The Transformation of Space and Time, re-interpretation of the Lorentz Factor and the Universal Speed Limit

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Abstract: In this paper we will examine how Newtonian Transformation of space and time between two Inertial Frames of Reference does not agree with the constant speed of light, as demonstrated theoretically by the Laws of Maxwell and later proven experimentally numerous times. Next, we will examine how the Lorentz Factor can be re-interpreted and finally based on the re-interpretation we will see what the Universal Speed Limit should be.

***IndexTerms* - Newtonian Mechanics, Transformation, Lorentz Factor, Re-Interpretation, Universal Speed Limit.**

I. INTRODUCTION

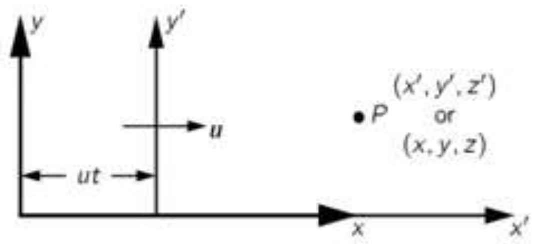
The paper starts with Newtonian concepts of transformation of space and time between two Inertial Frames that are moving at a constant velocity with respect to each other. However, these transformations cannot explain the constant speed of light as demonstrated by Maxwell's Theory and later proven by numerous experiments. This led to the requirements of a new way to transform space and time. We look at how the Lorentz Factor, a very important part of Special Theory of Relativity can be re-interpreted. And finally, based on this re-interpretation we determine what the Universal Speed Limit should be.

II. THEORETICAL DERIVATIONS AND KINEMATICS

2.1 Newtonian Mechanics and the constant speed of light.

Let us consider two Inertial Frames of Reference. Observer 1 resides in one Inertial Frame and Observer 2 resides in the other Inertial Frame. Let the Frame in which Observer 1 resides be designated as S = {x, y, z, t} and the Frame in which Observer 2 resides be designated as S' = {x', y', z', t'}. Further, let us suppose, that the Inertial Frame in which Observer 2 resides is moving in the x-direction with uniform velocity u. According to Newtonian Mechanics this can be illustrated by the following figure:

Figure 1**[1]**



Let it be assumed, that at t = 0, the two Inertial Frames of Reference coincided. After time t, Observer 2's origin has shifted by a distance ut. At time t, Observer 1 measures the co-ordinates of Event P as (x, y, z) while Observer 2 measures the co-ordinates of Event P as (x', y', z'). According to Newtonian Mechanics the following relation between the co-ordinates can be written down:

$$x' = x - ut \quad (1)$$
$$y' = y \quad (2)$$
$$z' = z \quad (3)$$
$$t' = t \quad (4)$$

Now, let Observer 1 release a beam of light parallel to the x-direction. Observer 1 sees the speed of light as c. If the beam of light was released from the origin of the Inertial Frame in which Observer 1 is residing at t = 0 and covers a distance x in time t, then the following can be written:

$$\frac{x - 0}{t - 0} = \frac{\Delta x}{\Delta t} = \frac{dx}{dt\ (when\ \Delta t\ tends\ to\ 0)} = c$$

Assuming, Newtonian Mechanics applies to speed of light and differentiating Equation 1 with respect to time we get:

$$\frac{dx'}{dt} = \frac{dx}{dt} - u = c - u$$

Thus, Observer 1 sees the speed of the beam of light as c, but Observer 2 who is in an Inertial Frame which is moving with a velocity u with respect to Observer 1 sees the speed of the same beam of light as c – u. However, Maxwell's Laws and further experimentation has proved that the speed of light is always constant for two different Inertial Frames of Reference which are moving with a uniform velocity with respect to each other **[2]**. Therefore, the space and time transformations, as provided by Newtonian Mechanics needed to be replaced and this led to the requirement for a new way of transforming space and time between Inertial Frames.

2.2 The Lorentz Transformation with respect to Space and a Re-Look at the Lorentz Factor.

Let the same situation, as depicted in Figure 1 be considered. The two Inertial Frames be S = {x, y, z, t} and S' = {x', y', z', t'}. Now, let us consider the spatial transformation, as related to the direction of motion (which is along the x-axis), as follows:

$$x' = A(x - ut) \qquad (5)$$

From Dimensional Analysis we can state, that A must be a dimension-less function and we can also state that for a value of u, A must be constant **[3]**. By the principle of Relativity, we can consider S' to be the rest Frame and S to be the Frame in motion. Therefore, Equation 5, can also be written as follows:

$$x = A(x' + ut') \qquad (6)$$

As, motion is being considered only along the x-direction the following can be written:

$$y' = y \qquad (7)$$

$$z' = z \qquad (8)$$

Now, let us consider, that from the origin of the Frame S, at t=0, Observer 1 releases a beam of energy parallel to the x-axis which has a very special property. **The speed of that beam is always measured to be the same by any two set of Inertial Frames under observation. In this case, both S and S' measure the speed of the beam as 'k'**. In Frame S, after time t the distance covered by the beam can be considered as x which can be written as follows:

$$x = kt \qquad (9)$$

The distance as measured from Frame S' can be written as follows:

$$x' = kt' \qquad (10)$$

Putting Equation 9 and Equation 10 in Equation 5, the following can be obtained: **[4]**

$$kt' = A(kt - ut) \quad (11)$$

$$t' = A\left(t - \frac{u}{k}t\right) \quad (12)$$

Putting Equation 9 and Equation 10 in Equation 6, the following can be obtained:

$$kt = A(kt' + ut') \ (13)$$

Taking t' common from the Right-Hand Side of Equation 13 and substituting the value of t' from Equation 12, we get the following **[5]**:

$$kt = A(k+u)A\left(1 - \frac{u}{k}\right)t$$

$$i.e, k = A^2(k+u)(\frac{k-u}{k})$$

$$k^2 = A^2(k+u)(k-u)$$

$$k^2 = A^2(k^2 - u^2)$$

$$A^2 = \frac{1}{1 - \frac{u^2}{k^2}}$$

$$A = \frac{1}{\sqrt{1 - \frac{u^2}{k^2}}}$$

Therefore, Equation 5 can be written as follows:

$$x' = \frac{1}{\sqrt{1 - \frac{u^2}{k^2}}}(x - ut) \qquad (14)$$

Equation 14 is like the Lorentz Transformation for space which is as follows:

$$x' = \gamma(x - ut)$$

Therefore, γ can be equated with A and can be re-interpreted as follows:

$$\gamma = \frac{1}{\sqrt{1 - \frac{(Relative\ Velocity\ between\ two\ Inertial\ Frames\ of\ Reference)^2}{(The\ velocity\ that\ is\ measured\ to\ be\ exactly\ same\ by\ two\ Inertial\ Frames\ of\ Reference\ which\ are\ moving\ at\ a\ Relative\ Velocity\ with\ respect\ to\ each\ other)^2}}}$$

Therefore, the Lorentz Factor can be re-interpreted as the *numeral one* ***(1) divided by*** *the* ***square root*** *of, the numeral one* ***(1) minus*** *the* ***square of the relative velocity between the two Inertial Frames of Reference*** *under consideration* ***divided by*** *the* ***square of the velocity of such a beam of energy whose velocity is measured to be exactly same by the two Inertial Frames of Reference which are moving at a constant velocity with respect to each other***. Depending on this new interpretation of the Lorentz Factor, we will determine what the Universal Speed Limit should be.

2.3 The Universal Speed Limit

Let, a constant positive Force, F, be applied to a particle with a positive rest mass m_0 from the time t = 0 to the time t = T along the positive x-direction **[6]**. Force can be defined as rate of change of momentum, which can be described as:

$$\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt} \qquad (15)$$

$$i.e, \boldsymbol{F}dt = d\boldsymbol{p} \qquad (16)$$

Relativistic momentum $(\boldsymbol{p})$ can be written as:

$$p = \gamma m_0 u \qquad (17)$$

Integrating both sides of Equation 16 the following is obtained:

$$\int_0^T \boldsymbol{F}dt = \int d(\gamma m_0 u) \qquad (18)$$

$$F[T-0] = \gamma m_0 u \qquad (19)$$

$$FT = \frac{1}{\sqrt{1-\frac{u^2}{k^2}}} m_0 u \qquad (20)$$

In the last article, we equated γ with A and in Equation 20, γ has been replaced with A. Now let us consider, a system of units where k = 1 and the velocity u can be represented as a percentage of k. Equation 20, thus takes the form **[7]**:

$$FT = \frac{1}{\sqrt{1-u^2}} m_0 u \qquad (21)$$

$$i.e, F^2T^2(1-u^2) = {m_0}^2u^2 \qquad (22)$$

$$F^2T^2 = {m_0}^2u^2 + F^2T^2u^2 \qquad (23)$$

$$F^2T^2 = u^2\,({m_0}^2 + F^2T^2) \qquad (24)$$

$$u^2 = \frac{F^2T^2}{({m_0}^2 + F^2T^2)} \qquad (25)$$

For u to be greater than k (which has been taken as 1), u^2 must be greater than 1. This scenario is possible only if,

$$F^2T^2 > {m_0}^2 + F^2T^2 \qquad (26)$$

$$i.e, {m_0}^2 < 0 \qquad (27)$$

However, at the beginning, we considered, the particle has positive rest mass, therefore Equation (27) is a mathematical impossibility. Further, from Equation 25, it can be stated, that, for a particle having positive rest mass, u^2 will always be less than 1. Or, in other words u < 1, or u < k. In Equation 25 if the rest mass is considered as 0 then the Equation becomes:

$$u^2 = \frac{F^2T^2}{(F^2T^2)} = 1 \qquad (28)$$

Therefore, it can be concluded that for particles having positive rest mass, the maximum velocity that can be reached will always be lesser than k (which is the velocity which is measured to be exactly same by two Inertial Frames of Reference). And for particles whose rest mass is 0, the velocity of propagation is k. **Therefore, k (which is the velocity which is measured to be exactly same by two Inertial Frames of Reference) is the Universal Speed Limit.** ***Thus, any velocity, that is measured to be the same and is unaffected by the relative velocity between two Inertial Frames that are moving at a constant velocity with respect to each other must be the Universal Speed Limit.***

Only the velocity of Electro-Magnetic Radiation (The speed of light) is always measured to be the same by any two Inertial Frames that are moving with a constant velocity with respect to each other. Thus, it can finally be concluded that the speed of light, in material free space, is the Universal Speed Limit.

REFERENCES

[1] Feynman, R. The Feynman Lectures on Physics.

[2] Dutta Roy, A. 2018. Electro-Magnetic Wave Velocity as observed by two different Inertial Frames in Source Free and Material Free Region. International Research Journal of Engineering and Technology (IRJET) Vol 5, Issue 9: 875 - 878

[3, 4, 5] Faraoni, V. 2013. Special Relativity, Undergraduate Lecture Notes in Physics, DOI: 10.1007/978-3-319-01107-3_2, © Springer International Publishing Switzerland.

[6, 7] Verma, H.C. Concepts of Physics, Vol 2.