of topology. The purpose is to introduce g*-binary regular closed and open sets and investigate some of their basic properties in g-binary topological space. Section 2 deals with the basic concepts of g-binary topology. In section 3 g*-binary regular closed and open sets are studied and investigated some of the basic properties. Throughout the paper $\wp(X)$ denotes the power set of X.

2. Preliminaries

Definition 2.1: Let X and Y are any two non-empty sets. A generalized binary topology (or g-binary topology) from X to Y is a binary structure $M_g \subseteq \wp(X) \times \wp(Y)$ that satisfies the following axioms:

i) $(\emptyset, \emptyset)$ and $(X, X) \in M_g$

ii) If $\{(A_\alpha, B_\alpha); \alpha \in \Delta\}$ is a family of members of M_g, then $(\bigcup_{\alpha\in\Delta} A_\alpha, \bigcup_{\alpha\in\Delta} B_\alpha) \in M_g$

If M_g is a generalized binary topology from X to Y, then the triplet (X, Y, M_g) is called a generalized binary topological space (g-binary topological space) and the members of $M_g$are called the g-binary open subsets of the g-binary topological space (X, Y, M_g). The elements of $X \times Y$ are called the g-binary points of g-binary topological space (X, Y, M_g).

Definition 2.2: Let (X, Y, M_g) be a g-binary topological space and $A \subseteq X, B \subseteq Y$. Then (A, B) is g-binary closed in (X, Y, M_g) if $(X\backslash A, Y\backslash B) \in M_g$.

Proposition 2.1: Let (X, Y, M_g) be a g-binary topological space. Then

i) (X, Y) and $(\emptyset, \emptyset)$ are g-binary closed sets.

ii) If $\{(A_\alpha, B_\alpha): \alpha \in \Delta\}$ is a family of g-binary closed sets, then $(\bigcap_{\alpha\in\Delta} A_\alpha, \bigcap_{\alpha\in\Delta} B_\alpha)$ is g-binary closed.

Definition 2.3: Let (X, Y, M_g) be a g-binary topological space and $(A, B) \subseteq (X, Y)$. Let $(A, B)^{1^*}{}_g = \bigcap\{A_\alpha: (A_\alpha, B_\alpha)$ is g-binary closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$ and Let $(A, B)^{2^*}{}_g = \bigcap\{B_\alpha: (A_\alpha, B_\alpha)$ is g-binary closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$.Then $(A, B)^{1^*}{}_g, (A, B)^{2^*}{}_g)$is g-binary closed and $(A, B) \subseteq (A, B)^{1^*}{}_g, (A, B)^{2^*}{}_g)$. The ordered pair $\left((A, B)^{1^*}{}_g, (A, B)^{2^*}{}_g)\right)$ is called g-binary closure of (A, B) and is denoted $gbcl(A, B)$ in the g-binary topology (X, Y, M_g) where $(A, B) \subseteq (X, Y)$.

Proposition 2.2: Let $(A, B) \subseteq (X, Y)$. Then (A, B) is g-binary closed in (X, Y, M_g) iff $(A, B) = gbcl(A, B)$.

Definition 2.4: Let (X, Y, M_g) be a g-binary topological space and $(A, B) \subseteq (X, Y)$. Let $(A, B)^{1^0}{}_g = \cup\{A_\alpha: (A_\alpha, B_\alpha)$ is g-binary open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$ and Let $(A, B)^{2^0}{}_g = \cup\{B_\alpha: (A_\alpha, B_\alpha)$ is g-binary open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$.Then $((A, B)^{1^0}{}_g, (A, B)^{2^0}{}_g)$ is g-binary open and $((A, B)^{1^0}{}_g, (A, B)^{2^0}{}_g) \subseteq (A, B)$. The ordered pair $((A, B)^{1^0}{}_g, (A, B)^{2^0}{}_g)$ is called g-binary interior of (A, B) and is denoted by $gbint(A, b)$.

Proposition 2.3: Let $(A, B) \subseteq (X, Y)$. Then (A, B) is g-binary open in (X, Y, M_g) iff $(A, B) = gbint(A, B)$.

3. g*-Binary Regular Closed and Open Sets

Definition 3.1: Let (X, Y, M_g) be a g-binary topological space. Let $(A, B) \in \wp(X) \times \wp(Y)$. Then (A, B) is g*-binary closed if $gbcl(A, B) \subseteq (V, U)$ whenver $(A, B) \subseteq (V, U)$ and (V, U) is g-binary open in (X, Y, M_g).

