Some quotient properties on degree of vertices and domination number of the dominating sets and the complimentary dominating sets of an interval graph G

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Abstract:

Interval graphs, their importance over the years can be seen in the increasing number of researchers trying to explore the field. Their sincere efforts are observed in establishing the relevance of the subject in practicality in coherence with reality. The domination theory, in particular various domination parameters in interval graphs are the main object of study in this paper. In particular we prove necessary quotient(4) properties under which the domination number of dominating sets as well as the complimentary dominating sets. In this paper "Some quotient properties on degree of vertices and domination number of the dominating sets and the complimentary dominating sets of an interval graph G".

Key words:

Interval family, Interval graph, Dominating set, Domination number, Complimentary dominating set, Complimentary domination number, Euclidean division algorithm.

I. Introduction

Interval graphs are rich in combinatorial structures and have found applications in several disciplines such as traffic control, ecology, biology, computer sciences and particularly useful in cyclic scheduling and computers storage allocation problems etc. Having a representation of graph with intervals or arcs can be helpful in combinatorial problems of the graph, such as isomorphism testing and finding maximum independent set and cliques of graphs.

A graph is said to be a simple graph if it has no loops and no parallel edges. A graph is said to be connected if there is a path between every pair of vertices, otherwise it is said to be disconnected graph. The number of edges incident with a vertex V is called degree of V and it is denoted by d(v)**.** Graphs considered in this paper are all undirected, connected and simple graphs. If A is a set, then the absolute compliment (6) of A (or simply the compliment of A) is the set of elements not in A. In other words, if U is the universe that contains all the elements under study, and there is no need to mention it because it is obvious and unique, then the absolute compliment of A is the relative compliment of A in U. That is $A^C = U \setminus A = \{x \in \cup / x \notin A\}$.The absolute compliment of A is usually denoted by A^C, other notations A^1, $\overline{A}$ etc. According Euclid's division lemma if we have two positive integers a and b, then there exists unique integers q and r which satisfies the condition a= bq + r where $0 \le r < b$. If a prime p divides the product ab of two integers a and b, then p must divide at least one of those integers a and b. For example: If p=11, a= 88, b= 72, then ab = 88 × 72= 6336, and since this is divisible by 11, the lemma implies that one or both of 88 or 72 must be as well. In fact, 88 = 11 × 8.

II. Preliminaries

Let I= $\{ I_1, I_2, I_3, I_4, ..., I_n \}$ be an interval family, where each I_i is an interval on the real line and $I_i = [a_i, b_i]$ for I = 1, 2, 3,...........,n. Here a_i is called the left end point and b_i is called the right end point. Without loss of generality, one can assume that, all end points of the intervals are distinct numbers between 1 and 2n. An interval of degree one is called a pendent interval. The intervals are named in the increasing order of their right end points. The graph G is an interval graph if there is one-to-one correspondence between the vertex set V and the interval family I. Two vertices of G are joined by an edge if and only if their corresponding intervals in I intersect. That is if $I_i = [a_i, b_i]$ and $I_j = [a_j, b_j]$, then I_i and I_j will intersect if $a_i < b_j$ or $a_j < b_i$.

A set D of vertices of G is dominating set of G if every vertices of G is dominated by at least one vertex of D. Equivalently: A set D of vertices of G is a dominating set(5) if every vertex in V(G) – D is adjacent to at least one vertex in D. The minimum cardinality among the dominating set of G is called the domination number (3,7) of G and denoted by $\gamma(G)$.

Let G (V, E) be any interval graph and D be any minimum dominating set then V – D is said to be compliment of a dominating set and is simply denoted as D^C. Let $\gamma(G)$ be the domination number(1,2) of the graph G. And $\gamma^C(G)$ be the compliment domination number of G and is the cardinality of D^C.

III. Pre-requisite results

Result 1: Let I= $\{ I_1, I_2, I_3, I_4, \ldots\ldots\ldots, I_n \}$ be any finite interval family and let G(V,E) be an interval graph corresponding to an interval family I such that every interval I_i, $i \neq n$ is intersect the next interval only. Suppose n = 3q+r, where r = 0, 1, 2 and q is any integer and if r = 0, then the domination number $\gamma(G)$= q and the Minimum dominating set D = $\{v_2, v_5, v_8, \ldots\ldots\ldots, v_{n-1}\}$.

