A COMPARITIVE STUDY ON REDUCTION OF LOWAND HIGH GAUSSIAN NOISE IN SURVEILLANCE IMAGES USING PARTICLE FILTERS

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Abstract – In this paper propose a noise removal method for reducing low and high noise in digital images. An efficient Rao-Blackwellized Particle Filter (RBPF) with MLE edge Preserving approach is used for improving the learning stage of the image structural model and guiding the particles to the most appropriate direction. It increases the efficiency of particle transitions. The proposal distribution is computed by conditionally Gaussian state space models and Rao-Blackwellized particle filtering with MLE edge Preserving. The discrete state of operation is identified using the continuous measurements corrupted by Gaussian noise. The analytical structure of the model is computed by the distribution of the continuous states. The posterior distribution can be approximated with a recursive, stochastic mixture of Gaussians. Rao-Blackwellized particle filtering is a combination of a particle filter (PF) and a bank of Kalman filters. The distribution of the discrete states is computed by using Particle Filters and the distribution of the continuous states are computed by using a bank of Kalman filters. The RBPF with MLE edge Preserving is very effective in eliminating noise when compared with particle filter. In this paper different performance metrics are evaluated for this type of noise removal technique. The metrics are Mean Square error, Root Mean square error, Peak Signal to Noise Ratio, Normalized absolute Error, and Normalized Cross Correlation, Mean Absolute Error and Signal to Noise Ratio. Experimental results prove that RBPF with MLE edge Preserving outperforms for degraded surveillance images.

Index Terms: Kalman filter, Particle filter, Rao-Blackwellized particle filter, Sampling Importance Resampling

I. INTRODUCTION

Restoration of degraded images has become an important and effective technique in aerial imaging, medical imaging and many other applications. The images obtained are a degraded version of the original image due to imaging process, atmosphere and the recording medium which introduce noises in the captured image. No imaging system gives images of perfect quality because of degradations caused by various reasons. So the image needs to be restored for subsequent computer processing and human viewing.

Rao-Blackwellized Particle Filter (RBPF) is an efficient Monte Carlo particle filter for restoring images. This algorithm finds the analytical structure of the model. The distribution of the continuous states is computed exactly by knowing the values of the discrete states. A particle filter (PF) which is used to compute the distribution of the discrete states and a bank of Kalman filters which is used to compute the distribution of the continuous states. Therefore, combine a particle filter (PF) with a bank of Kalman filters is known as Rao-Blackwellization, because it is related to the Rao-Blackwell formula. That is, approximate the posterior distribution with a recursive, stochastic mixture of Gaussians. The RBPF makes less estimation mistakes. The distribution of the discrete states is computed by RBPF. The Rao-Blackwellized particle filter is to improve the learning stage by estimating a posterior. Here Sampling Importance Resampling (SIR) filter is used for updating a set of samples. In the particle filtering, we use a weighted set of particles to approximate the posterior. This approximation can be updated recursively. The Gaussian density can be computed analytically by using marginal posterior density. This density satisfies the alternative recursion. The particle filter starts at a time with an unweight measure. For each particle we compute the importance weights using the information at time t. A resampling step selects only the correct particles to obtain the unweight measure. This yields an approximation of that is “concentrated” on the most likely hypothesis. Now use a weighted set of samples to represent the marginal posterior distribution. The marginal density is a Gaussian mixture that can be computed efficiently with a stochastic bank of Kalman filters. A Rao-Blackwellized filter that combines this marginalization and sampling. MRF filter is used for the image restoration, even though it has some drawbacks on their selection of filter parameter. Thus the application of RBPF with MLE edge preserving technique in this case is supposed to greatly improve the estimation efficiency and improve the restoration problem.

II. RAO-BLACKWELLIZED PARTICLE FILTER

If the model is of certain form the efficiency of SIR can be enhanced by using the theorem of Rao-Blackwell. The theorem states that if $g(X)$ is any kind of estimator of some parameter θ and $T(X)$ some sufficient statistic, then the conditional expectation of $g(X)$ given $T(X)$ is usually better estimate of θ and never worse. This kind of estimator transformation is often referred as Rao-Blackwellization. The Rao-Blackwellized particle filter (RBPF)is based on the idea that sometimes it is possible to compute some of the filtering equations in closed form and the other with Monte Carlo sampling instead of using sampling methods for all equations. According to Rao-Blackwell theorem this leads to estimators with less variance, which can be interpret as replacing a finite set of particles with a infinite set, which is always more accurate than any finite set. In this context the Rao-Blackwellized particle filtering refers to marginalized filtering of Markov models, which are conditionally Gaussian of form

