Construction of control chart for waiting time in (M/M/S): (∞/FCFS) Queuing model using process capability

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Abstract: Queuing theory has been practiced to a mixture of business condition related to customer participation. The firm gives service facility and attempts to remain the costs and time minimum to impart goodwill among customers. This requires the study of service facility to find the number of customers and their waiting time. Control chart technique may be applied to analyze the services and the successful performance of concerns. Control chart constructed for the time spent in the system provides the past design about expected waiting time, maximum waiting time and minimum waiting time which guarantees customer's satisfaction. Based on this aspect, the construction of control chart using process capability for M/M/S queuing model with infinite capacity is proposed in this paper.

Keywords: *Waiting time, Poisson arrival, Exponential service time, process capability and average run length.*

I. INTRODUCTION

In queuing system the customer satisfaction can be increased by constructing control charts for average queue length and providing control limits for this so as to make effective utilization of time. A common situation that occurs in everyday life is that of queuing or waiting in line. Queues are usually seen at bus stops, hospitals, bank counters, railway booking counters and so on. In general, queues form when the demand for service exceeds its supply. The queuing in relation to the time spent by customers to access services is becoming a major source of concern to service providers Poongodi and Muthulakshmi (2013). Providing too much of service capacity to operate a system involves loss to service providers whereas not providing enough service capacity results in excessive waiting time and cost to customers. The optimum solution may be obtained by predicting and reducing waiting times and adjusting staffing. Although extensive research exists on queueing systems, relatively little attention has been given to the statistical monitoring of operational performance, an area which may help achieve more efficient operations. For example, by keeping track of the cycle time each individual item experiences in a production system, it is possible to detect changes in service rates or identify irregular patterns of customer arrivals.

The analysis of time spent in the system by the control chart technique is suitable since Montgomery (2005) proposed a number of applications of control charts in assuring quality in manufacturing industries. Shore (2000) developed control chart for random queue length, N of M/M/1 queueing model by considering the first three moments. Khaparde and Dhabe (2010) constructed the control chart using method of weighted variance for random queue length N for M/M/1 queueing model analyzed waiting time in system of M/M/1 queuing model using control chart. In this paper, an attempt is made to construct Shewhart (1931) control chart using process capability for waiting time, W of M/M/S queueing model. This model finds applications in a number of fields where the system is having multiple parallel servers like assembly of machines, aircrafts, ATM centers and supermarkets.

II. CONCEPTS AND TERMINOLOGIES

2.1 Arrival pattern

Arrival pattern describes the manner in which the units arrive and join the system. The source from which the units come may be finite or infinite. A unit may arrive either singly or in a group. The arrival pattern is often measured in terms of the average number of arrivals per unit time.

2.2 Service pattern

Service pattern describes the manner in which the service is rendered to the arrivals. Customers may be served either singly or in batches. The time required for serving a unit is called service time and the mean service rate is denoted by μ. The service pattern may be stationary or non-stationary with respect to time and state dependent or independent with respect to number of customers waiting for service.

2.3 Queue discipline

Queue discipline refers to the manner in which customers are selected for service from the queue. The most common disciplines based on the arrivals of customers into the system are first come first served (FCFS) and last come first served (LCFS). Customers may also be served randomly irrespective of their arrivals to the system called service in random order (SIRO).

2.4 Upper specification limit (USL)

It is the greatest amount specified by the producer for a process or product to have the acceptable performance.

2.5 Lower specification limit (LSL)

It is the smallest amount specified by the producer for a process or product to have the acceptable performance.

2.6 Tolerance level (TL)

It is a statistical interval within which, with some confidence level, a specified proportion of a sampled population falls. It is the difference between USL and LSL, TL = USL-LSL.

2.7 Process capability (C_P)

Process capability compares the output of an in-control process to the specification limits by using capability indices (Montgomery, 2005). The comparison is made by forming the ratio of the spread between the process specifications to the spread of the process values, as measured by 6 process standard deviation units. i. e. $C_p = \frac{TL}{6\sigma} = \frac{USL-LSL}{6\sigma}$.

2.8 Average run length (ARL)

The average run length is the number of points that, on average, will be plotted on a control chart before an out of control condition is indicated (*www.micquality.com*).

If the process is in control:

$$ARL = \frac{1}{\alpha}$$

If the process is out of control:

$$ARL = \frac{1}{1-\beta}$$

where α is the probability of a Type I error and β the probability of a Type II error.

III. MODEL DESCRIPTION FOR M/M/S

M/M/S model has 'S' servers arranged in parallel where the service time at each counter is identical and follows the same exponential law. A customer can go to any of the free counters for service. The system has infinite capacity with First Come First Serve (FCFS) queue discipline. The customers arrive in a Poisson distribution with mean arrival rate λ and the mean service rate μ.

a. Steady state equations for M/M/S

Let $P_n(t)$ = Probability that there are n customers in the system (waiting and in service) at time t. If there are n customers in the queuing system at any point in time, then the following two cases may arise:

(i) If n < s, then there will be no queue and (s–n) number of servers will not be busy. The combined service rate will be $\mu_n = n\,\mu$, n < s.

(ii) If n ≥ s, then all the servers will be busy and the maximum number of customers in the queue will be (n – s). Then $\mu_n = s\,\mu$, n ≥ s.

