CONSISTENCY OF MAXIMUM PLANARITY IN RECTILINEAR CROSSING NUMBER OF COMPLETE ZERO DIVISOR GRAPH

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Abstract : Let R be a commutative ring and let Z(R) be its set of zero- divisors. We associate a graph $\boldsymbol{\Gamma}(\boldsymbol{R})$ to R with vertices $Z(R)^* = Z(R) - \{0\}$, the set of non- zero zero divisors of R and for distinct $u,v \in Z(R)^*$, the vertices u and v are adjacent if and only if uv = 0 . In this paper we survey some ways of transforming a non-planar graph into a planar for complete zero divisor graphs, and discuss measures to obtain the planarity of a graph. We also characterize both the minimum and maximum number of edge crossings possible in particular Zero divisor graph classes. First we define the maximum rectilinear crossing number (MRCN) of a graph G, denoted by CR($\boldsymbol{\Gamma}(\boldsymbol{Z_n})$.)) where we seek a straight line drawing maximizing the number of edge crossings and secondly we recall the minimum rectilinear crossing number of zero divisor graphs especially for complete graph. Ultimately we investigate, the Maximum planar sub graphs of these maximum and minimum Rectilinear crossing number of zero divisor graphs.

IndexTerms - Rectilinear Crossing number, planar graph, Zero Divisor Graph.

I. INTRODUCTION

A graph which can be drawn in the plane in such a way that edges meet only at points corresponding to their common ends is called a ***Planar*** graph, and such a drawing is called a ***Planar embedding*** of the graph. Let *G* be a graph drawn in the plane with the requirement that the edges are line segments, no three vertices are collinear, and no three edges may intersect in a point, unless the point is a vertex. Such a drawing is said to be a ***Rectilinear drawing*** of *G*. The rectilinear crossing number of *G*, denoted by $\overline{cr}(G)$, is the fewest number of edge crossings attainable over all rectilinear drawings of *G* [3]. Any such a drawing is called optimal. The idea of a zero divisor graph of a commutative ring was introduced by I. Beck in [1]. The zero divisor graph is very useful to find the algebraic structures and properties of rings. We mainly focus on D. F. Anderson and P. S. Livingston's zero divisor graphs.[2, 4, 10]

II. SOME DEFINITIONS

Definition - 1: If a and b are two non-zero elements of a ring Z_n such that $a.b = 0$, then 'a' and 'b' are the ***Zero divisors*** of commutative ring Z_n.

Definition - 2: If a graph $G' = (V, E')$ is a ***maximum planar subgraph*** of a graph $G = (V, E)$ such that there is no planar subgraph $G'' = (V, E'')$ of G with $|E''| > |E'|$, then G' is called a maximum planar subgraph of G.

III. CONSISTENCY OF MAXIMUM RECTILINEAR CROSSING NUMBER OF ZERO DIVISOR REGULAR GRAPHS

The problem of drawing a graph in the plane with a minimum number of edge crossings with straight line segments called the Rectilinear crossing number of zero divisor graph has been formulated in full generality [7,8,9] is a well-studied problem. The maximum Rectilinear crossing number of zero divisor regular graphs is meant for complete Zero divisor graph $\overline{CR}\left(\Gamma\left(Z_{p^2}\right)\right)$ which contains ($p-1$) vertices within it that implies $\overline{CR}\left(\Gamma\left(Z_{p^2}\right)\right)$ is a ($p-2$) -Regular graph and can be denoted by $R_{p-1,p-2}$, and is defined as follows.

$$\overline{CR}\left(\Gamma\left(Z_{p^2}\right)\right) = \overline{CR}\left(R_{p-1,p-2}\right) = \binom{p-1}{4}$$

The Maximum Rectilinear crossing number for the class , $R_{n,d}$ of all d -Regular graphs of order n is denoted by $\overline{CR}\left(R_{n,d}\right)$. Then from a theorem [6] we have

$$\overline{CR}\left(R_{n,d}\right) \geq \binom{n}{4} - \sum_{i=1}^{k-1} (n)(i-1)(n-2i)$$

where $d = n-1, k = \frac{1}{2}(n-d-1)$

We mean the Consistency is that reduction of the Maximum Rectilinear crossing number of zero divisor Regular graphs to the maximum induced planar subgraph, by framing an algorithm by removal of edges and also by removal of crossings, which will be clear from the following theorem.

