

Study on Anisotropic Metamaterials and its Properties

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ABSTRACT Anisotropic metamaterials are widely used in the field of topics because of their unique electromagnetic properties. These metamaterials can be made from multilayer metallo-electric structures. These properties depend not only on the wavelength and polarization but also the direction of the optical wave-vector. In this paper we have reviewed the recent progress on the parametric design of transportation devices.

INTRODUCTION Anisotropic metamaterials expressions for the anisotropic mass density were reported in the quasistatic limit and they depend on the cylinder's parameters and the lattice geometry. It can be shown that even neglecting the terms of multiple scattering interaction, mass anisotropy appears for non-symmetric lattices. The components of the effective mass density tensor are

$$\rho_{xx}^{*-1}(\omega) = \frac{1 - f^2 \eta^2(\omega)(A+1)^2}{1 + 2f\eta(\omega) + f^2 \eta^2(\omega)(1-A^2)} \rho_b^{-1} \quad 1$$

$$\rho_{yy}^{*-1}(\omega) = \frac{1 - f^2 \eta^2(\omega)(A-1)^2}{1 + 2f\eta(\omega) + f^2 \eta^2(\omega)(1-A^2)} \rho_b^{-1} \quad 2$$

Where A is the anisotropy factor introduced in [38] and $\eta(\omega)$ is given by equation (4.14) with $\rho_a = \rho_a(\omega)$. It is assumed that the coordinate axes are oriented along the principal axes of the tensor ρ_{ik} .

The expression for effective bulk modulus remains the same as that in the previous section, so that we can obtain the tensor for the effective speed of sound [38] as follows:

$$\frac{c_{ij}^{*2}}{c_h} = \rho_{ij}^{*-1} B^* \quad 3$$

Note that the anisotropic mass density tensor can have both the principal values of the same sign (negative or positive) or they can be of opposite signs.

Therefore, for the acoustic refractive index, we have

$$n^*(\theta) = \frac{1}{\pm \sqrt{c^{*2}(\theta)}} = \frac{1}{\pm \sqrt{C_{xx}^{*2} \cos^2 \theta + C_{yy}^{*2} \sin^2 \theta}} \quad 4$$

Where θ defines the direction of propagation of acoustic waves in the effective medium. The refractive index surface $n^*(\theta)$ can be either elliptical, hyperbolic or imaginary, and also positive or negative, depending on the sign of B^* and the components of the mass density tensor.

When both the diagonal components of the mass density tensor have the same sign, but have signs different from that of the effective bulk modulus, the refractive index becomes imaginary and there is no sound propagation. When the signs are the same as that of the effective bulk modulus, the refractive index becomes elliptical – or circular in the case of isotropic media – and we have normal refraction or negative refraction, depending on the sign of the components of the mass density tensor.

If the components of the mass density tensor are of different signs, the refractive index surface is hyperbolic, and the sign of the square root in equation (4.20) is the same as that of B^* . The propagation of waves in this medium will be determined by the component of the mass density tensor that has the same sign as B^* . In this case the refractive index surface is hyperbolic, positive or negative.

Figure (A) Summarizes all types of propagation regimes allowed in metamaterials, Note that, for a given frequency, it is not possible to have a positive index in one direction and a negative index in another direction, since the character of the propagation is determined by the bulk modulus, whose sign is determined by the operating frequency.

$B > 0$	$\rho_u < 0$	$\rho_u > 0$
$B < 0$		
$\rho_n > 0$	Hyperbolic Positive Hyperbolic Negative	Elliptic Positive No Propagation
$\rho_n < 0$	No Propagation Elliptic Positive	Hyperbolic Positive Hyperbolic Negative

Figure (A) : Summary of the different types of propagation inside acoustic metamaterials.

The mass density tensor and the scalar bulk modulus are plotted in figure B for a system of cylinders with $B_a = 0.02B_b$ and $\rho_a = 0.5\rho_b$. The underlying lattice is rectangular with $b = 2a$ and its filling fraction $f = 0.3771$, corresponding to cylinders with radius $R_a = 0.49a$. Figure C depicts the real part of the corresponding components of the sound speed tensor. We observe that from $\frac{\omega a}{2\pi c_b}$ to 0.05, the refractive index is elliptical and positive; then a region of non-propagation appears, where the bulk modulus is negative and the two components of the mass density tensor are positive, leading to a refractive index imaginary for all directions. Finally, two frequency regions with a negative hyperbolic index appear since the mass density tensors have two resonances and still a negative effective bulk modulus.