The governing equations of this model are

$$P_0(t+\Delta t) = P_0(t)\,(1 - \lambda\Delta t) + P_1(t)\,\mu\,\Delta t + o(\Delta t),\ n = 0$$

$$P_n(t+\Delta t) = P_n(t)\,\left(1-(\lambda+n\mu)\Delta t\right) + P_{n-1}(t)\lambda\Delta t + P_{n+1}(t)(n+1)\mu\Delta t + o(\Delta t),\ n < s \quad \text{and}$$

$$P_n(t+\Delta t) = P_n(t)\,\left(1-(\lambda+s\mu)\Delta t\right) + P_{n-1}(t)\lambda\Delta t + P_{n+1}(t)s\mu\Delta t + o(\Delta t),\ n \geq s$$

which gives

$$P_0^{|}(t) = -\lambda P_0(t) + \mu P_1(t),\ n = 0$$

$$P_n^{|}(t) = -(\lambda+n\mu)P_n(t) + \lambda P_{n-1}(t) + (n+1)\mu P_{n+1}(t),\ n < s$$

$$P_n^{|}(t) = -(\lambda+s\mu)P_n(t) + \lambda P_{n-1}(t) + s\mu P_{n+1}(t),\ n \geq s$$

The steady state equations corresponding to the above equations

$$0 = -\lambda P_0 + \mu P_1,\ n = 0$$

$$0 = -(\lambda+n\mu)P_n + \lambda P_{n-1} + (n+1)\mu P_{n+1},\ n < s$$

$$0 = -(\lambda+s\mu)P_n + \lambda P_{n-1} + s\mu P_{n+1},\ n \geq s$$

Let $\rho = \frac{\lambda}{s\mu}$ be the traffic intensity. The above result yields

$$P_n = \begin{cases} \frac{1}{n!}\left(\lambda/\mu\right)^n P_0, & n < s \\ \frac{1}{s!}\left(1/s^{n-s}\right)\left(\lambda/\mu\right)^n P_0, & n \geq s \end{cases}$$

where

$$P_0 = \left\{\sum_{n=0}^{s-1}\frac{(s\rho)^n}{n!} + \frac{(\rho s)^s}{s!(1-\rho)}\right\}^{-1}$$

b. Performance measures for M/M/S

(i) $E(L_s) =$ Average number of customers in the system

$$= \frac{\rho(\rho s)^s}{s!(1-\rho)^2}P_0 + \frac{\lambda}{\mu}$$

(ii) $E(L_q) =$ Average number of customers in the queue

$$= \sum_{n=1}^{\infty}(n-s)P_0$$

$$= \frac{\rho(\rho s)^s}{s!(1-\rho)^2}P_0$$

(iii) $E(W_s) =$ Average waiting time of a customer in the system

$$= \frac{(s\rho)^s}{s!s\mu(1-\rho)^2}P_0 + \frac{1}{\mu}$$

(iv) $E(W_q) =$ Average waiting time of a customer in the queue

$$= \frac{(s\rho)^s}{s!s\mu(1-\rho)^2}P_0$$

Let W denote the waiting time of a customer in the system which includes both the waiting time and the service time. The probability density function of the random variable W as in Gross and Harris (1998) is

$$f(w) = \frac{1}{(s - s\rho - 1)}\left[\left(s(1-\rho) - 1 + \frac{(\rho s)^s}{s!(1-\rho)}P_0\right)\mu e^{-\mu w} - \frac{(\rho s)^s}{s!(1-\rho)}P_0(s\mu - \lambda)e^{-(s\mu-\lambda)w}\right], \; w > 0$$

$$E(w) = \frac{(s\rho)^s}{s!s\mu(1-\rho)^2}P_0 + \frac{1}{\mu}$$

$$V(w) = \frac{1}{\mu^2}\left[2\left(1 + \frac{(\rho s)^s\left(\mu(s+1) - \lambda\right)}{\mu s!s^2(1-\rho)^3}P_0\right) - \left(1 + \frac{(\rho s)^s}{s!s(1-\rho)^2}P_0\right)^2\right]$$

For M /M /s queuing model the parameters of the control chart for waiting time of the customer in the system are given by

$$UCL = \frac{1}{\mu}\left\{\left(\frac{(s\rho)^s}{s!s(1-\rho)^2}P_0 + 1\right) + 3\left\{\sqrt{\left[2\left(1+\frac{(\rho s)^s(\mu(s+1)-\lambda)}{\mu s!s^2(1-\rho)^3}P_0\right)-\left(1+\frac{(\rho s)^s}{s!s(1-\rho)^2}P_0\right)^2\right]}\right\}\right\}$$

$$CL = \frac{(s\rho)^s}{s!s\mu(1-\rho)^2}P_0 + \frac{1}{\mu}$$

$$LCL = \frac{1}{\mu}\left\{\left(\frac{(s\rho)^s}{s!s(1-\rho)^2}P_0 + 1\right) - 3\left\{\sqrt{\left[2\left(1+\frac{(\rho s)^s(\mu(s+1)-\lambda)}{\mu s!s^2(1-\rho)^3}P_0\right)-\left(1+\frac{(\rho s)^s}{s!s(1-\rho)^2}P_0\right)^2\right]}\right\}\right\}$$

IV. METHODS AND MATERIALS

a. Control chart for waiting time (W) for M/M/S Model

Shewhart type control charts are constructed by approximating the statistic under consideration by a normal distribution. The parameters of the control chart (Poongodi and Muthulakshmi, 2013) are given by

$$UCL=E(W)+3\sqrt{V(W)}$$

$$CL=E(W)$$

$$LCL=E(W)-3\sqrt{V(W)}$$

For M /M /S queueing model the parameters of the control chart for waiting time of the customer in the system are given by

$$UCL = \frac{1}{\mu}\left\{\left(\frac{(s\rho)^s}{s!s(1-\rho)^2}P_0 + 1\right) + 3\left\{\sqrt{\left[2\left(1+\frac{(\rho s)^s(\mu(s+1)-\lambda)}{\mu s!s^2(1-\rho)^3}P_0\right)-\left(1+\frac{(\rho s)^s}{s!s(1-\rho)^2}P_0\right)^2\right]}\right\}\right\}$$

$$CL = \frac{(s\rho)^s}{s!s\mu(1-\rho)^2}P_0 + \frac{1}{\mu}$$

$$LCL = \frac{1}{\mu}\left\{\left(\frac{(s\rho)^s}{s!s(1-\rho)^2}P_0 + 1\right) - 3\left\{\sqrt{\left[2\left(1+\frac{(\rho s)^s(\mu(s+1)-\lambda)}{\mu s!s^2(1-\rho)^3}P_0\right)-\left(1+\frac{(\rho s)^s}{s!s(1-\rho)^2}P_0\right)^2\right]}\right\}\right\}$$

b. Waiting time control chart using process capability (Cp) for M/M/S Model

For a specified TL and C_p of the process (Radhakrishnan and Balamurugan, 2012), the value of σ (termed as σ_{cp}) is calculated from $C_p = \left(TL/6\sigma\right)$ using a computer program for various combinations of TL and C_p.