$$p(x_k|x_{k-1,}\lambda_{k-1}) = N(x_k|A_{k-1}(\lambda_{k-1})x_{k-1,}Q_{k-1}(\lambda_{k-1})) \quad (2.1)$$

$$p(y_k|x_k,\lambda_k) = N(y_k|H_k(\lambda_k)x_k,R_k(\lambda_k \quad (2.2)$$

$$p(\lambda_k|\lambda_{k-1}) = (\text{any given form}), \quad (2.3)$$

where $x_k$is the state, y_k the measurement and an arbitrary latent variable. $M(\lambda)$ denotes that matrix M is conditioned on the value of latent variable λ.If the prior distribution of x_k is also Gaussian the state variables x_k can be integrated out in closed form and only the latent variables λ_k need to be sampled. The resulting algorithm is summarized below.

A. Conditionally Gaussian Rao-Blackwellized particle filter

Given an importance distribution $\pi(\lambda_k|\lambda_{1:k-1}^{(i)}, y_{1:k})$, set of particles $\{w_{k-1}^{(i)}, \lambda_{k-1}^{(i)}, m_{k-1}^{(i)}, p_{k-1}^{(i)} : i = 1, \dots, N$ conditioanls on the previously drawn latent vales$\lambda_{k-1}^{(i)}$ *as*

1. Perform kalman filter for the means of $m_{k-1}^{(i)}$ and the covariance's $P_{k-1}^{(i)}$of particles $i = 1, \dots., N$ conditional on the previously drawn latent variables $\lambda_{k-1}^{(i)}$ as

$$m_{k-1}^{(i)} = A_{k-1}^{(i)}(\lambda_{k-1}^{(i)})m_{k-1}^{(i)} \quad (2.4)$$

$$P_k^{-(i)} = A_{k-1}(\lambda_{k-1}^{(i)})P_{k-1}^{(i)}A_{k-1}^T(\lambda_{k-1}^{(i)}) + Q_{k-1}(\lambda_{k-1}^{(i)}) \quad (2.5)$$

2. Draw new latent variables $\lambda_{k-1}^{(i)}$for each particle in $i = 1, \dots, N$from the corresponding importance distributions

$$\lambda_k^{(i)} \sim \pi(\lambda_k|\lambda_{1:k-1}^{(i)}, y_{1:k}) \quad (2.6)$$

3. Calculate new (unnormalized) weights as follows:

$$w_{k-1}^{*(i)} \alpha\, w_{k-1}^{*(i)} \frac{P(y_k|\lambda_{1:k}^{(i)}, y_{1:k-1})P(\lambda_k^{(i)}|\lambda_{k-1}^{(i)})}{\pi(\lambda_k^{(i)}|\lambda_{k-1}^{(i)}, y_{1:k})} \quad (2.7)$$

Where the likelihood term is the marginal measurement likelihood of the kalman filter

$$p(y_k|\lambda_{1:k}^{(i)}, y_{1:k-1}) = N(y_k|H_k(\lambda_k^{(i)})m_k^{-(i)}, H_k(\lambda_k^{(i)})P_k^{-(i)}H_K^T(\lambda_k^{(i)}) + R_k(\lambda_k^{(i)})) \quad (2.8)$$

Such that the model parameters in the kalman filter are conditioned on the drawn latent variable value$\lambda_k^{(i)}$

4. Normalize the weights to sum to unity as

$$w_k^{(i)} = \frac{w_k^{*(i)}}{\sum_{j=1}^N w_k^{*(i)}} \quad (2.9)$$

5. Perform Kalman filter updates for each of the particles conditional on the drawn latent variables$\theta_k^{(i)}$

$$v_k^{(i)} = y_k - H_k(\lambda_k^{(i)})m_k^- \quad (2.10)$$

$$S_k^{(i)} = H_k(\lambda_k^{(i)})P_k^{-(i)}H_K^T(\lambda_k^{(i)}) + R_k(\lambda_k^{(i)}) \quad (2.11)$$

