THE PATH AND STAR OF GALLAI FUZZY GRAPHS

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Abstract

In this paper, we investigate some theorems on path and star graphs of Gallai fuzzy graphs as well as we have also analyses the structures and fuzzy domination number.

Key words: fuzzy path, fuzzy star of Gallai fuzzy graph, fuzzy domination number.

AMS Classification: 03E72, 05C99**.**

1. Introduction

The study of dominating sets in graphs has been begun by Ore and Berge [1], theory of domination, independent and irredundance in graphs are introduced by E. J. Cockayne, et al., 1931 in [2]. In 1965, L.A. Zadeh [3] introduced a mathematical frame work is to be explain the concepts of uncertainty in real life through the publication of a seminal paper. In 1975, A. Rosenfeld [4] introduced the notation of fuzzy graph theoretic concept such as paths, cycles and connectedness. In 1996, Van Bang Le [5] discussed the paper of Gallai graphs and anti-Gallai graphs. However, S. Aparna Lakshmanan and S.B. Rao have also discussed about the Gallai graphs and anti-Gallai graphs in their published article [6]. Further, In [7], A. Somasundram and S. Somasundram deliberated the domination in fuzzy graphs. Moreover, A. Nagoorgani and P. Vadivel dealt with domination, independent and irredundance numbers in [8]. In our earlier work, we have discussed the concept of Gallai fuzzy graphs in [9, 10].

In this paper, we have discussed about some theorems on fuzzy path, fuzzy star and complete fuzzy graphs of Gallai fuzzy graph. In addition, the structures and fuzzy domination numbers are also discussed in this manuscript.

2. Preliminaries

A fuzzy graph with G as the underlying set is a finite non-empty unordered pair of G = (σ, μ), where σ: V → [0, 1] is a fuzzy subset, μ: → [0, 1] is a fuzzy relation on the fuzzy subset σ such that $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$ for all x, y Є V where ∧ and ∨ stands for minimum and maximum. The underlying crisp fuzzy graph of G = (σ, μ) is denoted by G^* = (V, E), where V = $\{x \epsilon V : \sigma(x) > 0\}$ and E =$\{(x, y) \epsilon VxV : \mu(x, y) > 0\}$, the fuzzy order P and fuzzy size of the fuzzy graph G = (σ, μ) are defined by p = $\Sigma_{v \epsilon V} \sigma(x)$ and q =$\Sigma_{x, y \epsilon E} \mu(x, y)$. Each pair μ = x, y of fuzzy vertices in σ is an fuzzy edge of Γ (G) and μ is said to join x and y are fuzzy adjacent vertices, fuzzy vertex x and fuzzy edge μ are fuzzy incident with each other as are σ and μ if two distinct fuzzy edges are incident with a common fuzzy vertex, then they are called fuzzy adjacent edges. A fuzzy edge e = xy of a fuzzy graph is a fuzzy effective edge if μ(x, y) = σ(x) ∧ (y). N(x) = { y ∈ V / μ(x, y) = σ(x) ∧ σ(y)} is called the open neighborhood of x and N[x] = N(x) ∪{x} is the closed neighborhood of x.

Definition 2.1

Let G be a fuzzy graph and u be a fuzzy vertex in G then there exists a fuzzy vertex v such that (u, v) is a fuzzy edge then we say that u dominates v.

Definition 2.2

Let G = (σ, μ) be a fuzzy graph. A subset D of V is said to be fuzzy dominating set of G if for every v ∈ V-D, there exists u ∈ D such that u dominates v.

Definition 2.3

A fuzzy dominating set D of a fuzzy graph G is called minimal fuzzy dominating set of G, if for every fuzzy vertex v ∈ D, D-{v} is not a fuzzy dominating set.

Definition 2.4

The Minimum fuzzy cardinality among all minimal fuzzy dominating sets in Γ(G) is called fuzzy domination number of Γ(G) and is denoted by $\gamma(\Gamma(G))$.

Definition: 2.5

A path of length n is a sequence of distinct vertices $u_0, u_1, u_2 \ldots\ldots u_n$ such that fuzzy $\mu(u_{i-1}, u_i) > 0$, i=1, 2 ….n is called a fuzzy path.

Definition: 2.6

A fuzzy star is a fuzzy tree whose unique maximum spanning tree is a star. A fuzzy star G = (σ, μ) with σ*, |σ ∗|= n +1 is denoted FS_n.