Result 2: Let I= $\{ I_1, I_2, I_3, I_4, \ldots\ldots\ldots, I_n \}$ be any finite interval family and let G(V,E) be an interval graph corresponding to an interval family I such that every interval I_i, $i \neq n$ is intersect the next interval only. Suppose n = 3q+r, where r = 0, 1, 2 and q is any integer and if r = 1, then the domination number $\gamma(G)$= q + 1 and the Minimum dominating set D= $\{v_2, v_5, v_8, \ldots\ldots\ldots, v_n\}$ where v_n is the last interval of I.

Result 3: Let I= $\{ I_1, I_2, I_3, I_4, \ldots\ldots\ldots, I_n \}$ be any finite interval family and let G(V,E) be an interval graph corresponding to an interval family I such that every interval I_i, $i \neq n$ is intersect the next interval only. Suppose n=3q+r, where r = 0, 1, 2 and q is any integer and if r = 2, then the domination number $\gamma(G)$= q+1 and Minimum dominating set D is either D= $\{v_2, v_5, v_8, \ldots\ldots\ldots, v_{n-1}\}$ or D= $\{v_2, v_5, v_8, \ldots\ldots\ldots, v_n\}$.

IV. Main theorems

THEOREM 1: Let I= $\{ I_1, I_2, I_3, I_4, \ldots\ldots\ldots, I_n \}$ be any finite interval family and let G be an interval graph corresponding to an interval family I. D be a minimum dominating set and D^c is a compliment dominating set of an interval graph G. And $\gamma(G)$ and $\gamma^C(G)$ be the domination numbers of an interval graph G with respect to D and D^C respectively then $\gamma(G) + \gamma^C(G) = n$.

Proof: Let I= $\{ I_1, I_2, I_3, I_4, \ldots\ldots\ldots, I_n \}$ be an interval family and let G be an interval graph corresponding to an interval family I. Our aim to show that $\gamma(G) + \gamma^C(G) = n$.

By Euclidean division algorithm, n = 3q + r, where r = 0, 1, 2 and q is any integer and n is a number of intervals in I.

If r = 0 then D = $\{v_2, v_5, v_8, \ldots\ldots\ldots, v_{n-1}\}$ and $\gamma(G)$= q since the result 1.

Then D^C = $\{v_1, v_3, v_4, v_6, \ldots\ldots, v_n\}$ and

$$\gamma^C(G) = |D^C|$$

$$= |V - D|$$

$$= |V| - |D|$$

$\gamma^C(G)$ = n−q

Therefore $\gamma(G)+\gamma^C(G)=q+n-q$

= n

Therefore $\gamma(G)+\gamma^C(G)=n$

If r = 1, then D= $\{v_2, v_5, v_8, ..., v_n\}$ and $\gamma(G)$= q + 1 since the result 2.

And hence D^C = $\{v_1, v_3, v_4, v_6, ..., v_{n-1}\}$ and

$\gamma^C(G)=|D^C|$

$=|V - D|$

$= |V| - |D|$

= n - (q + 1)

$\gamma^C(G)$ = n – q - 1

Therefore $\gamma(G)+\gamma^C(G)=q+1+n-q-1$ = n

Therefore $\gamma(G)+\gamma^C(G)=n$

If r = 2, then D= $\{v_2, v_5, v_8, ..., v_{n-1}\}$ or D= $\{v_2, v_5, v_8, ..., v_n\}$.

Suppose D= $\{v_2, v_5, v_8, ..., v_{n-1}\}$ and $\gamma(G)$= q + 1 since the result 3.

And hence D^C = $\{v_1, v_3, v_4, v_6, ..., v_n\}$ and

$\gamma^C(G)=|D^C|$

$=|V - D|$

$= |V| - |D|$

= n - (q + 1)

$\gamma^C(G)$ = n – q - 1

Therefore $\gamma(G)+\gamma^C(G)=q+1+n-q-1$

= n

Therefore $\gamma(G)+\gamma^C(G)=n$

OR

Suppose D= $\{v_2, v_5, v_8, ..., v_n\}$and $\gamma(G)$= q + 1 since the result 3.

