Continuous Self-Maps on some Metric Space

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ABSTRACT: The main aim of this paper is to represent the continuity of self mapping on some metric space. Continuity under some fixed circle theorem on metric space, uniqueness theorems and existence of the self mapping with fixed circle.

Keywords-Continuous mapping, fixed circle, uniqueness mapping.

Introduction

In mathematics, a function defining the distance between elements of a set is known as a metric space. Geometrically, the world 'metric' is used to refer a structure defined only on differentiable functions which is more properly termed a metric tensor.

The function is continuous only iff there is small changes in input result to output result .If it is not, then the function is said to be discontinuous.

A function is said to be continuous at any point if it satisfies the conditions given below:

(i) Left hand limit of f = Right hand limit of f
(ii) The function is defined at x = a
(iii) Limit of f at 'a'= f(a)

On other hands, a function f is continuous at x=a provided all three of the following conditions

(i) f (a) exists

(ii) limit $f\ (x)$ exists $x \to$ a
(iii) f (a) = if $f\ (x)$
$x \to$ a

It is important to describe the behavior of the properties of a dynamical system under the replacement of the interval by a more complicated graph. We prove that the limit sets of self-maps of the circle‘s’ retain certain properties of those of the interval.

Definition :-

A metric on a set X is a function d : X x X → R is also called distance function where R is the set of real numbers and x, y, z in X. This function is required to satisfy the following conditions.

(i) $d(x, y) \geq 0$ (Non – negativity)

(ii) $d(x, y) = 0$ if $f(x) = y$ (identity)

(iii) $d(x, y) = d(y, x)$ (symmetry)

(iv) $d(x, z) \leq d(x, y) + d(y, z)$ (Triangle inequality)

The first condition is implied by the others. A metric d on X is called intrinsic if any two points x and y in X can be joined by a curve with length arbitrarily close to d (x, y)

For set on which an addition + : X x X →X is defined, d is called a translation invariant metric if

$$d(x, y) = d(x+a, y+a) \qquad x, y, a \in X$$

Mappings :-

A mapping f from a metric space (x, d) to another (y, e) is an isometry if it is distance preserving: that is $E[(f(x_1), f(x_2)] = d(x_1, x_2)$

A mapping f from a metric space (x, d) to another (y, e) is continuous at x in X if for all real $\in > 0$ there exists $\delta > 0$ such that

$d(x^1, x) < \delta \Rightarrow e(f(x^1), f(x) < \in$ and continuous if it is continuous at every point of x.

Some fixed – circle theorem on metric space

The fixed point theory and its applications to various areas of science are well known. By geometric interpretation, " we represent the existence and uniqueness theorems for fixed circle of self- mappings on metric spaces" .

Theorem (1) :- let (X, d) be a complete metric space and T : X → X. If there exists a lower Semi continuous functions ψ mapping x into the non-negative real numbers

d(x, Tx) $\leqslant \psi(x) - \psi(T_{(x)})$ where $x \in X$

Then T has a fixed point

Proof - By Rhoades' condition

d (Tx, Ty) < Max $\{d\ (x, y), d\ (x, Tx), d\ (y, Ty), d\ (x, Ty), d(y, Tx)\}$

x,y ∈ X and x ≠ y

In special metric spaces, mapping with fixed points have been used in neural networks as activation functions. For example Mobius transformations is a rational function of the form

$$T(z) = \frac{az+b}{cz+d} \qquad \ldots\ldots(i)$$

Here a, b, c, d are complex numbers and satisfying ad-bc ≠ 0

Thus, we conclude, "the fixed points of a neural network were determined by the fixed points of the employed activation function". By guaranteed, "the existence of the fixed points of an activation function were underlying Mobius transformation of one or two fixed points." On the other side, there are some examples of functions which fix a circle. Let C be the metric space with the usual metric

d (z,w) = $|z - w|$

z, w ∈ C

Let the mapping T be defined as Tz = $\frac{1}{z}$ z ∈ C

The mapping T fixes the unit circle Co.

Existance of the self-mapping with fixed circle.

Some example of mappings which have or not fixed circles Now , we define by definition as – “Suppose (X, d) is a metric space and the circle C_{x0}, r = $\{x \in X: d\,(x_0, r) = r\}$. For a self mapping T : X→ X if T_Z = x for all x∈C_{x0}, then we call the circle (C_{x0},r) as the fixed circle of T.

Theorem: - Suppose (X, d) is a metric space and also (C_{x0}, r) is any circle on X. Also we suppose the mapping.

$\psi : X \to [0,\infty)$, $\psi(x) = d(x, x_0)$; $x \in X$

If there exists a self mapping T : X→ X satisfying (C1) $d(x_1\, Tx) < \psi(x) - \psi(T_x)$ and (C2) $d(Tx, x_0) \geq r$, $x \in (C_{x0}, r)$.