Definition: 2.7

A fuzzy graph G = (σ, μ) is complete fuzzy graph if μ(x, y) = σ(x) ∧ (y) for all x, y ϵ V.

3. The path and star graphs of Gallai fuzzy graphs.

In this section, we have discussed the definitions on Gallai fuzzy graphs and some important theorems of Gallai fuzzy graphs as follows.

Definition: 3.1

The Gallai fuzzy graph Γ(G) of a fuzzy graph G has the fuzzy edges of G as its fuzzy vertices and two distinct fuzzy edges of G are fuzzy incident in G, but do not span a fuzzy triangle in G. The line fuzzy graph L (G) of a fuzzy graph G has the fuzzy edges of G as its fuzzy vertices and two distinct fuzzy edges of G are adjacent in L (G) if they are fuzzy incident in G in [9, 10].

This concept of Gallai fuzzy graph will be apply to the simple connected fuzzy path, star and complete fuzzy graph G. Also to analyses the construction and fuzzy domination number of Gallai fuzzy graph.

Theorem: 3.2

Let P_n ($n \geq 2$) be a fuzzy path of length n-1. Then the Gallai fuzzy graph of P_n, Γ(P_n) is a fuzzy path, P_{n-1} of length n-2.

Proof:

Let P_n ($n \geq 2$) be the fuzzy path of length n-1. Let V(P_n) = { $u_0, u_1, u_2 \ldots\ldots u_{n-1}, u_n$ } be a vertex set of P_n such that u_i, u_{i+1} ϵ E(P_n), for all i=1,2,…..,n-1.We prove the result Γ(P_n) = P_{n-1}, for an integer n, by induction method.Let n = 2, then P_2 is the fuzzy path of length 1.By the definition of Gallai fuzzy graph, V (Γ(P_2)) = {u_1u_2}.Hence Γ (P_2) = P_1, since there is no adjacent edge in P_2 , for the edge u_1u_2 in E(P_n).
Hence the result follows for n = 2.Let n =3, then P_3 is the fuzzy path of length 2.Let V(P_3) = {u_1, u_2, u_3}.It is clear that P_3 is a triangle free graph.Then V(Γ(P_4)) = {u_1u_2, u_2u_3} since the edges u_1u_2 and u_2u_3 are adjacent in P_4, the vertices u_1u_2 and u_2u_3 are adjacent in Γ(P_3).Therefore Γ (P_3) contains two adjacent vertices only.
This implies that Γ(P_3) = P_2, path of length 2.Hence the result follows for n = 3.Assume that the result is true for n < K.Let P_k be the path of length k-1.Let V(P_k) = { $u_1, u_2, u_3 \ldots\ldots u_k$} be the vertex set of P_k such that $u_i u_{i+1}$ ϵ E(P_k), for all i=1,2,........k-1. Let P ' = P_k - e , where e = $u_i u_{i+1}$ for some i = 2,3,……k-2. Then P ' is the union of two paths P_m and P_{k-m}, for 1< m< k.Since m < k and k-m < k, Γ(P_m) = P_{m-1} and Γ(P_{k-m}) = P_{k-m-1}, by induction hypothesis. That is Γ(P ') = P_{m-1} U P_{k-m-1}, since P '+e contains one edge more than P ", Γ(P '+e) has one vertex more than Γ(P'), since e does not from a triangle in P_n and also e is adjacent to the edges u_{i-1} u_i and u_{i+1} u_{i+2}.The vertex e is adjacent to the vertex u_{i-1} u_i of V(P_m) and the vertex u_{i+1} u_{i+2} of V(P_{k-m}), by the definition of Gallai fuzzy graph. Hence, | V (Γ(P ′ + e))| = m-1+k-m-1+1= k-1. Therefore Γ(P ′ + e) has a k-1 vertices and Γ(P ′ + e) = P_{k-1}. That is Γ(P_k) = P_{k-1}, path of the length k-1. Hence, the theorem has proved.