Theorem 1: The consistency of Maximum Rectilinear crossing number of $\overline{CR}\left(\Gamma\left(Z_{p^2}\right)\right)$, for an outer planar graph,

(i)When removing outermost edges, the edge planarity is,

$$\overline{CR}\left(\Gamma(Z_{p^2})\right) = \frac{(p-1)d}{24} - \sum_{n=1}^{d-2} n$$

(ii)When removing the edges involved in crossing is,

$$\overline{CR}\left(\Gamma(Z_{p^2})\right) = \frac{(p-1)d}{24}[3pd - 9d - 2d^2 + 2]$$

(iii) When removing the minimum number of inner edges to get an maximum induced planar subgraph is,

$$P\left(\Gamma(Z_{p^2})\right) = \frac{(p-1)d}{24}[3pd - 9d - 2d^2 + 2] - 1\sum_{n=1}^{d-2} n - 2\sum_{n=1}^{d-3} n + \cdots + (d-2)$$

Proof: The vertex set of $\Gamma(Z_{p^2})$ is,

$V\left(\Gamma(Z_{p^2})\right) = \{p, 2p, .., p(p-1)\}$. Then $\left|V\left(\Gamma(Z_{p^2})\right)\right| = p - 1$.

The edge set is $E\left(\Gamma(Z_{p^2})\right) = \begin{pmatrix} (p,2p),(p,3p)..(p,p(p-1)) \\ (2p,3p)\ldots(2p,p(p-1)) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ (p(p-2),p(p-1)) \end{pmatrix}$

Now consider the Rectilinear drawing of $\Gamma(Z_{p^2})$ where the vertices are arranged as those of a convex n-gon. Step by step we delete all diagonals of lengths 1,2,…k-1. Let us assume that $E_1\left(\Gamma(Z_{p^2})\right)$ be the set of all edges which does not involve in crossings. That is the edge set,

$$E\left(\Gamma(Z_{p^2})\right) = \{(p,2p),(2p,3p),..(p(p-1),p)\}$$

These edges contributes the (p-1) edges of lengths 1,2,..k-1. We proceed by counting the number of crossings we remove from the drawing by now deleting the (p-1) edges of length k. Therefore,

$$\overline{CR}(R_{p-1,p-2}) \geq \binom{p-1}{4} - \sum_{i=1}^{k-1}(p-1)(i-1)(p-1-2i)$$

$$= \binom{p-1}{4} - \frac{1}{6}(p-1)(k-1)(k-2)(3(p-1)-4k)$$

Where $d = p-2, k = \frac{(p-d)}{2}$. Substituting k in the closed form of the sum above, we obtain the desired result.

$$\overline{CR}(R_{p-1,p-2}) = \binom{p-1}{4} - \frac{1}{6}(p-1)(k-1)(k-2)(3p-3-4k)$$

$$= \frac{(p-1)(p-2)(p-3)(p-4)}{24} - \frac{p-1}{6}\left(\frac{p-d}{2}-1\right)\left(\frac{p-d}{2}-2\right)\left(3p-3-4\left(\frac{p-d}{2}\right)\right)$$

$$= \frac{(p-1)}{24}[(p-2)(p-3)(p-4) - (p-d-2)(p-d-4)(3p-3-2p+2d)]$$

$$= \frac{(p-1)}{24}[(p^3 - 9p^2 + 26p - 24) - (p^3 - 9p^2 - 3pd^2 + 26p + 9d^2 - 2d + 2d^3)]$$

$$= \frac{(p-1)d}{24}[3pd - 9d - 2d^2 + 2]$$

Now deleting the edges in $\overline{CR}(R_{p-1,p-2})$ we get,

$\overline{CR}(R_{p-1,p-2}) - \overline{CR}\left(E_1\left(\Gamma(Z_{p^2})\right)\right) = \frac{(p-1)d}{24}[3pd - 9d - 2d^2 + 2] - \overline{CR}\left(E_1\left(\Gamma(Z_{p^2})\right)\right)$

Here we conclude that $\overline{CR}(R_{p-1,d})$ is same as $\overline{CR}(R_{p-1,p-2}) - \overline{CR}\left(E_1\left(\Gamma(Z_{p^2})\right)\right)$

Hence the condition $E^0 = \{\overline{CR}(G) = \overline{CR}(G-e), e \in E\}$ is satisfied which is evident from the following cases.