Then we have to prove that the circle(C_{x0}, r) is a fixed circle of T.

Proof :- Let us consider the mapping ψ defined by ψ : X→ [0, ∞), $\psi(x) = d(x, x_0)$. Suppose consider any arbitrary point x∈ (C_{x0},r). We show that Tx = x where x ∈ (C_{x0}, r)

By condition (C_1), we get

$d(x, Tx) \leq \psi(x) - \psi(Tx) = d(x, x_0) - d(Tx, x_0) = r - d(Tx, x_0)$ ------(ii)

Because of the condition ‘C2’ the point ‘Tx’ should be lies on or exterior of the circle (C_{x0}, r). Then we have two cases, If $d(Tx, x_0) > r$ then using (ii) we have a contradiction. Therefore, $d(Tx, x_0) = r$. In this case, using (ii) we get

$d(x, Tx) \leq r - d(Tx, x_0) = r - r = 0$ and so Tx = x

Hence, we obtain Tx = x ;x ∈ (C_{x0},r) consequently, the self mapping T fixes the circle (C_{x0}, r).

Now, we give a fixed circle example

Example :- Let (x, d) be a metric space and x/α be constant such that (x/α , x_0) > r

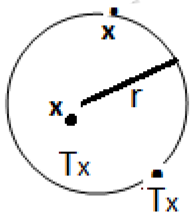
Let us consider a circle C_{x0}, r and define the self mapping T: X → X as

$$Tx = \begin{cases} x : x \in C_{x0,}r \\ \alpha \quad : otherwise \end{cases}$$

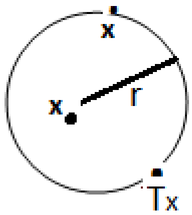
for all x ∈ X. Then it can be easily seen that the conditious (C1) and (C2) are satisfied. Thus ,The Circle (C_{x0}, r) is a fixed circle of 'T.'

By the following examples of self-mapping which satisfies the condition (C1) and does not satisfy the condition (C2)

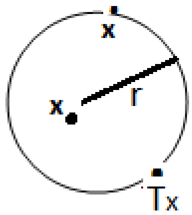
Example (ii) – Let (X, d) be any metric space, (C_{x0}, r) be any circle on X and the self mapping T : X → X be defined as $T_x = x_0$,



(A) The Condition (C1)



(B) The condition (C2)



(C) The condition (C1) ∩ (C2)

The geometric interpretation of the conditions (C1) and (C2)

For all x ∈ X. Then the self- mapping T satisfies the condition (C1) but does not satisfy the condition (C2). Thus 'T' does not fix the circle (C_{x0},r.)

Example (iii) :- Suppose (X, d) is a any metric space and (C_{x0}, r) is any circle on X. Also Suppose α be chosen such that d (α , x_0) = ρ > r and consider the self mapping T : X → X for all x ∈ X. Then the self – mapping T satisfies the condition 'C2' and does not satisfies the condition 'C1'. Therefore, 'T' does not fix the circle (C_{x0}, r.)

Uniqueness Theorems

We investigate the uniqueness of the fixed circle. Notice that the fixed circle (C_{x0}, r) is not necessarily unique in theorem (i) and (ii) we can give the following result.

Proposition :- Suppose (X, d) is a metric space. For any given circle (C_{x0}, r) and (C_{x0}, ρ) there exists at least one self-mapping T of X such that T fixes the circles (C_{x0}, r) and (C_{x1}, ρ).

Proof :- Let (C_{x0}, r) and (C_{x1}, ρ)be any circles on X. Let us define the self-mapping

T : X → X as

$$Tx = \begin{cases} x : x \in C_{x0,r} \ U \ Cx_1,\rho \\ \quad \alpha : otherwise \end{cases}$$

all x ∈ X

Where α is constant satisfying d (α , x_0) ≠ r and d (α, x_1) ≠ ρ . Let us define the

Mapping ψ_1, ψ_2: X → [0, ∞) as

ψ_1, (x) = d (x, x_0) and ψ_2 (x) = d (x, x_1) ; x ∈ X.

Clearly, the conditions (C1) and (C2) are satisfied by T for the circles (C_{x0}, r) and (C_{x1}, ρ)with the mappings, ψ_1(x) and ψ_2(x) respectively.

Clearly (C_{x0}, r) and (C_{x1},) are the fixed circles of T by theorem (i)

Noted that the circles (C_{x0}, r) and (C_{x1},) do not have to be disjoint.

We apply “uniqueness condition for the fixed circles in theorem (ii)”

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