Domination Parameters of Some Graph Transformation

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ABSTRACT:

Let $G = (V, E)$ be an undirected simple graph. *The domination parameters of a graph G of order n has been already introduced. It is defined as $D \subseteq V(G)$ is a dominating set of G, if every vertex $v \in V - D$ is adjacent to atleast one vertex in D.* The theory of domination and different domination parameters have studied in earlier. In this paper we have established some theorems and properties of domination parameters to the transformation of path, cycle and star graph.

Key words: Graph, Domination, Transformation.

Introduction: 1.0

A graph $G = (V, E)$, where V is a finite set of elements, called vertices and E is a set of unordered pairs of distinct vertices of G called edges. The degree of a vertex v in G is the number of edges incident on it. Every pair of its vertices are adjacent in G is said to be complete, the complete graph on$'n'$ vertices is denoted by K_n.

Let u and v be the vertices of a graph G,$a\, u - v$ walk of G is an alternating sequences $u = u_o, e, u, e_2, u_2, \ldots . u_{n-1}, e_n, v_n = v$ of vertices and edges beginning with vertex u and ending with vertex v such that $e_i = u_{i-1}u_i$ for all $i = 1,2, \ldots\ldots, n$. The number of edges in a walk is called its length. A walk in which all the vertices are distance in called a path. A path on$'n'$ vertices is denoted by P_n. A closed path is called a cycle, a cycle on $'n'$ vertices is denoted by C_n. Let $G = (V, E)$ be a simple connected graph, for any vertex $v \in V$, the open neighborhood is the set $N(v) = \{u \in V / uv \in E\}$ and the closed neighborhood of v is the set $N[v] = N(v) \cup \{u\}$. For a set $S \subset V$, the open neighborhood of S is $N(S) = \bigcup N(v), v \in s$ and the closed neighborhood of S is $N[s] = N(S) \cup S$.

Definition 1.1:

` A set D ⊆ V is a dominating set of G if every vertex v ∈ V – D is adjacent to at least one vertex of D. We call a dominating set D is a minimal if there is no dominating set
D' ⊆ V (G) with D′ ⊂ D and D′ ≠ D. Further we call a dominating set D is minimum if these is no dominating set D ⊆ V(G) with | D' | < | D|. The cardinality of a minimum dominating set is called the domination number denoted by $\gamma(G)$ and the minimum dominating set D of G is also called a γ- set.

Definition 1.2:_Generalized the concept of total graphs to a transformation graph G^{xyz} with x, y,;{−,+}, where G^{+++} is the total graph of G, and G^{---} is its complement. Also, G^{--+},$^{-+-}$ and G^{-++} are the complement of G^{++-}, $G^{+-+}$and G^{+--} respectively.

A dominating set D is said to be a total dominating set if every vertex in V is adjacent to some vertex in D. The total domination number of G denoted by γ_t (G) is the minimum cardinality of a total dominating set.

Definition 1.3:

A dominating set D of a graph G is an independent dominating set, if the induced sub graph <D> has no edges. The independent domination number γ_i (G) is the minimum cardinality of a independent dominating set.

Definition 1.4:

A dominating Set D is said to be connected dominating set, if the induced sub graph <D> is connected. The connected domination number $\gamma_c(G)$ is the minimum cardinality of a connected dominating set.

Lemma 2.1

Let G be a connected graph with $\delta(G) \geq 2$, them $\gamma(G) + \gamma'(G) = n$ if and only if $G = P_4$ or C_4.

Lemma 2.2

Let G be a connected graph with $\delta = 1$ and $\Delta = n$ then $\gamma(G) + \gamma'(G) = n + 1$ if and only if $G = k_{1, n}$.

Lemma 2.3

For any tree with $n \geq 2$ with more than two pendent vertices then there exists a vertex $v \in V$ such that $\gamma(T - v) = \gamma(T)$.

Theorem: **2.4**

Let G any path graph P_n, G^{---} is the transformation of G, then.

$\gamma(G^{--}) = \gamma_t(G^{---}) = \gamma_c(G^{---}) = \gamma_{ns}(G^{---}) = 2$ and $\gamma_i(G^{---}) = 3$

Proof:

Let $G = P_n$ } $V(G) = \{v_i/\ 1 \leq i \leq n\}, E(G) = \{e_i = v_i v_{i+1},/\ 1 \leq i \leq n-1\}$

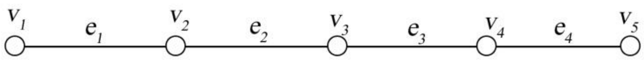
are the vertices and edges of G then $V(G^{---})$ ==$\{v_i, e_i/\ 1 \leq i \leq n$, j=1,2,…, $\overline{(n-1)}$ } is the vertex set of the transformation of G and $|V(G^{---})| = 2n - 1$; $d(v_1) = d(v_n) = 2n - 4$,

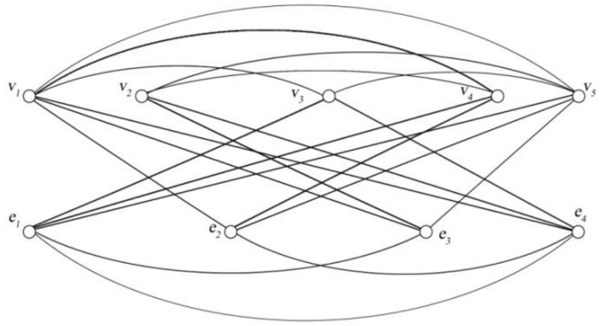
$d(v_i) = 2n - 6/\ 2 \leq i \leq n - 1. d(e_1) = d(e_n) = 2n - 5, d(e_j) = 2n - 6\ /\ 2 \leq j \leq n - 2$

for all $v_{i,}, e_j \in V(G^{---})$.