Case (i): Let $p = 5$

Therefore $d = p - 2 = 5 - 2 = 3$

$\overline{CR}(\Gamma(Z_{25})) = \overline{CR}(R_{p-1,d}) = \overline{CR}(R_{4,3}) = 1 = \frac{1}{2}[2]$

$= \frac{12}{24}[45 - 27 - 18 + 2]$

$$= \frac{4\times3}{24}[3\times5\times3 - 9\times3 - 2\times9 + 2] = \frac{(p-1)d}{24}[3pd - 9d - 2d^2 + 2]$$

$$\overline{CR}\left(E_1\left(\Gamma(Z_{p^2})\right)\right) = \overline{CR}(5,10) + \overline{CR}(10,15) + \overline{CR}(15,20) = 0$$

$$\text{Therefore, } \overline{CR}(R_{p-1,d}) - \overline{CR}\left(E_1\left(\Gamma(Z_{p^2})\right)\right) = 1 - 0 = 1$$

Case (ii): Let $p = 7$

Therefore $d = p - 2 = 7 - 2 = 5$

$\overline{CR}(\Gamma(Z_{49}))=\overline{CR}(R_{p-1,d}) = \overline{CR}(R_{6,5}) = 15 = \frac{30}{24}$ [12]

$=\frac{30}{24}[105 - 45 - 50 + 2]$

$=\frac{6\times5}{24}[3 \times 7 \times 5 - 9 \times 5 - 2 \times 25 + 2] = \frac{(p-1)d}{24}[3pd - 9d - 2d^2 + 2]$

$\overline{CR}\left(E_1\left(\Gamma(Z_{p^2})\right)\right) = \overline{CR}(7,14) + \overline{CR}(14,21) + \overline{CR}(35,42) = 0$

Therefore, $\overline{CR}(R_{p-1,d}) - \overline{CR}\left(E_1\left(\Gamma(Z_{p^2})\right)\right) = 15 - 0 = 15$

Let $E_2\left(\Gamma(Z_{p^2})\right)$ be the set of all edges which involves in crossings for an outer planar graph is given by,

$$E_2\left(\Gamma(Z_{p^2})\right) = \{(p,3p),..(p,p(p-2)),(2p,4p)..(2p,p(p-2)),..,(p(p-3)),p(p-1)\}$$

We know that as the edges of $E_1\left(\Gamma(Z_{p^2})\right)$ doesnot involve in crossings, all the vertex has exactly has d-2 edges which involves in crossing. Let us start with vertex p. Initially, all the edges are drawn to every vertex adjacent to p, which has zero crossings. The next vertex $2p$ has the edge set, $E_2^1\left(\Gamma(Z_{p^2})\right) = \{(2p,4p)..(2p,5p),..(2p,p(p-1))\}$ which has (d-2) edges. The vertex 3p has the edge set, $E_2^2\left(\Gamma(Z_{p^2})\right) = \{(3p,5p)..(3p,6p),..(3p,p(p-1))\}$ which has (d-3) edges. Continuing this way, we are left with the edge set, $E_2^3\left(\Gamma(Z_{p^2})\right) = \{p(p-3),p(p-1))\}$ with one edge. We know that for an outerplanar d-regular graph there are $\frac{(p-1)d}{2}$ edges. So summing up all the edges which involves in crossing, we get,

$$(d-2) + (d-3)+..+1 = \sum_{n=1}^{d-2} n$$

Therefore we can find the number of edges which doesnot involve in crossing is,

$$\frac{(p-1)d}{2} - \sum_{n=1}^{d-2} n$$

Which means $\frac{(p-1)d}{2} - (p-1) = \frac{(p-1)(d-2)}{2}$ edges involve in crossing, which is evident from the following cases. If p=5, then d=2 is a planar graph. So assuming for $p > 7$,

Case (i): Let p=7

Then

$$d = 5 \Rightarrow \frac{(p-1)d}{2} - \sum_{n=1}^{3} n$$

$$= \frac{6 \times 5}{2} - (3 + 2 + 1) = 15 - 6 = 9$$

$\Rightarrow$ 9 edges doesnot involve in crossing for $\Gamma(Z_{49})$.

Case (ii): Let p=11

Then

$$d = 9 \Rightarrow \frac{(p-1)d}{2} - \sum_{n=1}^{7} n$$

$$= \frac{10 \times 9}{2} - (7 + 6+..+2 + 1) = 45 - 28 = 17$$

$\Rightarrow$ 17 edges does not involve in crossing for $\Gamma(Z_{121})$.