$$k_k^{(i)} = P_k^{-(i)}H_k^T(\lambda_k^{(i)})S_k^{-1} \quad (2.12)$$

$$m_k^{(i)} = m_k^{-(i)} + k_k^{(i)}v_k^{(i)} \quad (2.13)$$

$$P_k^{(i)} = p_k^{-(i)} - k_k^{(i)}S_k^{(i)}[K_k^{(i)}] \quad (2.14)$$

6. If the effective number of particles (**Kitagawa, 1996**) is too low, perform resampling. After the set of particles have been acquired the filtering distribution can approximated as

$$p(x_k, \lambda_k|y_{1:k}) \approx \sum_{i=1}^N w_k^{(i)}\, \delta(\lambda_k - \lambda_k^{(i)})N(x_k|m_k^{(i)}, p_k^{(i)}) \quad (2.15)$$

B. Kalman Filter Algorithm

The aim is to compute the marginal posterior distribution of the discrete states$P(z_{0:t}|y_{1:t})$. This distribution can be derived from the posterior distribution by standard marginalization. The posterior density satisfies the following recursion.

$$P(x_{0:t}, z_{0:t}|y_{1:t}) = P(x_{0:t-1}, z_{0:t-1}|y_{1:t-1}) \times \frac{P(y_t|x_t, z_t)P(x_t, z_t|x_{t-1}, z_{t-1})}{P(y_t|y_{1:t-1})} \quad (2.16)$$

This recursion involves intractable integrals. The density is Gaussian and it can be computed analytically using the marginal posterior density.

$$P(z_{0:t}|y_{1:t}) = P(z_{0:t-1}|y_{1:t-1})\frac{P(y_t|y_{1:t-1}, z_{0:t})P(z_t|z_{t-1})}{P(y_t|y_{1:t-1})} \quad (2.17)$$

The continuous probability distributions and discrete distributions admit densities. To represent the marginal posterior distribution using a weighted set of samples

$$\hat{P}\,N(z_{0:t}|y_{1:t}) = \sum_{i=1}^N w_t^{(i)}\delta_{z_{0:t}^{(i)}}(z_{1:t}) \quad (2.18)$$

The marginal density is a Gaussian mixture that can be computed efficiently with a stochastic bank of Kalman filters.

$$\widehat{P}\,N(x_{0:t}|y_{1:t}) = \sum_{i=1}^{N} w_t^{(i)} P(x_{0:t}|y_{1:t}, z_{0:t}^{(i)}) \quad (2.19)$$

A Rao-Blackwellized filter that combines this marginalization and sampling of z_t. The RBPF is similar to the PF, but we only sample the discrete states. Then for each sample of the discrete states, we update the mean and covariance of the continuous states using exact computations. In particular, we sample $z_t^{(i)}$ and then propagate the mean $\mu_t^{(i)}$ and covariance $\Sigma_t^{(i)}$ of x_t with a Kalman filter as follows:

$$\mu_{t|t-1}^{(i)} = A(z_t^{(i)})\mu_{(t-1|t-1)}^{(i)} + F(z_t^{(i)})u_t$$

$$\Sigma_{t|t-1}^{(i)} = A(z_t^{(i)})\Sigma_{t-1|t-1}^{(i)}A(z_t^{(i)})^T + B(z_t^{(i)})B(z_t^{(i)})^T$$

$$C(z_t^{(i)})\Sigma_{t|t-1}^{(i)}C(z_t^{(i)})^T + D(z_t^{(i)})D(z_t^{(i)})^T$$

$$y_{t|t-1}^{(i)} = C(z_t^{(i)})\mu_{(t|t-1)}^{(i)} + G(z_t^{(i)})u_t$$

$$\mu_{t|t}^{(i)} = \mu_{t|t-1}^{(i)} + \Sigma_{t|t-1}^{(i)}C(z_t^{(i)})^T S_t^{-1(i)}(y_t - y_{t|t-1}^{(i)})$$

$$\Sigma_{t|t}^{(i)} = \Sigma_{t|t-1}^{(i)} - \Sigma_{t|t-1}^{(i)}C(z_t^{(i)})^T S_t^{-1(i)}C(z_t^{(i)})\Sigma_{t|t-1}^{(i)}$$

Where

$\mu_{t|t-1} \triangleq E(x_t|y_{1:t-1}), \mu_{t|t} \triangleq E(x_t|y_{1:t}), \mu_{t|t-1} \triangleq E(x_t|y_{1:t-1}), \Sigma_{t|t-1} \triangleq \mathrm{cov}\,(x_t|y_{1:t-1}), \Sigma_{t|t} \triangleq \mathrm{cov}\,(x_t|y_{1:t})$

And $S_t \triangleq \mathrm{cov}\,(y_t|y_{1:t-1})$.