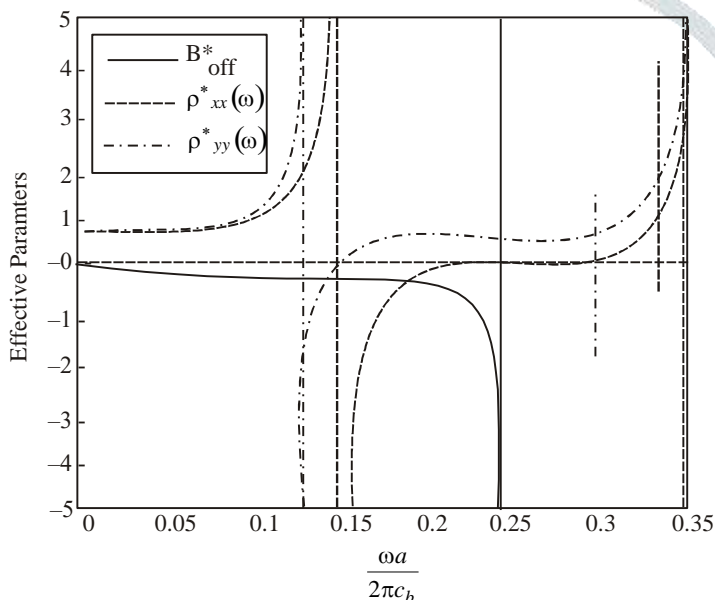


Figure B. Effective parameters, normalized to those of the background, of fluid-like cylinders distributed in a rectangular lattice with $b = 2a$. The parameters of the cylinders are $\rho_a = 0.5\rho_b$, $B_a = 0.02B_b$ and $R_a = 0.49a$.

Note that the two narrow multipolar resonances observed at about $\frac{\omega a}{2\pi c_b} = 0.3$ and 0.35 are located in a frequency region where the homogenization hypothesis is not valid. The thin horizontal line is a guide to the eyes defining the zero of the vertical axis.

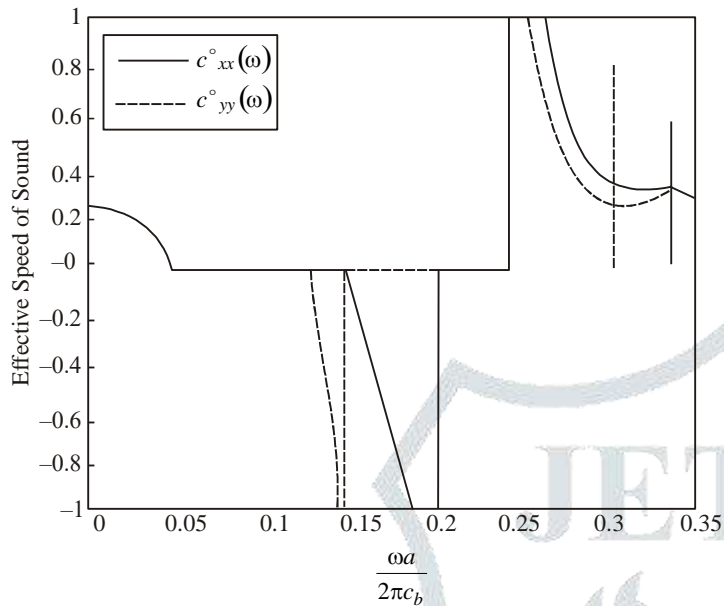


Figure C. Real part of the effective speed of the sound tensor of the medium described in figure B as a function of frequency. Note that we can have both components positive, or one negative and the other imaginary (the real part is equal to zero in this plot). It is impossible to have one component negative and the other positive. The horizontal dashed line is a guide to the eyes.

Multiple scattering interactions have been included within this analysis; however, their effects are negligible at low frequencies. Note also that as $\rho_a = 0.5\rho_b$, we have that $\eta = -\frac{1}{3}$, which is three times smaller than the air-water case, where $\eta = -1$, which make multiple scattering interactions irrelevant in general.

This simple example shows the variety and complexity of the propagation characteristics of anisotropic metamaterials. It is obvious that correct design of both elliptic and hyperbolic refractive indices should be done by properly engineering the constituent scatterers as well as the chosen underlying lattice.

References —

1. Li. C., Q. Sui, and F. Li. "Complex guided wave solution of grounded dielectric slab made of metamaterials," Progress In Electromagnetics Research, PIER 51, 187-195, 2005.
2. Grigorenko, A. N. "Negative refractive index in artificial metamaterials," Opt. Lett., Vol. 31, No. 2483, 2006.
3. Stratton, J. A., Electromagnetic Theory, MCGraw-Hill, New York, 1941.
4. Torrent D and Sanchez-Dehesa J Anisotropic mass density by radially periodic fluid structures Phys., Rev. Lett 105, 174301, 2010.
5. Sui, Q. C. Li. L.L.Li, and F. Li. "Experimental study of lambda/4 monopole antennas in a left-handed meta-material." Progress In electromagnetic Research, PIER 51, 283-293. 2005.

6. Censor, D., "Non-relativistic scattering: pulsating interfaces." Progress In Electromagnetics Research, PIER 54, 263, 2005.
7. C. J. Chapman, "The asymptotic theory of dispersion relations containing Bessel functions of imaginary order," Proc. R. Soc. A 468, 4008-4023 (2012).
8. M. Yan and N. A. Mortensen, "Hollowcore infrared fiber incorporating metalwire metamaterial," Opt. Express 17, 14851-14864 (2009).
9. J. G. Pollock and A. K. Iyer, "Miniaturized circular-waveguide probe antennas using metamaterial liners," IEEE Trans. Antennas Propagat., 63, 428-433 (2015).
10. C. R. Simovsky, P. A. Belov, A. V. Atrashenko, and Yu. S. Kivshar, "Wire metamaterials; physics and applications," Adv. Mat. 24, 4229-4248 (2012).