On removing $\sum_{n=1}^{d-2} n$ edges, the graph $R_{p-1,d}$ has (d-2)+(p-1) edges. Since the outermost (p-3) edges doesnot involve in any crossings, so neglecting (p-3) edges we get a star graph $S_{1,d-3}$ which is a planar graph. Note that removal of the edges, should not make any vertex isolated.

Now we calculate the number of crossings on removing the edges of $E_2^1, E_2^2,..E_2^{p-4}$. As E_2^1 has (d-2) edges, the crossings involved are $1[(d-2) + (d-3)+..+1], E_2^2$ has (d-3) edges, the crossings involved are $2[(d-3) + (d-4)+..+1]$, E_2^3 has (d-4) edges, the crossings involved are $3[(d-4) + (d-5)+..+1]$. Continuing this process upto E_2^{p-4} which has (d-2) crossings. Summing up all, $1[(d-2) + (d-3)+..+1] + 2[(d-3) + (d-4)+..+1]$+$3[(d-4) + (d-5)+..+1] + (d-2)(1)$. Therefore

$$E_2^1 + E_2^2+..+E_2^{p-4} = 1\sum_{n=1}^{d-2} n + 2\sum_{n=1}^{d-2} n +..+(d-2)$$

Finally to obtain the planarity, we subtract $E_2^1 + E_2^2+..+E_2^{p-4}$ from the total crossing , $\overline{CR}(R_{p-1,d})$. That is,

$$P\left(\Gamma(Z_{p^2})\right) = \frac{(p-1)d}{24}[3pd - 9d - 2d^2 + 2] - 1\sum_{n=1}^{d-2} n - 2\sum_{n=1}^{d-3} n + \cdots + (d-2)$$

Therefore we can obtain , $\overline{CR}(R_{p-1,d}) =, \overline{CR}(S_{1,d-3})$=0. This can be proved from the following cases by induction.

Case (i): Let p=5, then d=3

$$E\left(\Gamma\left(Z_{p^2}\right)\right) = E\left(\Gamma(Z_{25})\right) = \{(5,10),(5,15),(5,20),(10,15),(10,20)(15,20)\}$$

$= n\left[E\left(\Gamma(Z_{25})\right)\right] = 6 = \frac{4\times3}{2} = \frac{(p-1)d}{2}$

$$E_1\left(\Gamma(Z_{25})\right) = \{(5,10),..(10,15),(15,20),(5,20)\} \Rightarrow n\left[E_1\left(\Gamma(Z_{25})\right)\right] = 4 = (p-1)$$

$$E_2\left(\Gamma(Z_{25})\right) = \{(10,20)\} \Rightarrow n\left[E_2\left(\Gamma(Z_{25})\right)\right] = 1 = (d-2)$$

$$E_3\left(\Gamma(Z_{25})\right) = \{(5,15)\} \Rightarrow n\left[E_3\left(\Gamma(Z_{25})\right)\right] = 1 = (d-2)$$

Therefore $n\left[E\left(\Gamma(Z_{25})\right)\right] - n\left[E_1\left(\Gamma(Z_{25})\right)\right] = 2 = 6-4$

$$= \frac{(p-1)d}{2} - (p-1) = \frac{(p-1)(d-2)}{2} = \frac{(5-1)(3-2)}{2} = \frac{4\times1}{2}$$

= 2 edges involve in crossing. Therefore removing $n\left[E_2\left(\Gamma(Z_{25})\right)\right] = 1 = (d-2)$ edges,

$$\Rightarrow n\left[E\left(\Gamma(Z_{25})\right)\right] - n\left[E_1\left(\Gamma(Z_{25})\right)\right] - n\left[E_2\left(\Gamma(Z_{25})\right)\right]$$

$$= 1 = 2-1 = 6-4-1$$

$$= \frac{(p-1)(d-2)}{2} - \sum_{n=1}^{d-2} n = n\left[E_3\left(\Gamma(Z_{25})\right)\right]$$

Now,

$$\overline{CR}\left[E_2\left(\Gamma(Z_{25})\right)\right] = 1 = \sum_{n=1}^{d-2} n$$