Hence, using the prior proposal for z_t, find that the importance weights for z_t are given by the predictive density.

$$P(y_t|y_{1:t-1}, z_{1:t}) = N(y_t; y_{t|t-1}, S_t)$$

Here, the KF and PF parts interact heavily and steps in the respective algorithms are mixed, therefore it is difficult to clearly see the problem structure and how to use standard components in the filtering. Also, the time update and measurement update of the various filters are neither completely separated nor straight forwardly explained.

RBPF Standard Formulation:

Step 1: Initialization: $For\ i = 1, \dots, N, x_{0|-1}^{p,(i)} \sim px_0^p(x_0^p)$ and set $\{x_{0|-1}^{k,(i)}, P_{0|-1}^{(i)}\} = \{\bar{x}_0^k, P_0\}$. Let t=0.

Step 2: PF measurement update: $For\ i = 1, \dots, N$, evaluate and normalize the importance weights using to the likelihood.

$$p(y_t|X_t^p, Y_{t-1}) = N\left(h(x_t^p) + H(x_t^p)\hat{x}_{t|t-1}^k, H(x_t^p)P_{t|t-1}\left(H(x_t^p)\right)^T + R_t\right)$$

Sample the particle

Step 3: PF time update and Kalman filter update

3.1: KF measurement update

$$\hat{x}_{t|t-1}^{k,(i)} = \hat{x}_{t|t-1}^{k,(i)} + K_t\left(y_t - h(x_t^p) - H(x_t^p)\hat{x}_{t|t-1}^{k,(i)}\right)$$

$$P_{t|t} = P_{t|t-1} - K_t M_t K_t^T$$

$$M_t = H(x_t^p)P_{t|t-1}H_t^T(x_t^p) + R_t$$

$$K_t = P_{t|t-1}H^T(x_t^p)M_t^{-1}$$

3.2: PF time update:

$For\ i = 1, \dots, N,$

$$x_{t+1}^{p,(i)} \sim N\left(f^p(x_t^{p,(i)}) + F^p(x_t^p)\hat{x}_{t|t}^{k,(i)}, F^p(x_t^p)P_{t|t}\left(F^p(x_t^p)\right)^T + Q_t^p\right)$$

3.3: KF time update,

$$\hat{x}_{t+1|t}^{k,(i)} = F^k\hat{x}_{t|t}^{k,(i)} + f^k(x_t^p) + L_t\left(x_{t+1|t}^{p,(i)} - f^p\left(x_t^{p,(i)}\right) - F^p(x_t^p)\hat{x}_{t|t}^{k,(i)}\right)$$

$$P_{t+1|t} = k(x_t^p)P_{t|t}\left(F^k(x_t^p)\right)^T + Q_t^k - L_t M_t L_t^T$$

$$M_t = F^k(x_t^p)P_{t|t}\left(F^k(x_t^p)\right)^T + Q_t^p$$

$$L_t = F^k(x_t^p)P_{t|t}\left(F^p(x_t^p)\right)^T M_t^{-1}$$

Step 4: Set $t: t+1$ and repeat from step 2.

A. Maximum Likelihood Estimation (MLE)

The measured intensity of a pixel can be considered as a random variable that takes a value I in the space of observations Ω. Given an observed I, the probability that it resulted from a true intensity I' of scene radiance is given by the conditional probability density $P(I|I')$. Alter et al. [2] presents a similarity measure based on ML estimation. From two observations, it first estimates the true intensity I^*, and then derives similarity as the product of conditional probabilities:

$$I^* = \underset{I'}{\mathrm{argmax}}\, p(I_1|I')p(I_2|I') \quad (2.20)$$

$$S_{ML}(I_1, I_2) = p(I_1|I^*)p(I_2|I^*) \quad (2.21)$$

The ML similarity S_{ML} is defined as the likelihood of two intensity observations resulting from the single true intensity I^* that gives the greatest likelihood