$$UCL = \left(\frac{(s\rho)^s}{s!s\mu(1-\rho)^2}P_0 + \frac{1}{\mu}\right) + 3\sigma_q$$

$$CL = \frac{1}{\mu} + \frac{(s\rho)^s}{s!s\mu(1-\rho)^2}P_0$$

$$LCL = \left(\frac{(s\rho)^s}{s!s\mu(1-\rho)^2}P_0 + \frac{1}{\mu}\right) - 3\sigma_q$$

V. ILLUSTRATION

The following numerical illustration with certain selected values of λ and μ is constructing the control charts using process capability for mean waiting time based on M/M/S model as follows:

Table 1.1: Shewhart control chart and control chart using process capability for parameters μ=4 and s=2

Arrival rate (λ)	Service rate (μ)	Number of service	Busy time	P_0	Shewhart Control Chart	Control Chart using process capability

		channel (s)	(ρ)					($\sigma_q = 0.0035$)	
					LCL	CL	UCL	LCL	UCL
0.25	4	2	0.0313	0.9376	-0.5001	0.2502	1.0006	0.2397	0.2607
0.50	4	2	0.0625	0.8759	-0.5006	0.2510	1.0025	0.2405	0.2615
0.75	4	2	0.0938	0.8155	-0.5014	0.2522	1.0058	0.2417	0.2627
1.00	4	2	0.1250	0.7568	-0.5026	0.2539	1.0104	0.2434	0.2644
1.25	4	2	0.1563	0.7002	-0.5044	0.2560	1.0164	0.2455	0.2665
1.50	4	2	0.1875	0.6460	-0.5067	0.2586	1.0239	0.2481	0.2691
1.75	4	2	0.2188	0.5944	-0.5096	0.2616	1.0329	0.2511	0.2721
2.00	4	2	0.2500	0.5455	-0.5132	0.2652	1.0435	0.2547	0.2757
2.25	4	2	0.2813	0.4993	-0.5177	0.2691	1.0559	0.2586	0.2796
2.50	4	2	0.3125	0.4560	-0.5231	0.2736	1.0702	0.2631	0.2841
2.75	4	2	0.3438	0.4153	-0.5295	0.2785	1.0865	0.2680	0.2890
3.00	4	2	0.3750	0.3774	-0.5372	0.2840	1.1052	0.2735	0.2945
3.25	4	2	0.4063	0.3420	-0.5465	0.2900	1.1265	0.2795	0.3005
3.50	4	2	0.4375	0.3090	-0.5575	0.2967	1.1510	0.2862	0.3072
3.75	4	2	0.4688	0.2784	-0.5708	0.3042	1.1792	0.2937	0.3147