Therefore $P\left(\Gamma(Z_{25})\right) = \overline{CR}\left[E\left(\Gamma(Z_{25})\right)\right] - \overline{CR}\left[E_2\left(\Gamma(Z_{25})\right)\right] = 0 = 1-1$

$$= 1 - 1\sum_{n=1}^{d-2} n$$

$$= \frac{4\times3}{24}[45-27-18+2] - 1\sum_{n=1}^{d-2} n$$

$$= \frac{(5-1)\times3}{24}[3\times5\times3 - 9\times3 - (2\times9) + 2] - 1\sum_{n=1}^{d-2} n$$

$$\frac{(p-1)d}{24}[3pd - 9d - 2d^2 + 2] - 1\sum_{n=1}^{d-2} n - 2\sum_{n=1}^{d-3} n + \cdots + (d-2) = P\left(\Gamma\left(Z_{p^2}\right)\right)$$

Case (ii): Let p=7, then d=5

$$E\left(\Gamma\left(Z_{p^2}\right)\right) = E\left(\Gamma(Z_{49})\right) = \{(7,14),..,(7,42),(14,21),..,(14,42),(21,28),..,(21,42),(28,42),(35,42)\}$$

$= n\left[E\left(\Gamma(Z_{49})\right)\right] = 15 = \frac{6\times5}{2} = \frac{(p-1)d}{2}$

$$E_1\left(\Gamma(Z_{25})\right) = \{(7,14),(14,21),(21,28),(28,35),(35,42),(7,42)\}$$

$$\Rightarrow n\left[E_1\left(\Gamma(Z_{49})\right)\right] = 6 = (p-1)$$

$$E_2\left(\Gamma(Z_{25})\right) = \{(14,28),(14,35),(14,42),(21,35),(21,42),(28,42)\} \Rightarrow n\left[E_2\left(\Gamma(Z_{25})\right)\right] = 6$$

$$E_2^1\left(\Gamma(Z_{49})\right) = \{(14,28),(14,35),(14,42)\} \Rightarrow n\left[E_2^1\left(\Gamma(Z_{49})\right)\right] = 3 = (d-2)$$

$$E_2^2\left(\Gamma(Z_{49})\right) = \{(21,35),(21,42)\} \Rightarrow n\left[E_2^2\left(\Gamma(Z_{49})\right)\right] = 2 = (d-3)$$

$$E_2^3\left(\Gamma(Z_{49})\right) = \{(28,42)\} \Rightarrow n\left[E_2^3\left(\Gamma(Z_{49})\right)\right] = 1 = (d-4)$$

$$E_3\left(\Gamma(Z_{49})\right) = \{(7,21),(7,28),(7,35)\} \Rightarrow n\left[E_3\left(\Gamma(Z_{49})\right)\right] = 3 = (d-2)$$

Therefore $n\left[E\left(\Gamma(Z_{49})\right)\right] - n\left[E_1\left(\Gamma(Z_{49})\right)\right] = 9 = 15-6$

$$= \frac{(p-1)d}{2} - (p-1) = \frac{(p-1)(d-2)}{2} = \frac{(7-1)(5-2)}{2} = \frac{6\times3}{2}$$

= 9 edges involve in crossing. Therefore removing $n\left[E_2\left(\Gamma(Z_{49})\right)\right] = 6$ edges,

$$= n[E_2^1 + E_2^2 + E_2^3]\left(\Gamma(Z_{49})\right) = 3+2+1 = (d-2)+(d-3)+1 = \sum_{n=1}^{d-2} n$$

$$\Rightarrow n\left[E\left(\Gamma(Z_{49})\right)\right] - n\left[E_1\left(\Gamma(Z_{49})\right)\right] - n\left[E_2\left(\Gamma(Z_{49})\right)\right] = 3 = 9-6 = 15-6-6$$

$$= \frac{(p-1)(d-2)}{2} - \sum_{n=1}^{d-2} n = n\left[E_3\left(\Gamma(Z_{49})\right)\right]$$

Now, $\overline{CR}\left[E_2\left(\Gamma(Z_{49})\right)\right] = \overline{CR}[E_2^1 + E_2^2 + E_2^3]\left(\Gamma(Z_{49})\right) = 1(3+2+1) + 2(2+1) + 3(1)$

$$= 1\sum_{n=1}^{d-2} n + 2\sum_{n=1}^{d-3} n + 3\sum_{n=1}^{d-4} n$$

Therefore $P(\Gamma(Z_{49})) = \overline{CR}[E(\Gamma(Z_{49}))] - \overline{CR}[E_2(\Gamma(Z_{49}))]$

$$= 0 = 15 - 15\sum_{n=1}^{d-2} n$$

$$= \frac{6\times 5}{24}[105 - 45 - 50 + 2] - 6 - 6 - 3$$

$$= \frac{(p-1)d}{24}[3pd - 9d - 2d^2 + 2] - 1\sum_{n=1}^{d-2} n - 2\sum_{n=1}^{d-3} n - 3\sum_{n=1}^{d-4} n = P\left(\Gamma(Z_{p^2})\right)$$