And hence D^C = $\{v_1, v_3, v_4, v_6, ..., v_{n-1}\}$ and

$$\gamma^C(G)=|D^C|$$

$$=|V - D|$$

$$= |V| - |D|$$

$$= n - (q + 1)$$

$$\gamma^C(G) = n – q - 1$$

Therefore $\gamma(G)+\gamma^C(G)=q+1+n-q-1$

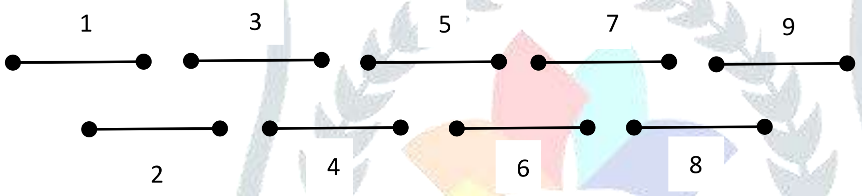
$$= n$$

Therefore $\gamma(G)+\gamma^C(G)=n$

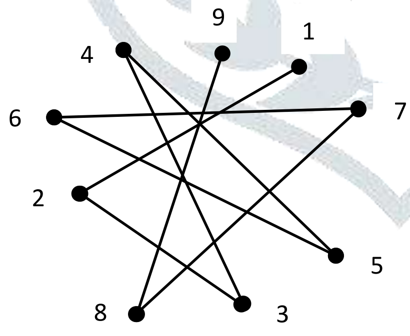
Hence theorem is proved at r = 0, 1, 2.

V. Experimental problems

Illustration 1: Let I= $\{ I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8, I_9 \}$ be an interval family and let G(V,E) be an interval graph corresponding to an interval family I is as follows.



Interval family I



Interval graph G

Here n= 9

And then n= 3 × 3 + 0

This is of the form n= 3q + r

Where q= 3, r = 0

If r = 0 then D = {2, 5, 8}, since result1

Then $\gamma(G) = 3$

And $D^C = \{1, 3, 4, 6, 7, 9\}$

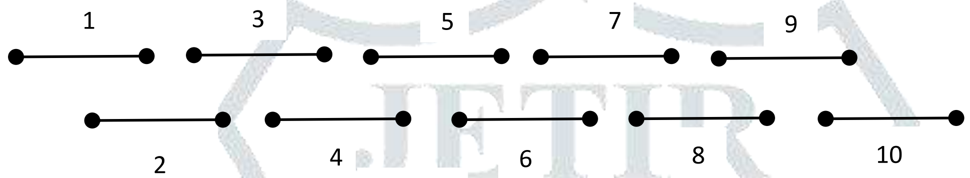
Then $\gamma^C(G) = 6$

Therefore $\gamma(G) + \gamma^C(G) = 3 + 6 = 9 = n$

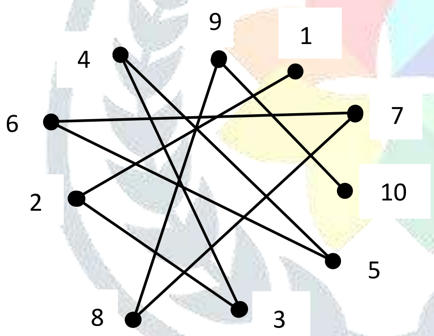
That is $\gamma(G) + \gamma^C(G) = n$

Hence theorem1 is verified at r = 0.

Illustration 2: Let I= $\{ I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8, I_9, I_{10} \}$ be an interval family and let G(V, E) be an interval graph corresponding to an interval family I is as follows:



Interval family I



Interval graph G

Here n = 10

And n = 3 × 3 + 1 = 10

This is of the form n = 3q + r

Where q = 3, r = 1

If r = 1 then D = {2, 5, 8, 10}, since result 2

Then $\gamma(G) = 4$

And $D^C = \{1, 3, 4, 6, 7, 9\}$

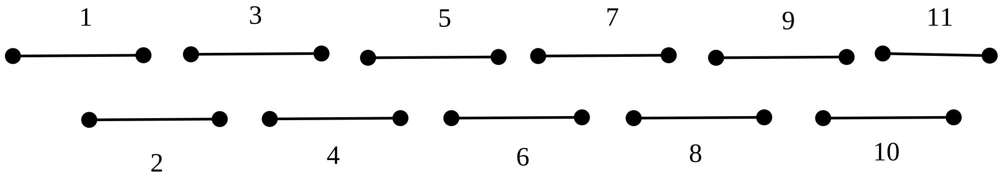
Then $\gamma^C(G) = 6$

Therefore $\gamma(G) + \gamma^C(G) = 4 + 6 = 10 = n$

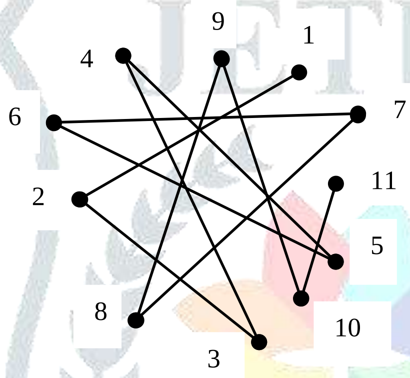
That is $\gamma(G)+\gamma^{C}(G)=n$

Hence theorem1 is verified at r = 1.

Illustration 3: Let I= $\{I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8, I_9, I_{10}, I_{11}\}$ be an interval family and let G(V, E) be an interval graph corresponding to an interval family I is as follows:



Interval family I



Interval graph G

Here n = 11

And n = 3 × 3 + 2

This is of the form n = 3q + r

Where q = 3, r = 2

If r = 2 then D = {2, 5, 8, 10} or D = {2, 5, 8, 11}, since result 3.

Suppose D = {2, 5, 8, 10}

Then $\gamma(G)$ = 4

And D^C = {1, 3, 4, 6, 7, 9, 11}

Then $\gamma^{C}(G)$ = 7

Therefore $\gamma(G)+\gamma^{C}(G)$ = 4 + 7 = 11

Hence $\gamma(G)+\gamma^{C}(G)=n$.

And suppose D = {2, 5, 8, 11}

Then $\gamma(G)$ = 4

And $D^C = \{1, 3, 4, 6, 7, 9, 10\}$

Then $\gamma^C(G) = 7$

Therefore $\gamma(G) + \gamma^C(G) = 4 + 7 = 11 = n$

$$\therefore \quad \gamma(G) + \gamma^C(G) = n$$

Hence theorem1 is verified at r = 2.

THEOREM 2: Let I=$\{ I_1, I_2, I_3, I_4, \ldots\ldots\ldots, I_n \}$ be any finite interval family and let G(V,E) be an interval graph corresponding to an interval family I such that every interval I_i, $i \neq n$ is intersect the next interval only and suppose n= 3q + r, where r = 0, 1, 2 and q is any integer then $\sum_{v \in D} d(v) + \sum_{v \in D^C} d(v) = 2n - 2.$

Proof: Let I=$\{ I_1, I_2, I_3, I_4, \ldots\ldots\ldots, I_n \}$ be any finite interval family and let G(V,E) be an interval graph corresponding to an interval family I such that every interval I_i, $i \neq n$ is intersect the next interval only, where $V = \{v_1, v_2, v_3, \ldots\ldots, v_n\}$.If there is one-to-one correspondence between the vertex set V and interval family I.

Suppose n= 3q+r where r = 0, 1, 2.

Case (i): Suppose r = 0 then n = 3q + 0 = 3q

If r = 0, then D= $\{v_2, v_5, v_8, \ldots\ldots\ldots, v_{n-1}\}$ and $\gamma(G)$= q, since the result1

And $\quad D^C = \{v_1, v_3, v_4, \ldots\ldots\ldots, v_{n-2}, v_n\}$

Then $\quad \gamma^C(G) = n - \gamma(G)$

$= 3q - q$

$\gamma^C(G) = 2q$

$$\sum_{v \in D} d(v) = d(v_2) + d(v_5) + d(v_8) + \ldots\ldots\ldots. + d(v_{n-1})$$

$= 2 + 2 + 2 + \ldots\ldots\ldots + 2$

$= 2q$

$$\sum_{v \in D} d(v) = 2q \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\sum_{v \in D^C} d(v) = d(v_1) + d(v_3) + d(v_4) + \ldots\ldots\ldots. + d(v_{n-2}) + d(v_n)$$