III. PROPOSED RBPF WITH MLE EDGE PRESERVING ALGORITHM

Sequential importance sampling step,

Step 1: For $i = 1, \dots, N$,

Set $\hat{\mu}_{t|t-1}^{(i)} \triangleq \mu_{t|t-1}^{(i)}, \hat{\Sigma}_{t|t-1}^{(i)} \triangleq \Sigma_{t|t-1}^{(i)},$ (3.1)

Sample $\hat{z}_t^{(i)} \sim P_r(z_t|z_{t-1}^{(i)})$ (3.2)

Step 2: For $i = 1, \dots, N$, evaluate and normalize the importance weights

$w_t^{(i)} \alpha P(y_t|y_{1:t-1}\hat{z}_t^{(i)})$ (3. 3)

Selection step

Step 1: Multiply/Discard particles

$$\left\{\hat{\mu}_{t|t-1}^{(i)}, \hat{\Sigma}_{t|t-1}^{(i)}, \hat{z}_t^{(i)}\right\}_{i=1}^{N} \quad (3.4)$$

With respect to high/low importance weights $w_t^{(i)}$ to obtain N particles

Here the low/high weights are update by the probability function, it is obtained by MLE

$$\left\{\mu_{t|t-1}^{(i)}, \Sigma_{t|t-1}^{(i)}, z_t^{(i)}\right\}_{i=1}^{N} \quad (3.5)$$

Updating step

Step 1:For $i = 1, \ldots, N$, use one step of the Kalman recursion to compute the minimum statistics

$$\left\{\mu_{t+1|t}^{(i)}, \Sigma_{t+1|t}^{(i)}, y_{t+1|t}^{(i)}, S_{t+1}^{(i)}\right\} \quad (3.6)$$

Given $\left\{z_t^{(i)}, \mu_{t|t-1}^{(i)}, \Sigma_{t|t-1}^{(i)}\right\}$ (3.7) Edge preserving:

Step 1: apply the edge preserving function using equation 2.20 and 2.21 in the updating step

IV. EXPERIMENTAL RESULTS

The performance of the proposed algorithm is tested with different noise levels. Each time the test image is corrupted by different additive white Gaussian noise standard deviation 10 and 50. These surveillance noisy images are denoised by two algorithms and the performance difference between the particle filter and the proposed approach measured by the parameters PSNR, MSE, RMSE, NAE, NCC, MAE and SNR. Noise removal for surveillance image is done by using proposed RBPF with MLE edge Preserving. Extensive comparative studies for surveillance image for both proposed RBPF algorithm with MLE edge Preserving to the existing standard particle filter.

A. Performance Metrics

Noise removal in the surveillance images is done using proposed edge preserving particle filter and Standard Particle Filter. The results were evaluated using performance metrics such as PSNR, MSE, RMSE, NAE, NCC, MAE and SNR

Let us take

$x = \text{double}(\text{input image})$,

$y = \text{double}(\text{filtered image})$,

$z = \text{abs}(x - y)$

Mean Square Error

$(\text{MSE}) = \sqrt{\text{mean}(\text{mean}(Z^2))}$ Higher value of MSE means that the quality of image is poor.

$\text{RMSE} = \sqrt{\text{MSE}}$ RMSE is similar to MSE; if the value of RMSE is small it means that the image quality of the restored image is good.

Peak signal to Noise Ratio

$(\text{PSNR}) = 25 * \log 10\left(\frac{25}{\sqrt{\text{MSE}}}\right)$

Higher value of PSNR means the restored image is of better quality.

Let us take difference $= x - y$

Normalized Absolute Error

$(\text{NAE}) = \frac{\text{sum}(\text{sum}(\text{abs})\text{difference})))}{\text{sum}(\text{sum}(\text{abs}(x)))}$

The smaller value of NAE means that the removal of noise in the image is better.

Let us assume $x_1 = \text{sum}\left(\text{sum}(x^2)\right)$

Normalized Cross Correlation

$(\text{NCC}) = \frac{\text{sum}(\text{sum}(x*y)}{x_1}$

If the Normalized Cross Correlation tends to 1, then the image quality will be better.

Mean Absolute Error is a quantity used to measure the sum of absolute difference between input image and the filtered image to the number of pixels present in the image. Mean absolute error is the average of an absolute error, where f_i is the filtered image and y_i is the input image.

Maximum Absolute Error

$(\text{MAE}) = \frac{1}{n}\sum_{i=1}^{n}\left|f_i - y_i\right|$

Lower value of MAE means better result in image quality.