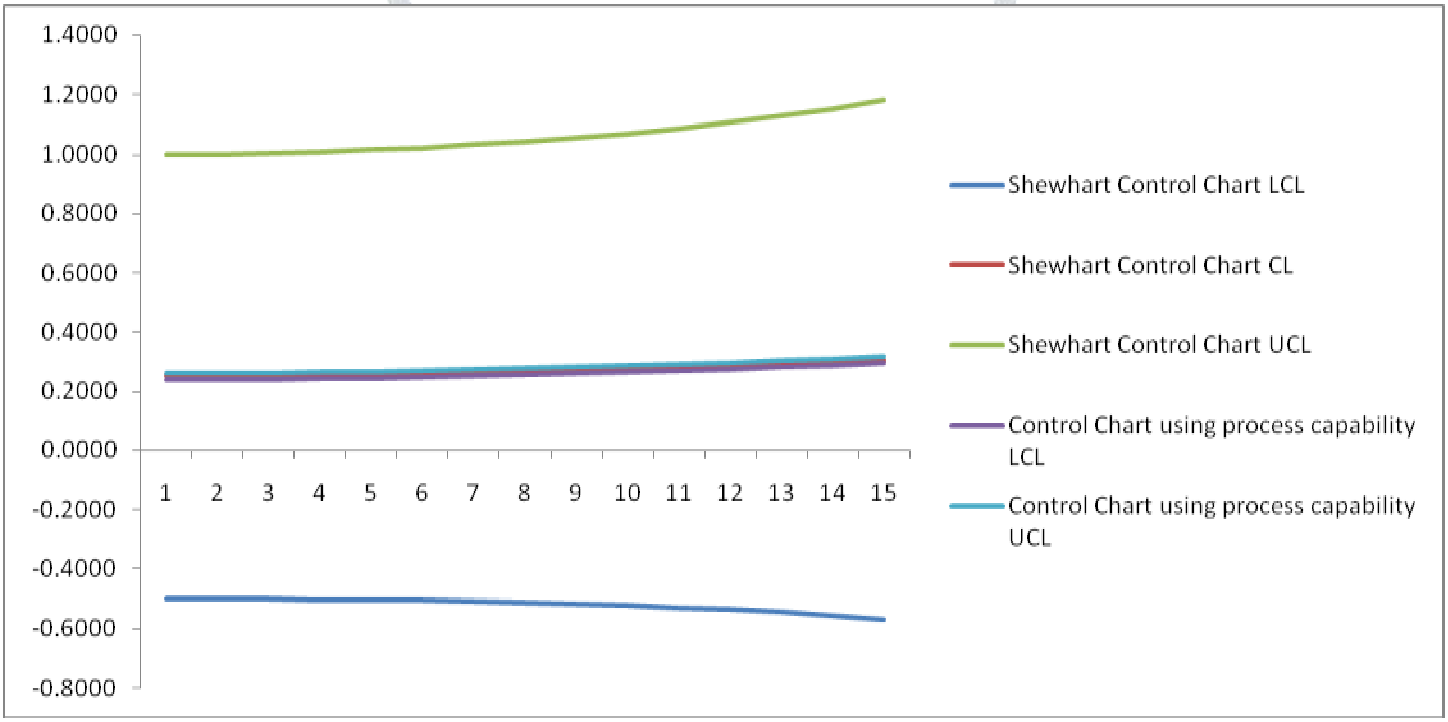

Figure 1.1: Shewhart control chart and control chart using process capability for parameters μ=4 and s=2

From the above Table 1.1, it shows that the increasing in arrival rate with constant service rate increases the average waiting time and the expected upper limit of waiting time and it is originated from the Figure 1.1 that the control limits interval of 3σ using process capability is smaller than the control limits interval of Shewhart. It is clear that the existing approach is not in good quality as expected, accordingly a modification and improvement is needed in the queueing system.

The average run length (ARL) and the false alarm rate are obtained as follows:

Table A: Average run length (ARL) for control charts

multiple of σ	Shewhart control chart	Control chart using process capability

0.5	155.39	155.22
1	43.94	43.90
1.5	14.98	14.97
2	6.31	6.30
2.5	3.24	3.24
3	2.00	2.00

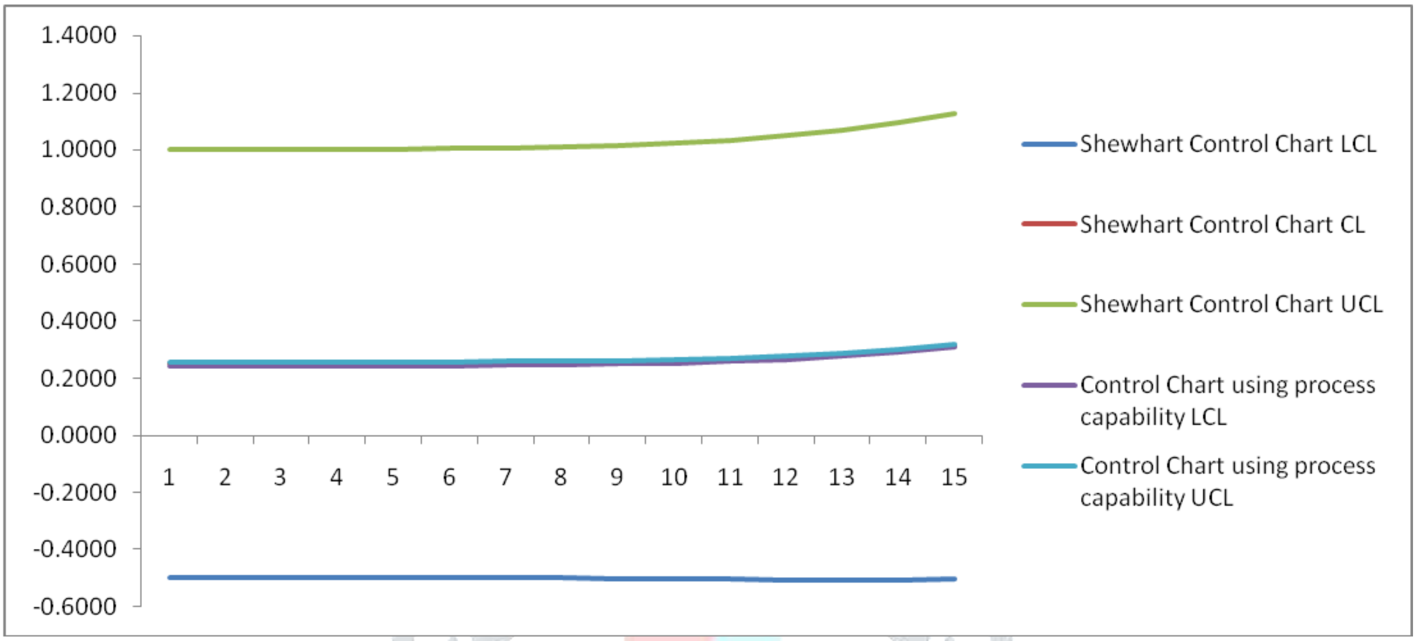

Figure 1.2: Shewhart control chart and control chart using process capability for parameters μ=4 and s=3

Table 1.2: Shewhart control chart and control chart using process capability for parameters μ=4 and s=3