$= 1 + 2 + 2 + \ldots\ldots\ldots + 2 + 1$

$= (2 + 2 + 2 + \ldots\ldots\ldots + 2) - 2$

$= 2(2q) - 2$

$$\sum_{v \in D^C} d(v) = 4q - 2 \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From (1) and (2)

$$\sum_{v \in D} d(v) + \sum_{v \in D^C} d(v) = 2q + 4q - 2$$

$= 6q - 2 = 2(3q) - 2 = 2n - 2$

Therefore $\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 2n - 2.$

Case (ii): Suppose r = 1 then n = 3q + 1

If r= 1 then D= $\{v_2, v_5, v_8, \ldots\ldots\ldots, v_n\}$ and $\gamma(G)$= q + 1, since result 2.

And D^C= $\{v_1, v_3, v_4, \ldots\ldots\ldots\ldots, v_{n-1}\}$

Then $\gamma^C(G) = n - \gamma(G)$

$\gamma^C(G) = 3q + 1 - (q + 1) = 2q$

$\sum_{v\in D} d(v) = d(v_2) + d(v_5) + d(v_8) + \ldots\ldots\ldots\ldots + d(v_n)$

$= 2 + 2 + 2 + \ldots\ldots\ldots\ldots + 2 + 1$

$= 2 + 2 + 2 + \ldots\ldots\ldots\ldots + 2 + 2 - 1 = 2(q + 1) - 1$

$= 2q + 2 - 1 = 2q + 1$

$\sum_{v\in D} d(v) = 2q + 1$ ……………………………………………………………………(3)

$\sum_{v\in D^C} d(v) = d(v_1) + d(v_3) + d(v_4) + \ldots\ldots\ldots\ldots + d(v_{n-1})$

$= 1 + 2 + 2 + \ldots\ldots\ldots\ldots + 2$

$= (2 + 2 + 2 + 2 + \ldots\ldots\ldots + 2) - 1$

$= 2(2q) - 1 = 4q - 1$

$\sum_{v\in D^C} d(v) = 4q - 1$ ……………………………………………………………………(4)

From (3) and (4)

$\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 2q + 1 + 4q - 1 = 6q = 2(3q) = 2(3q + 1 - 1) = 2(n - 1) = 2n - 2$

Therefore $\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 2n - 2.$

Case (iii): Suppose r = 2 then n = 3q + 2

If r= 2 then the domination number $\gamma(G)$= q + 1 and

The minimum dominating set is either D = $\{v_2, v_5, v_8, \ldots\ldots\ldots, v_{n-1}\}$ or D= $\{v_2, v_5, v_8, \ldots\ldots\ldots, v_n\}$

Sub case(i): Suppose D= $\{v_2, v_5, v_8, \ldots\ldots\ldots, v_{n-1}\}$ and $\gamma(G)$= q + 1

And D^C = $\{v_1, v_3, v_4, \ldots\ldots\ldots\ldots, v_{n-2}, v_n\}$

Then $\gamma^C(G) = n - \gamma(G)$

$= (3q + 2) - (q + 1)$

$\gamma^C(G)$ = 2q + 1

$$\sum_{v\in D} d(v) = d(v_2) + d(v_5) + d(v_8) + ... + d(v_{n-1})$$

= 2 + 2 + 2 + ...…... + 2

= 2 (q + 1)

$\sum_{v\in D} d(v)$ = 2q + 2 .... (5)

$$\sum_{v\in D^C} d(v) = d(v_1) + d(v_3) + d(v_4) + ... + d(v_{n-2}) + d(v_n)$$

= 1 + 2 + 2 + ...…. + 2 + 1 = (2 + 2 + 2 + ... + 2) − 2

= 2 (2q + 1) – 2

$\sum_{v\in D^C} d(v)$ = 4q .... (6)

From (5) and (6)

$\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v)$ = 2q + 2 + 4q = 6q + 2

= 6q + 4 − 2 = 2 (3q + 2) − 2 = 2n − 2

Therefore $\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 2n - 2.$

Sub case(ii): Suppose D= $\{v_2, v_5, v_8, ..., v_n\}$ and $\gamma(G)$ = q + 1

Then D^C = $\{v_1, v_3, v_4, ..., v_{n-1}\}$ and

$\gamma^C(G) = n - \gamma(G)$

= (3q + 2) − (q + 1)