Signal to Noise Ratio denoted as SNR is defined as power of original signal to power of the corrupted signal.

Signal-to-Noise Ratio

$(\text{SNR}) = 10 \log_{10}\left(\frac{P\ \text{signal}}{P\ \text{noise}}\right)$

B. Results for Surveillance image

10 % and 50% of Noise level is added to Surveillance image. Comparison of RBPF with MLE edge Preserving to the standard particle filter using performance metrics are given below .

TABLE 4.1: PERFORMANCE METRICS FOR NOISE LEVEL 10% AND 50%

Performance Metrics	Noise Level 10%		Noise Level 50%	
	Existing Standard Particle Filter	Proposed RPBF with MLE edge Preserving	Existing Standard Particle Filter	Proposed RPBF with MLE edge Preserving
PSNR	22.506	30.0159	21.366	29.1990
MSE	6.3336	2.9363	7.1217	3.1979
NAE	0.1048	0.0486	0.1177	0.0529
SNR	18.638	18.6389	18.638	18.6386
MAE	12.569	5.8308	14.125	6.3499
NCC	0.9750	0.9913	0.9701	0.9904
Execution Time	2.0714	32.8536	2.1392	23.4360

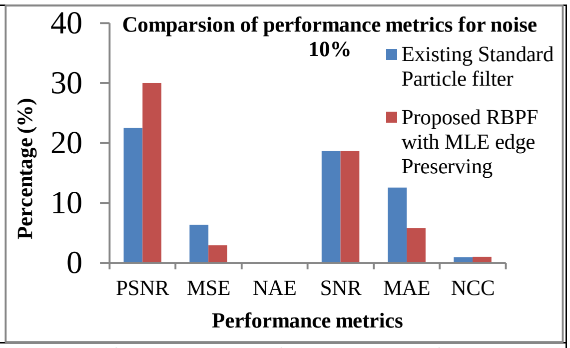

Fig 4.1 : Performance metrics of 10% Noise level for surveillance image

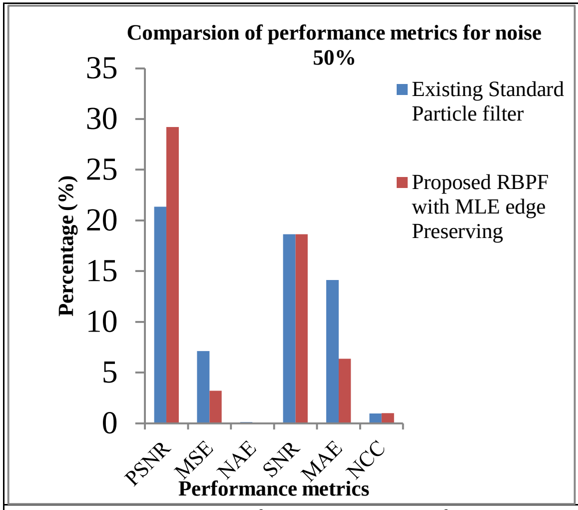

Fig. 4. 2: Performance metrics of 50% Noise level for surveillance image

The PSNR, MSE, NAE, SNR, MAE and NCC values are given in Table 4. 1. From the results shows that for noisy images (10% and 50%), proposed RBPF with MLE edge preserving algorithm obtains comparable PSNR, MSE, NAE, SNR, MAE and NCC values to the existing standard particle filter algorithm. Performance evaluation involves a qualitative criterion that reflects the ability of the proposed RBPF with MLE edge preserving algorithm suppress noise while preserving image details is given in Figure 4. 1 and 4.2. It is clear that RBPF with MLE edge preserving technique consistently outperforms existing standard particle filter and achieves the best results.

V. CONCLUSION

In this paper, proposed a denoising method motivated by previous analysis of particle filter. Insights from that study are used here to derive a high-performance denoising algorithm. Proposal Rao-Blackwellized particle filters (RBPF) with MLE edge preserving that exploits image denoising. The performance of RBPF with edge preserving approach is experimentally verified on a surveillance image and noise levels of 10% and 50%. The results presented here demonstrate that proposed method is compared to existing standard particle filter. Experimental validation of RBPF with MLE edge preserving method and compare it, visually and quantitatively, to existing standard particle filter with some performance metrics and concluded that our method suppresses the noise effectively, while retaining the fines details compared to existing denoised method of standard particle filter.

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