Arrival rate (λ)	Service rate (μ)	Number of service channel (s)	Busy time (ρ)	P_0	Shewhart Control Chart			Control Chart using process capability ($\sigma_q = 0.0018$)	
					LCL	CL	UCL	LCL	UCL
0.25	4	3	0.0208	1.0645	-0.5000	0.2500	1.0000	0.2446	0.2554
0.50	4	3	0.0417	1.1332	-0.5000	0.2500	1.0001	0.2446	0.2554
0.75	4	3	0.0625	1.2063	-0.5000	0.2501	1.0003	0.2447	0.2555
1.00	4	3	0.0833	1.2841	-0.5000	0.2503	1.0007	0.2449	0.2557
1.25	4	3	0.1042	1.3670	-0.5001	0.2507	1.0015	0.2453	0.2561
1.50	4	3	0.1250	1.4555	-0.5002	0.2514	1.0030	0.2460	0.2568
1.75	4	3	0.1458	1.5498	-0.5004	0.2525	1.0053	0.2471	0.2579
2.00	4	3	0.1667	1.6506	-0.5007	0.2541	1.0090	0.2487	0.2595
2.25	4	3	0.1875	1.7585	-0.5012	0.2566	1.0144	0.2512	0.2620
2.50	4	3	0.2083	1.8742	-0.5020	0.2601	1.0222	0.2547	0.2655
2.75	4	3	0.2292	1.9987	-0.5029	0.2652	1.0333	0.2598	0.2706
3.00	4	3	0.2500	2.1330	-0.5040	0.2722	1.0484	0.2668	0.2776
3.25	4	3	0.2708	2.2786	-0.5048	0.2819	1.0687	0.2765	0.2873
3.50	4	3	0.2917	2.4369	-0.5047	0.2952	1.0951	0.2898	0.3006
3.75	4	3	0.3125	2.6100	-0.5019	0.3132	1.1283	0.3078	0.3186

From the above Table 1.2, it shows that the increasing in arrival rate with constant service rate increases the average waiting time and the expected upper limit of waiting time and it is originated from the Figure 1.2 that the control limits interval of 3σ using process capability is smaller than the control limits interval of Shewhart.

Table 1.3: Shewhart control chart and control chart using process capability for parameters μ=4 and s=4

Arrival rate (λ)	Service rate (μ)	Number of service channel (s)	Busy time (ρ)	P_0	Shewhart Control Chart			Control Chart using process capability ($\sigma_q = 0.0002$)	
					LCL	CL	UCL	LCL	UCL
0.25	4	4	0.0156	1.0645	-0.5000	0.2500	1.0000	0.2494	0.2506
0.50	4	4	0.0313	1.1332	-0.5000	0.2500	1.0000	0.2494	0.2506
0.75	4	4	0.0469	1.2063	-0.5000	0.2500	1.0000	0.2494	0.2506
1.00	4	4	0.0625	1.2842	-0.5000	0.2500	1.0000	0.2494	0.2506
1.25	4	4	0.0781	1.3673	-0.5000	0.2500	1.0001	0.2494	0.2506
1.50	4	4	0.0938	1.4559	-0.5000	0.2501	1.0002	0.2495	0.2507
1.75	4	4	0.1094	1.5506	-0.5000	0.2502	1.0003	0.2496	0.2508
2.00	4	4	0.1250	1.6518	-0.4999	0.2504	1.0007	0.2498	0.2510
2.25	4	4	0.1406	1.7601	-0.4999	0.2506	1.0012	0.2500	0.2512
2.50	4	4	0.1563	1.8761	-0.4999	0.2510	1.0020	0.2504	0.2516
2.75	4	4	0.1719	2.0006	-0.4998	0.2517	1.0032	0.2511	0.2523
3.00	4	4	0.1875	2.1343	-0.4997	0.2527	1.0051	0.2521	0.2533
3.25	4	4	0.2031	2.2781	-0.4997	0.2541	1.0078	0.2535	0.2547
3.50	4	4	0.2188	2.4329	-0.4995	0.2561	1.0117	0.2555	0.2567
3.75	4	4	0.2344	2.6001	-0.4993	0.2589	1.0171	0.2583	0.2595

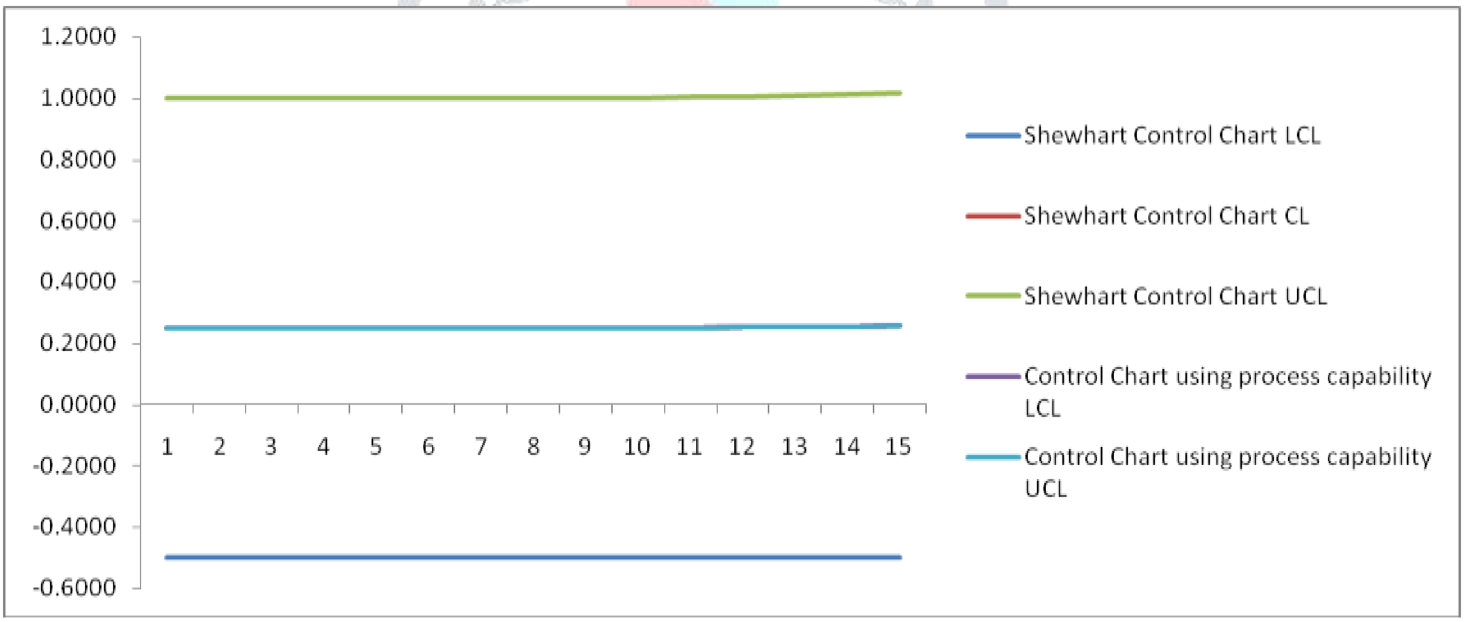

Figure 1.3: Shewhart control chart and control chart using process capability for parameters μ=4 and s=4