$\gamma^C(G)$ = 2q + 1

$$\sum_{v\in D} d(v) = d(v_2) + d(v_5) + d(v_8) + ... + d(v_n)$$

= 2 + 2 + 2 + ……… + 1

= 2 + 2 + 2 + ……….. + 1 + 1 − 1

= 2 + 2 + 2 + ……….. + 2 − 1

= 2(q + 1) − 1 = 2q + 2 − 1 = 2q + 1

$\sum_{v\in D} d(v)$ = 2q + 1.. (7)

$$\sum_{v\in D^C} d(v) = d(v_1) + d(v_3) + d(v_4) + ... + d(v_{n-1})$$

= 1+ 2 + 2 + …..... + 2 = (2 + 2 + 2 + ...…. + 2)− 1

= 2 (2q + 1)− 1= 4q + 2 − 1

$$\sum_{v\in D^C} d(v) = 4q + 1\ldots\ldots (8)$$

From (7) and (8)

$$\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 2q + 1 + 4q + 1 = 6q + 2 = 6q + 4 - 2 = 2(3q + 2) - 2 = 2n - 2$$

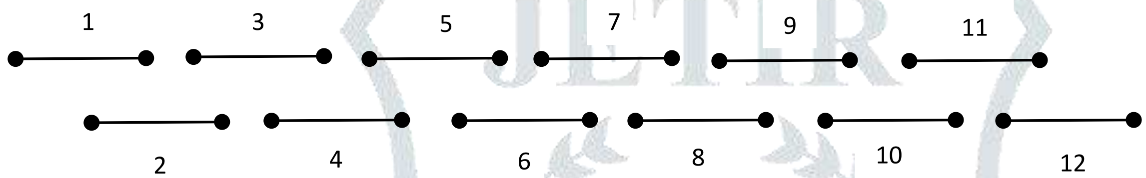
Therefore $\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 2n - 2.$

Hence theorem 2 is proved.

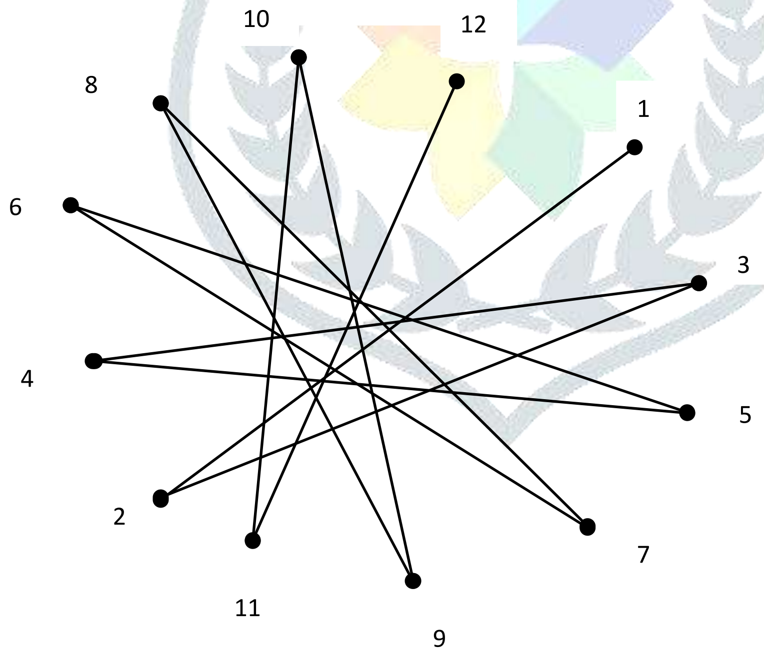
VI. Experimental problems

Illustration 4:

Let I= $\{ I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8, I_9, I_{10}, I_{11}, I_{12} \}$ be an interval family and let G(V, E) be an interval graph corresponding to an interval family I is as follows:



Interval family I



Interval graph G

Here n = 12

And then n = 3 × 4 + 0

This is of the form n = 3q + r

Where q = 4 and r = 0

If r = 0 then D = {2, 5, 8, 11}, since result 1.