Example 3.1: Let $X = \{1,2,3\}$ and $Y = \{a, b\}$. Then $M_g = \{(\emptyset, \emptyset), (\{\emptyset\}, \{b\}), (\{X\}, \{a\}), (\{1,2\}, \{Y\}), (X, Y)\}$ is g-binary topology. Clearly $(\emptyset, \emptyset), (\{X\}, \{a\}), (\{\emptyset\}, \{b\})$ and (X, Y) are g-binary closed sets in (X, Y, M_g). Let $(A, B) = (\{2\}, \{Y\}) \in \wp(X) \times \wp(Y)$. Then $gbcl(A, B) = (X, Y) \subseteq (X, Y)$ where $(A, B) = (\{2\}, \{Y\}) \subseteq (X, Y)$ and (X, Y) is g-binary open. Therefore $(A, B) = (\{2\}, \{Y\})$ is g*-binary closed. In this way we can prove all g-binary closed sets are g*-binary closed.

Proposition 3.1: Every g-binary closed set in a g-binary topological space is g*-binary closed.

Definition 3.2: Let (X, Y, M_g) be a g-binary topological space. Let $(A, B) \subseteq (X, Y)$. Then (A, B) is called g-binary regular open if $(A, B) = gbint(gbcl(A, B))$ and g-binary regular closed if $(A, B) = gbcl(gbint(A, B))$.

Proposition 3.2: Every g-binary regular open set in a g-binary topological space is g-binary open.

Proof: Let (A, B) be a g-binary regular open in g-binary topological space (X, Y, M_g). Therefore, we have $(A, B) = gbint(gbcl(A, B))$. This implies$(A, B) \subseteq gbint(A, B)$. But $gbint(A, B) \subseteq (A, B)$. Thus $(A, B) = gbint(A, B)$. Hence (A, B) is g-binary open.

Remark 3.1: Converse of Proposition 3.2 need not be true shown in Example 3.2.

Example 3.2: Let $X = \{1,2\}$, $Y = \{a, b, c\}$ and $M_g = \{(\emptyset, \emptyset), (\{1\}, \{a\}), (\{2\}, \{b, c\}), (\{X\}, \{a, c\}), (X, Y)\}$. Clearly M_g is g-binary topology from X to Y. Then $(\emptyset, \emptyset), (\{1\}, \{a\}), (\{2\}, \{b, c\})$ and (X, Y) are g-binary closed sets. Clearly the set $(\{X\}, \{a, c\})$ is g-binary open but not g-binary regular open.

Definition 3.3: Let (X, Y, M_g) be a g-binary topological space. Let $(A, B) \subseteq (X, Y)$. Then (A, B) is called g*-binary regular closed if $gbcl(A, B) \subseteq (U, V)$, whenever $(A, B) \subseteq (U, V)$ and (U, V) g-binary regular open in (X, Y, M_g).
Proposition 3.3: Every g*-binary closed set in a g-binary topological space is g*-binary regular closed.
Proof: Let (A, B) be a g*-binary closed in g-binary topological space (X, Y, M_g). Suppose $(A, B) \subseteq (U, V)$ where (U, V) g-binary regular open. Then by Proposition 3.1, we have (U, V) is g-binary open. Since (A, B) is g*-binary closed, we have $gbcl(A, B) \subseteq (U, V)$. Therefore (A, B) is g*-binary regular closed.
Remark 3.2: Converse of Proposition 3.3 need not be true shown in Example 3.3.
Example 3.3: Let $X = \{1,2\}$, $Y = \{a, b, c\}$ and $M_g = \{(\emptyset, \emptyset), (\{1\}, \{a\}), (\{2\}, \{b, c\}), (\{X\}, \{a, c\}), (X, Y)\}$. Clearly M_g is g-binary topology from X to Y. Then $(\emptyset, \emptyset)$, $(\{1\}, \{a\})$, $(\{2\}, \{b, c\})$ and (X, Y) are g-binary closed sets. Further g-binary regular open sets are $(\emptyset, \emptyset)$, $(\{1\}, \{a\})$, $(\{2\}, \{b, c\})$ and (X, Y). Consider the set $(\{X\}, \{a\})$. Therefore $gbcl(\{X\}, \{a\}) = (X, Y) \subseteq (X, Y)$, where $(\{X\}, \{a\}) \subseteq (X, Y)$ and (X, Y) is g-binary regular open. Clearly the set $(\{X\}, \{a\})$ is g*-binary regular closed but not g*-binary closed, because $gbcl(\{X\}, \{a\}) = (X, Y) \nsubseteq (\{X\}, \{a, c\})$, where $(\{X\}, \{a, c\})$ is g-binary open set.
Definition 3.4: Let (X, Y, M_g) be a g-binary topological space. Let $(A, B) \subseteq (X, Y)$. Then (A, B) is called g*-binary regular open if $(X\backslash A, Y\backslash B)$ is g*-binary regular closed.
Example 3.4: Let $X = \{1,2\}$, $Y = \{a, b, c\}$ and $M_g = \{(\emptyset, \emptyset), (\{1\}, \{a\}), (\{2\}, \{b, c\}), (\{X\}, \{a, c\}), (X, Y)\}$. Clearly M_g is g-binary topology from X to Y. Let $(A, B) = (\{1\}, \{a, c\})$. Then $(X\backslash A, Y\backslash B) = (\{2\}, \{b\})$ is g*-binary regular closed because $gbcl(\{2\}, \{b\}) = (\{2\}, \{b, c\}) \subseteq (\{2\}, \{b, c\})$ where $(\{2\}, \{b, c\})$ is are g-binary regular open. Therefore $(A, B) = (\{1\}, \{a, c\})$ is g*-binary regular open.