From the above Table 1.3, it shows that the increasing in arrival rate with constant service rate increases the average waiting time and the expected upper limit of waiting time and it is originated from the Figure 1.3 that the control limits interval of 3σ using process capability is smaller than the control limits interval of Shewhart. Furthermore the comparison between Figures 1.1, 1.2 and 1.3 increases in number of servers decreases the average waiting time and lesser variation in the control chart using process capability compared the existing control chart.

From the following Tables 1.4, 1.5 and 1.6 give the parameters of the control chart for a constant arrival rate λ and various values of the service rate μ and the number of servers s=2, 3 and 4 respectively.

Table 1.4: Shewhart control chart and control chart using process capability for parameters λ=3 and s=2

Arrival rate (λ)	Service rate (μ)	Number of service channel (s)	Busy time (ρ)	P_0	Shewhart Control Chart			Control Chart using process capability ($\sigma_q = 0.0076$)	
					LCL	CL	UCL	LCL	UCL
3	5.00	2	0.3000	0.4730	-0.4166	0.2174	0.8514	0.1946	0.2402
3	5.25	2	0.2857	0.4930	-0.3950	0.2055	0.8060	0.1827	0.2283
3	5.50	2	0.2727	0.5116	-0.3756	0.1949	0.7654	0.1721	0.2177
3	5.75	2	0.2609	0.5291	-0.3580	0.1854	0.7288	0.1626	0.2082
3	6.00	2	0.2500	0.5455	-0.3422	0.1768	0.6957	0.1540	0.1996
3	6.25	2	0.2400	0.5608	-0.3277	0.1689	0.6656	0.1461	0.1917
3	6.50	2	0.2308	0.5752	-0.3144	0.1618	0.6380	0.1390	0.1846
3	6.75	2	0.2222	0.5888	-0.3022	0.1553	0.6127	0.1325	0.1781
3	7.00	2	0.2143	0.6016	-0.2909	0.1492	0.5894	0.1264	0.1720
3	7.25	2	0.2069	0.6136	-0.2805	0.1437	0.5679	0.1209	0.1665
3	7.50	2	0.2000	0.6250	-0.2708	0.1385	0.5479	0.1157	0.1613
3	7.75	2	0.1935	0.6358	-0.2618	0.1338	0.5293	0.1110	0.1566
3	8.00	2	0.1875	0.6460	-0.2533	0.1293	0.5119	0.1065	0.1521
3	8.25	2	0.1818	0.6556	-0.2454	0.1251	0.4957	0.1023	0.1479
3	8.50	2	0.1765	0.6648	-0.2380	0.1212	0.4805	0.0984	0.1440

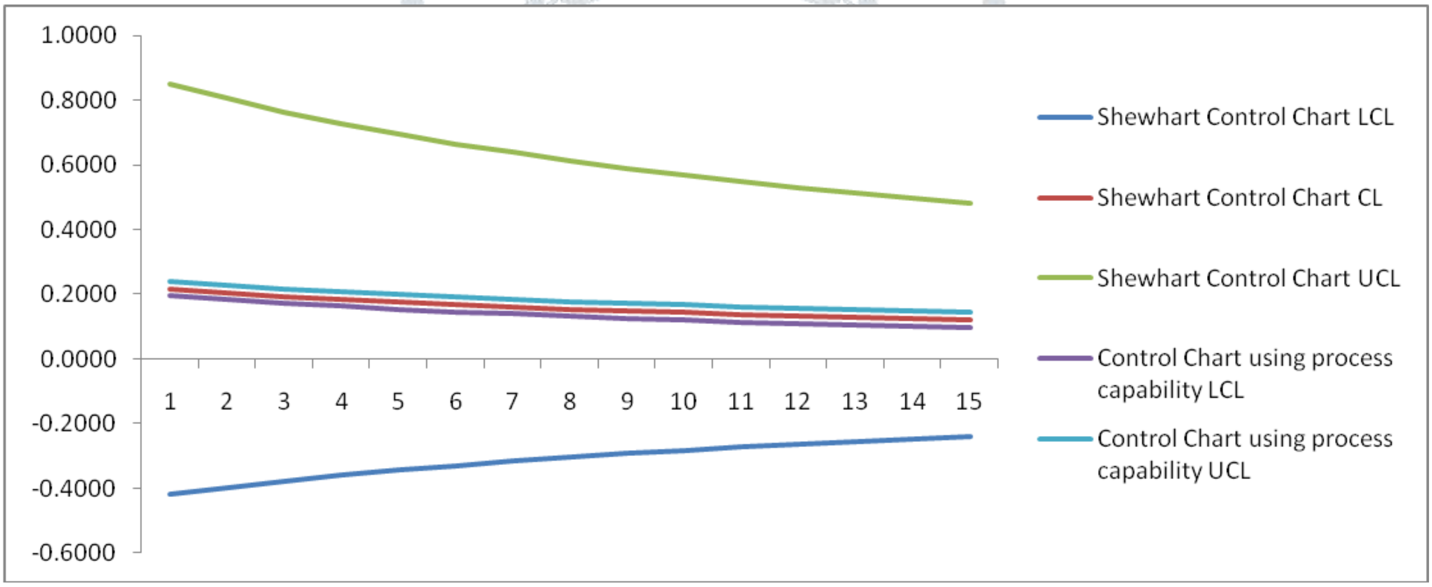

Figure 1.4: Shewhart control chart and control chart using process capability for parameters λ=3 and s=2

From the above Table 1.4, it shows that the increasing in service rate with constant arrival rate decreases the average waiting time and the expected upper limit of waiting time and it is originated from the Figure 1.4 that the control limits interval of 3σ using process capability is smaller than the control limits interval of Shewhart. It is clear that the existing approach is not in good quality as expected, accordingly a modification and improvement is needed in the queueing system.