Example: 3.3

The fuzzy path P_n, $n \geq 2$ is as shown below

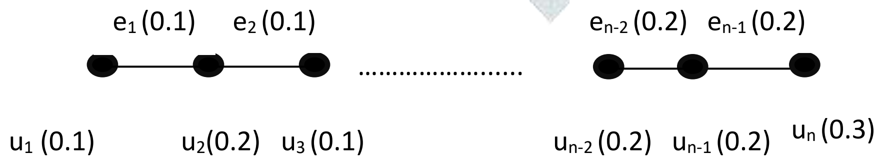


Fig : 3.3.1

The Gallai fuzzy path Γ (P_n) = P_{n-1} is as shown below

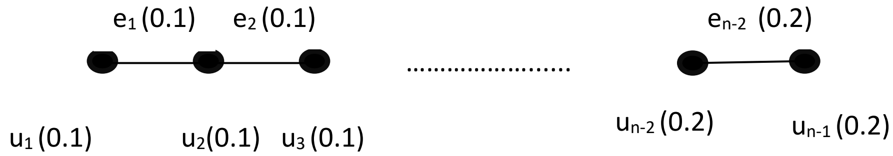


Fig : 3.3.2 Γ(P_n) = P_{n-1} and $\gamma(\Gamma(P_3)) = 0.1$

Remark: 3.4

In this previous theorem, if the edge e can be taken the form of either $u_1 u_2$ or $u_{k-1} u_k$, then either P_m or P_{m-k} is P_1.Since there is no edge in P_1, we cannot be find the Gallai graph, $\Gamma (P_n)$ for $n = 1$.

Theorem: 3.5

Let $K_{1,n}$ ($n \geq 1$) be a fuzzy star. Then the Gallai fuzzy graph of $K_{1,n}$, $\Gamma(K_{1,n})$ is a complete fuzzy graph, K_n of n vertices.

Proof:

Let $K_{1,n}$ ($n \geq 1$) be a fuzzy star with the bipartition (X, Y), where x = { x } and Y = {$y_1, y_2, \ldots\ldots y_n$}. Then $E(K_{1,n}) = \{xy_1, xy_2 \ldots\ldots. xy_n\}$.By the definition of Gallai fuzzy graph, $V(\Gamma (K_{1,n})) = \{ xy_1, xy_2, \ldots\ldots. xy_n \}$.Since each pair of edges xy_i and xy_j are adjacent in $K_{1,n}$, also, $K_{1,n}$ is a triangle free graph. Every pair of vertices in $\Gamma(K_{1,n})$ are adjacent .Hence, $\Gamma (K_{1,n})$ is a complete fuzzy graph with , $| V (\Gamma (K_{1,n})) | = | E (K_{1,n}) |$ vertices. Therefore $\Gamma (K_{1,n}) = K_n$. Hence, it proved.