Proposition 3.5: Every g-binary open set in a g-binary topological space is g*-binary open.
Proof: Let (X, Y, M_g) be a g-binary topological space. Let (A, B) be g-binary open in (X, Y, M_g). Then $(X\backslash A, Y\backslash B)$ is g-binary closed in (X, Y, M_g). By proposition 3.1 $(X\backslash A, Y\backslash B)$ is g*-binary closed in (X, Y, M_g). Therefore (A, B) is g*-binary open in (X, Y, M_g).
Remark 3.3: The converse of Proposition 3.5 need not true shown in example 3.5.
Example 3.5: Let $X = \{1,2,3\}$ and $Y = \{a, b\}$. Then $M_g = \{(\emptyset, \emptyset), (\{\emptyset\}, \{b\}), (\{1,3\}, \{Y\}), (\{X\}, \{a\}), (X, Y)\}$ is g-binary topology. Clearly $(\emptyset, \emptyset), (\{\emptyset\}, \{b\}), (\{X\}, \{a\})$ and (X, Y) are g-binary closed sets in (X, Y, M_g). Suppose $(A, B) = (\{\emptyset\}, \{Y\})$ and $(X\backslash A, Y\backslash B) = (\{X\}, \{\emptyset\})$. Clearly $(X\backslash A, Y\backslash B) = (\{X\}, \{\emptyset\}) \subseteq (\{X\}, \{a\})$ and $gbcl(X\backslash A, Y\backslash B) = gbcl(\{X\}, \{\emptyset\}) = (\{X\}, \{a\}) \subseteq (\{X\}, \{a\})$. Therefore $(X\backslash A, Y\backslash B) = (\{X\}, \{\emptyset\})$ is g*-binary closed, implies $(A, B) = (\{\emptyset\}, \{Y\})$ is g*-binary open but not g-binary open.
Proposition 3.6: Every g*-binary open set in a g-binary topological space is g*-binary regular open.
Proof: Let (A, B) be a g*-binary open in g-binary topological space (X, Y, M_g). Then $(X\backslash A, Y\backslash B)$ g*-binary closed in (X, Y, M_g). Then by Proposition 3.3, $(X\backslash A, Y\backslash B)$ is g*-binary regular closed. Therefore (A, B) is g*-binary regular open.
Remark 3.4: Converse of Proposition 3.6 need not be true shown in Example 3.6.
Example 3.6: Let $X = \{1,2\}$, $Y = \{a, b, c\}$ and $M_g = \{(\emptyset, \emptyset), (\{1\}, \{a\}), (\{2\}, \{b, c\}), (\{X\}, \{a, c\}), (X, Y)\}$. Clearly M_g is g-binary topology from X to Y. Let $(A, B) = (\{\emptyset\}, \{b, c\})$. Then $(X\backslash A, Y\backslash B) = (\{X\}, \{a\})$ is g*-binary regular closed because $gbcl(\{X\}, \{a\}) = (X, Y) \subseteq (X, Y)$ where (X, Y) is are g-binary regular open. Therefore $(A, B) = (\{\emptyset\}, \{b, c\})$ is g*-binary regular open. Now $gbcl(\{X\}, \{a\}) = (X, Y) \nsubseteq (\{X\}, \{a, c\})$, where $(\{X\}, \{a, c\})$ is g-binary open set. Therefore $(\{X\}, \{a\})$ is not g*-binary closed. This implies that $(A, B) = (\{\emptyset\}, \{b, c\})$ is not g*-binary open.
From above discussion we have the following result:

g-binary closed $\Rightarrow (\nLeftarrow)$ g*-binary closed $\Rightarrow (\nLeftarrow)$ g*-binary regular closed.
g-binary open $\Rightarrow (\nLeftarrow)$ g*-binary open $\Rightarrow (\nLeftarrow)$ g*-binary regular open.

Conclusion:
In g-binary topology g*-binary regular closed and g*-binary regular open sets are introduced and some specific relationships are also established.

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