Table 1.5: Shewhart control chart and control chart using process capability for parameters λ=3 and s=3

Arrival rate (λ)	Service rate (μ)	Number of service channel (s)	Busy time (ρ)	P_0	Shewhart Control Chart			Control Chart using process capability ($\sigma_q = 0.0071$)	
					LCL	CL	UCL	LCL	UCL
3	5.00	3	0.2000	1.8269	-0.4013	0.2069	0.8150	0.1856	0.2282
3	5.25	3	0.1905	1.7745	-0.3820	0.1958	0.7736	0.1745	0.2171
3	5.50	3	0.1818	1.7283	-0.3644	0.1861	0.7365	0.1648	0.2074

3	5.75	3	0.1739	1.6873	-0.3484	0.1773	0.7030	0.1560	0.1986
3	6.00	3	0.1667	1.6506	-0.3338	0.1694	0.6726	0.1481	0.1907
3	6.25	3	0.1600	1.6176	-0.3204	0.1623	0.6449	0.1410	0.1836
3	6.50	3	0.1538	1.5878	-0.3080	0.1557	0.6194	0.1344	0.1770
3	6.75	3	0.1481	1.5607	-0.2965	0.1497	0.5959	0.1284	0.1710
3	7.00	3	0.1429	1.5359	-0.2859	0.1442	0.5742	0.1229	0.1655
3	7.25	3	0.1379	1.5133	-0.2760	0.1390	0.5541	0.1177	0.1603
3	7.50	3	0.1333	1.4925	-0.2668	0.1343	0.5354	0.1130	0.1556
3	7.75	3	0.1290	1.4732	-0.2582	0.1298	0.5179	0.1085	0.1511
3	8.00	3	0.1250	1.4555	-0.2501	0.1257	0.5015	0.1044	0.1470
3	8.25	3	0.1212	1.4390	-0.2425	0.1218	0.4861	0.1005	0.1431
3	8.50	3	0.1176	1.4236	-0.2354	0.1182	0.4717	0.0969	0.1395

From the above Table 1.5, it shows that the increasing in service rate with constant arrival rate decreases the average waiting time and the expected upper limit of waiting time and it is originated from the Figure 1.5 that the control limits interval of 3σ using process capability is smaller than the control limits interval of Shewhart.

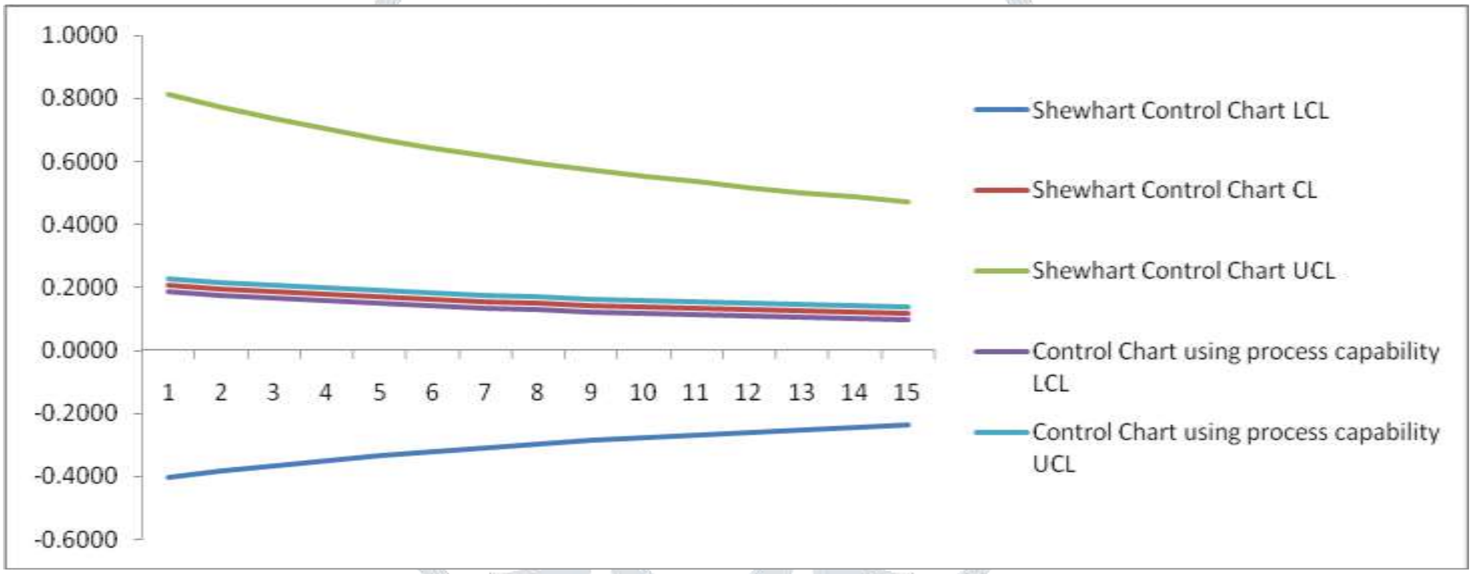

Figure 1.5: Shewhart control chart and control chart using process capability for parameters λ=3 and s=3