IV. CONSISTENCY OF MINIMUM RECTILINEAR CROSSING NUMBER OF ZERO DIVISOR GRAPHS

Theorem 2: The consistency of Minimum number of Rectilinear crossing of $\overline{cr}\left(\Gamma(Z_{p^2})\right)$ obtained, when removing the minimum number of edges to get an maximum induced planar subgraph is, $P\left[E\left[\overline{cr}\left(\Gamma(Z_{p^2})\right)\right]\right] = \frac{(p-1)(p-2)}{2} - 3(p-1) + 6$.

Proof:

From theorem [5,9] we know that the Minimum number of Rectilinear crossing of $\overline{cr}\left(\Gamma(Z_{p^2})\right)$ can be obtained by placing the vertices according to non-collinearity. That is each of three vertices forms a triangle one inside the other and each of four vertices forms a convex polyhedron. To find the consistency of $\overline{cr}\left(\Gamma(Z_{p^2})\right)$we proceed by removing the edges gradually to make the graph a maximum planar induced graph. From the above theorem, it is clear that the number of edges denoted by $n\left[\overline{cr}\left(\Gamma(Z_{p^2})\right)\right]$ and equivalent to $\frac{(p-1)(p-2)}{2}$. There are $6 - 3(p-1)$which doesnot indulge in the planarity of the graph. So the remaining edge contributes the minimum number of crossings. That is $\frac{(p-1)(p-2)}{2} - 3(p-1) + 6$ shall be removed from $\overline{cr}\left(\Gamma(Z_{p^2})\right)$ to make the graph a maximum induced planar subgraph denoted by , $P\left[E\left[\overline{cr}\left(\Gamma(Z_{p^2})\right)\right]\right]$.

Case (i): Let $p = 5$,

$$E\left(\Gamma(Z_{p^2})\right) = E(\Gamma(Z_{25})) = \{(5,10),(5,15),(5,20)(10,15),(10,20),(15,20)\}$$

$$n[E(\Gamma(Z_{25}))] = 6 = \frac{4\times 3}{2} = \frac{(p-1)(p-2)}{2}$$

Since $\Gamma(Z_{25})$ is planar, no edges are removed.

$$P\left[E\left[\overline{cr}\left(\Gamma(Z_{p^2})\right)\right]\right] = 0 = 6 - 6 = 6 - 12 + 6 = \frac{(p-1)(p-2)}{2} - 3(p-1) + 6$$

Case (i): Let $p = 7$,

$$E\left(\Gamma(Z_{p^2})\right) = E(\Gamma(Z_{49})) = \{(7,14),..(7,42),(14,21)(14,42),(21,28),(21,42),(28,35),(28,42),(35,42)\}$$

$$n[E(\Gamma(Z_{49}))] = 15 = \frac{6\times 5}{2} = \frac{(p-1)(p-2)}{2}$$

The edges are drawn until the graph does not changes its planarity. $P\left[\overline{cr}\left(\Gamma(Z_{p^2})\right)\right]$ exists when the following 12 edges remains in the graph. That is, $n\left\{P\left[\overline{cr}\left(\Gamma(Z_{p^2})\right)\right]\right\} = \{(7,14),..(7,42),(14,21)(14,42),(21,42),(28,35),(28,42),(35,42)\}$

=12. Therefore, $n[E(\Gamma(Z_{49}))] - n\left\{P\left[\overline{cr}\left(\Gamma(Z_{p^2})\right)\right]\right\} = P\left[E\left[\overline{cr}\left(\Gamma(Z_{p^2})\right)\right]\right] = 3$

$$= 15 - 12 = 15 - 18 + 6 = \frac{6\times 5}{2} - 3\times 6 + 6$$

$$= \frac{(p-1)(p-2)}{2} - 3(p-1) + 6$$

V .CONCLUSION

In this paper we find maximum planar subgraph from a complete graphs, especially for zero divisor graphs in any rectilinear drawing of G. We infer from the above formulae that the removal of edges involved in crossings leading to a planar graph can be applied in any cabel networks, oil pipelines or diodes in a transistor. Suppose there arise a situation to remove any connections that crosses the network or disturbs in transmitting signals, so that the network becomes a complete planar one, without disturbing any nodes or vertices.

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