Example: 3.6

The fuzzy star $K_{1,n}$ $n \geq 1$ is as shown below

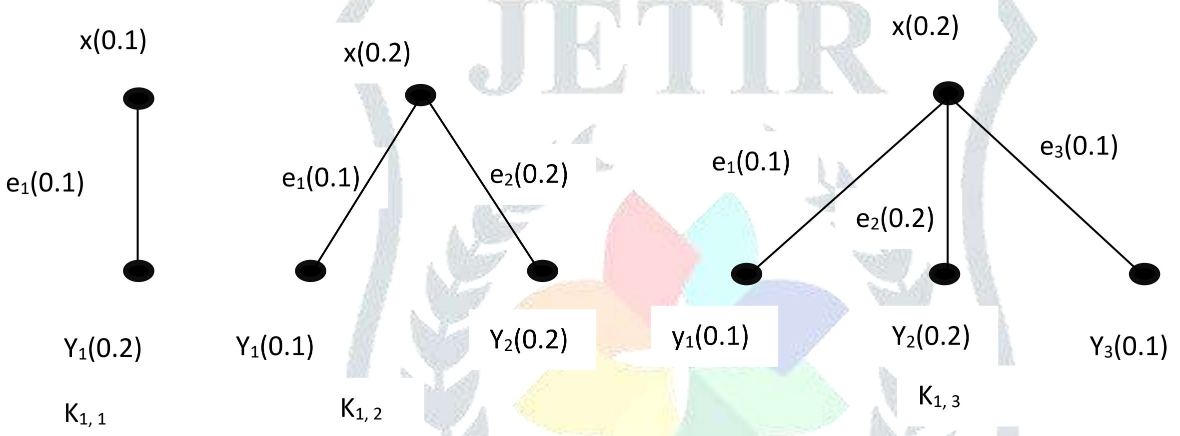


Fig: 3.6.1

The complete Gallai fuzzy graph is as shown below

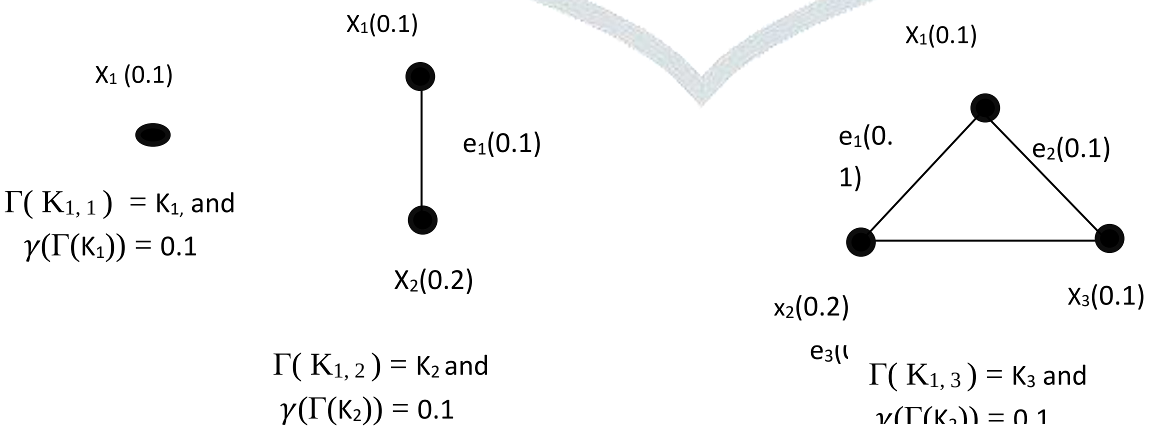


Fig: 3.6.2

Theorem: 3.7

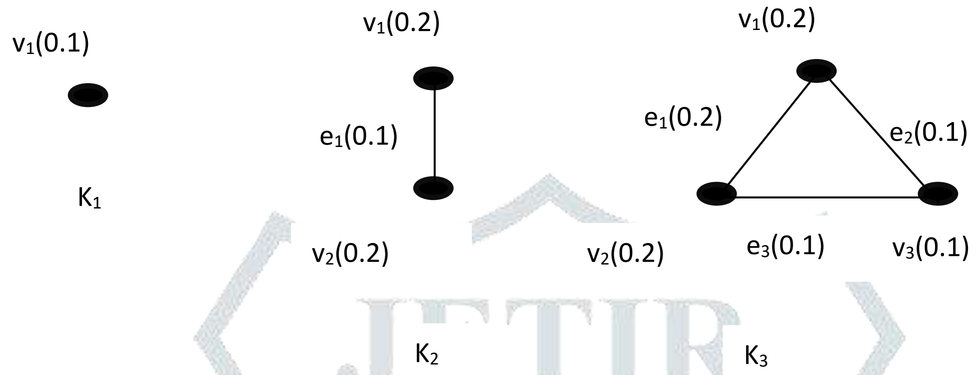
Let K_n be a complete fuzzy graph with n vertices then the Gallai fuzzy graph of K_n, $\Gamma(K_n)$ is a totally disconnected fuzzy graph, $\overline{K}_m$ with $m = \frac{n(n-1)}{2}$ vertices .

Proof:

Let K_n be a complete fuzzy graph with n verticesLet $V(K_n) = \{v_1, v_2, \ldots, v_n\}$ be the vertex set of K_n such that $V_i V_j \in E(K_n)$, for all $i = 1,2,\ldots n$; $j = 1,2,\ldots n$ and $i \# j$. Then, by the definition of the Gallai fuzzy graph, $V(\Gamma(K_n)) = \{ v_i v_j \setminus i = 1,2,\ldots n$ and $j = 1,2,\ldots n$; $i \# j \}$.It is clear that every pair of edges are adjacent in K_n. But each edge $v_i v_j$ in K_n is the triangle of K_n. Therefore no two vertices of $\Gamma(K_n)$ is adjacent. Hence, $\Gamma(K_n)$ is a totally disconnected fuzzy graph with $|E(K_n)|$ vertices. Since $|E(K_n)| = \binom{n}{2} = \frac{n(n-1)}{2}$, $\Gamma(K_n) = \overline{K}_m$, where $m = \frac{n(n-1)}{2}$ Hence, proved.

Example: 3.8

The complete fuzzy graph with n vertices is as shown below



The complete Gallai fuzzy graph is as shown below

Fig: 3.7.1

Fig: 3.7.2

Remark: 3.9

In the above example, since there is no edge in $\Gamma(K_2) = K_1$ and $\Gamma(K_3) = \overline{K}_3$ cannot be occurred the domination number.

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