Table 1.6: Shewhart control chart and control chart using process capability for parameters λ=3 and s=4

Arrival rate (λ)	Service rate (μ)	Number of service channel (s)	Busy time (ρ)	P_0	Shewhart Control Chart			Control Chart using process capability ($\sigma_q = 0.0069$)	
					LCL	CL	UCL	LCL	UCL
3	5.00	4	0.1500	1.8287	-0.3999	0.2007	0.8013	0.1800	0.2214
3	5.25	4	0.1429	1.7762	-0.3809	0.1910	0.7629	0.1703	0.2117
3	5.50	4	0.1364	1.7298	-0.3636	0.1822	0.7280	0.1615	0.2029
3	5.75	4	0.1304	1.6886	-0.3478	0.1742	0.6962	0.1535	0.1949
3	6.00	4	0.1250	1.6518	-0.3333	0.1669	0.6671	0.1462	0.1876
3	6.25	4	0.1200	1.6187	-0.3200	0.1602	0.6403	0.1395	0.1809
3	6.50	4	0.1154	1.5887	-0.3077	0.1540	0.6157	0.1333	0.1747
3	6.75	4	0.1111	1.5615	-0.2963	0.1483	0.5928	0.1276	0.1690
3	7.00	4	0.1071	1.5367	-0.2857	0.1430	0.5716	0.1223	0.1637
3	7.25	4	0.1034	1.5139	-0.2758	0.1380	0.5519	0.1173	0.1587
3	7.50	4	0.1000	1.4930	-0.2667	0.1334	0.5335	0.1127	0.1541

3	7.75	4	0.0968	1.4738	-0.2581	0.1291	0.5162	0.1084	0.1498
3	8.00	4	0.0938	1.4559	-0.2500	0.1250	0.5001	0.1043	0.1457
3	8.25	4	0.0909	1.4394	-0.2424	0.1213	0.4849	0.1006	0.1420
3	8.50	4	0.0882	1.4240	-0.2353	0.1177	0.4706	0.0970	0.1384

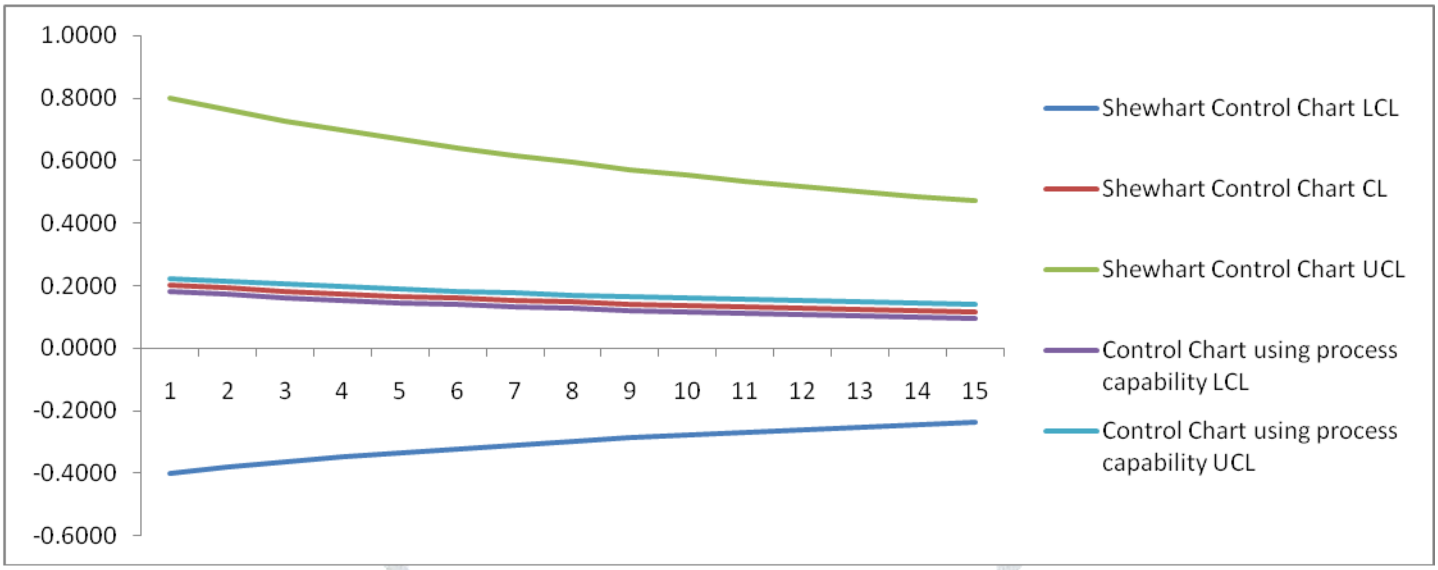

Figure 1.6: Shewhart control chart and control chart using process capability for parameters λ=3 and s=4

From the above Table 1.6, it shows that the increasing in service rate with constant arrival rate decreases the average waiting time and the expected upper limit of waiting time and it is originated from the Figure 1.6 that the control limits interval of 3σ using process capability is smaller than the control limits interval of Shewhart. Furthermore the comparison between Figures 1.4, 1.5 and 1.6 increases in number of servers decreases the average waiting time and lesser variation in the control chart using process capability compared the existing control chart.

VI. CONCLUSION

The outcome of numerical example shows that the proposed method leads better than the existing control charts and the control limits interval of control chart using process capability is smaller than the control limits interval of Shewhart. It is clear that the performance of the system is in shortage than the requirement based on the control chart using process capability. The proposed control chart using process capability for average waiting time for M/M/S model will increase the fulfilment and self-assurance of the consumers. It also reveals that the mean waiting control chart using process capability is compatible, better performance and efficient than the Shewhart 3σ control chart through the average run length (ARL) for approximately symmetric distributions based on the numerical illustration.

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