And then D^C = {1, 3, 4, 6, 7, 9, 10, 12}

$$\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = \{d(2) + d(5) + d(8) + d(11)\} + \{d(1) + d(3) + d(4) + d(6) + d(7) + d(9) + d(10) + d(12)\}$$

$$= 2 + 2 + 2 + 2 + 1 + 2 + 2 + 2 + 2 + 2 + 2 + 1$$

$$= 22$$

$$= 2(12) - 2$$

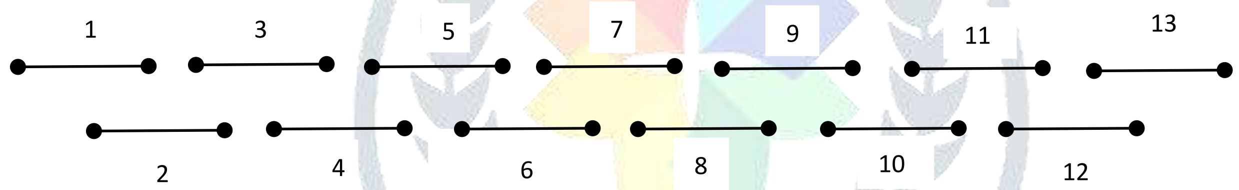
$$= 2n - 2$$

$$\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 2n - 2.$$

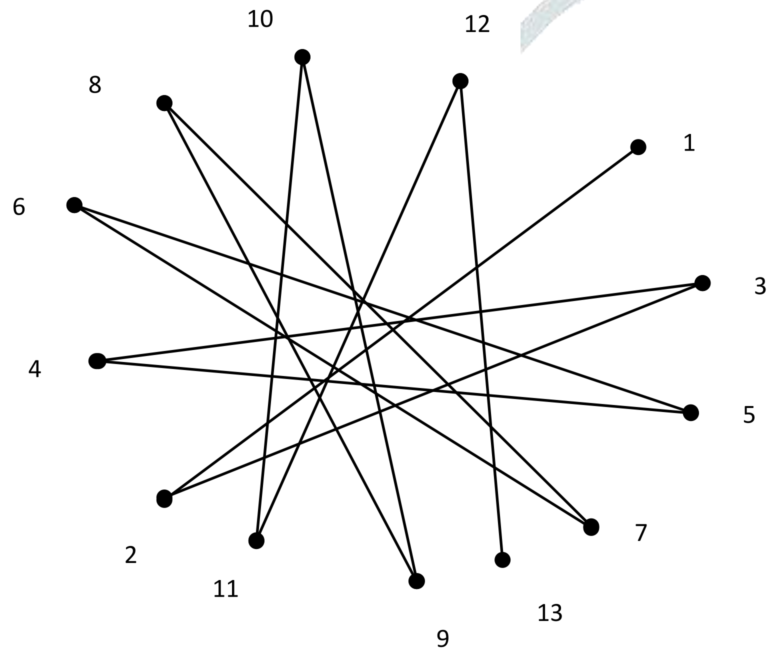
Hence theorem 2 is verified at r = 0.

Illustration 5:

Let I= $\{I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8, I_9, I_{10}, I_{11}, I_{12}, I_{13}\}$ be an interval family and let G(V, E) be an interval graph corresponding to an interval family I is as follows:



Interval family I



Interval graph G

Here n = 13

And then n = 3 × 4 + 1

This is of the form n = 3q + r

Where q = 4 and r = 1

If r = 1 then D = {2, 5, 8, 11, 13}, since result 2.

And then D^C = {1, 3, 4, 6, 7, 9, 10, 12}

$$\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = \{ d(2) + d(5) + d(8) + d(11) + d(13)\} + \{ d(1) + d(3) + d(4) + d(6) + d(7) + d(9) + d(10) + d(12) \}$$

= 2 + 2 + 2 + 2 + 1 + 1 + 2 + 2 + 2 + 2 + 2 + 2 + 2

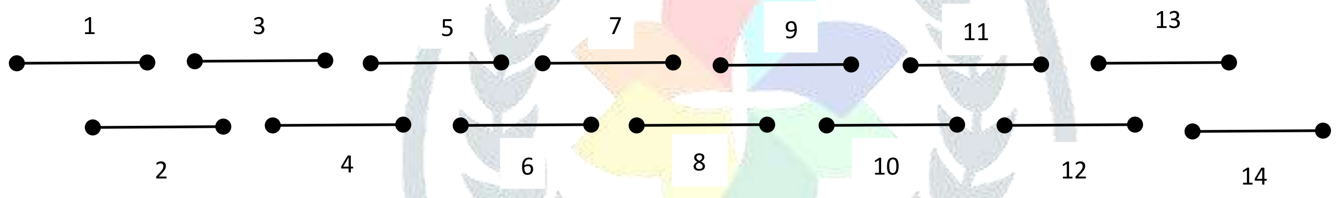
= 24 = 2(13) – 2 = 2n – 2

$$\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 2n - 2.$$

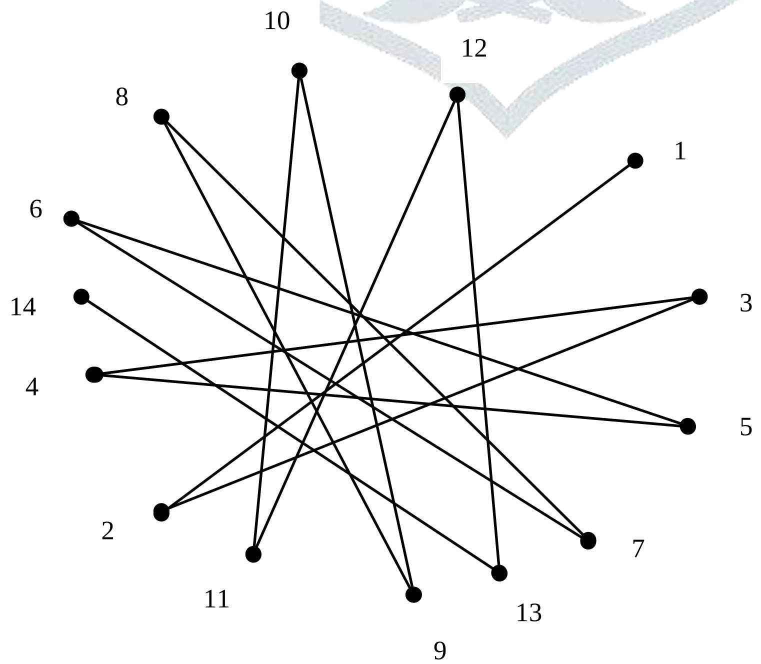
Hence theorem 2 is verified at r = 1.

Illustration 6:

Let I= $\{ I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8, I_9, I_{10}, I_{11}, I_{12}, I_{13}, I_{14} \}$ be an interval family and let G(V, E) be an interval graph corresponding to an interval family I is as follows:



Interval family I



Interval graph G

Here n = 14

And then n = 3 × 4 + 2

This is of the form n = 3q + r

Where q = 4 and r = 2

If r = 2 then D = {2, 5, 8, 11, 13}

or D = {2, 5, 8, 11, 14}, since result 3.

Sub case (i) :

Suppose D= {2, 5, 8, 11, 13}

And then D^C = {1, 3, 4, 6, 7, 9, 10, 12, 14}

$$\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = \{ d(2) + d(5) + d(8) + d(11) + d(13) \} + \{d(1) + d(3) + d(4) + d(6) + d(7) + d(9) + d(10) + d(12) + d(14) \}$$

$$= 2 + 2 + 2 + 2 + 2 + 1 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 1$$

$$= 26$$

$$= 2(14) - 2$$

$$\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 2n - 2.$$

Sub case (ii) :

Suppose D = {2, 5, 8, 11, 14}

And then D^C = {1, 3, 4, 6, 7, 9, 10, 12, 13}

$$\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = \{ d(2) + d(5) + d(8) + d(11) + d(14)\} + \{ d(1) + d(3) + d(4) + d(6) + d(7) + d(9) + d(10) + d(12) + d(13) \}$$

$$= 2 + 2 + 2 + 2 + 1 + 1 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2$$

$$= 26$$

$$= 2(14) - 2$$

$$= 2n - 2$$

Therefore $\sum_{v\in D} d(v) + \sum_{v\in D^C} d(v) = 2n - 2.$

Hence theorem 2 is verified at r = 2.

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