For example G= P_5 and its Transformation

G^{---} is given in figure: 1





$G = P_5$ (Figure: 1) (G^{---})

If $n > 3, N(v_1) = V(G^{---}) - \{v_2, e_1\} / v_1, v_2, e_1 \in V(G^{---})$and $\{v_2, e_1\} \in N(v_n)$ for all $e_1, v_2, v_n, \in V(G^{---})$.Hence, $\{v_1, v_n\}$ is a dominating set of G^{---}.Since each v_i, $2 \leq i \leq n - 1$ is adjacent with v_{i-1} and v_{i+1} and incident with e_{i-1} and e_i in G. Hence, $v_{i-1}, v_{i+1}, e_{i-1}$ *and* $e_i \notin N(v_i)$in G^{---}.Similarly each $e_i \in E(G)$ is adjacent with e_{i-1}, e_{i+1} is incident with $v_{i,}$ and v_{i+1} in G.Hence, $e_{i-1}, e_{i+1}, v_{i,}$ and $v_{i+1} \notin N(e_i)$ in G^{---}.Therefore, either $\{v_i\}$ or $\{e_i\}$ is not a dominating set of G^{---}. Hence $\{v_1, v_n\}$ is the dominating set with minimum cardinality. Therefore, $\gamma(G^{---}) = 2$. Since, $v_1$and v_n are independent in G therefore, v_1 and v_n are adjacent in G^{---} which shows that $\{v_1, v_n\}$ is a comeeted and total dominating set of G^{---}.Clearly, $G^{---} - \{v_1, v_n\}$ is a conneeted sub graph in G^{---} . Therefore, $\{v_1, v_n\}$ is a non split dominating set of G^{---}. Let $D_i = \{v_i, e_{i-1}, e_i/\ 1 < i < n\}$, clearly, all vertices of $G^{---} - D_i$ is adjacent with the elements of D_i and the elements of D_i are independent in G^{---}, hence D_i is the independent dominating set of G^{---} and $|D_i| = 3$.

Hence, $\gamma(G^{---}) = \gamma_c(G^{---}) = \gamma_{ns}(G^{---}) = 2$ and $\gamma_i(G^{---}) = 3$

Theorem: **2.5**

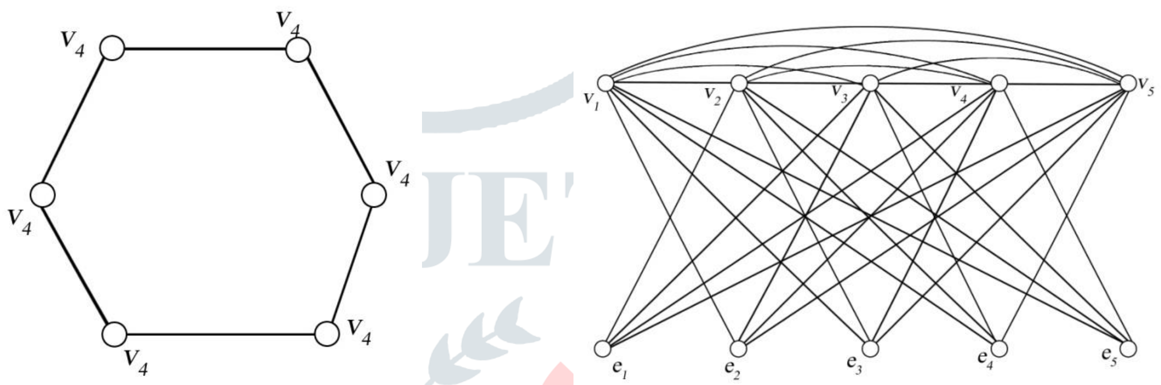
Let $G = C_n$ be any cycle containing $'n'$ vertices then $\gamma(G^{---}) \leq 3$

Proof: Let $G = C_n$, then vertex and the edge set of G is

$V(G) = \{v_i / \ 1 \leq i \leq n\}, E(G) = \{e_i = v_i v_{i+1}, e_n = v_1 v_n / \ 1 \leq i \leq n-1\}$

Let G^{---} is the transformation G then $V(G^{---}) = \{v_i, e_i / \ 1 \leq i \leq n\}$

For example, let $G = C_6$ and its transformation G^{---} is given as below.



$G = c_6$ (G^{---})

Figure: 2

$d(v_i) = d(e_i) = 2n - 5$ for all $v_i, e_i \in G^{---}$.

If $n = 4$, $D(G^{---}) = \{v_2, v_3, v_4\}$, $D_i(G^{---}) = \{v_1, v_2, e_1\}$,

$D_c(G^{---}) = D_t(G^{---}) = D_{ns}(G^{---}) = \{v_2, v_3, e_1\}$ are the respective dominating set of G^{---}.

If $n = 5$, $D(G^{---}) = D_t(G^{---}) = D_c(G^{---}) = \{v_1, e_3\}$ and

$D_i(G^{---}) = \{v_2, v_3, e_2\}$ are the respective dominating sets of G^{---}

If $n > 5$, $D(G^{---}) = D_c(G^{---}) = D_t(G^{---}) = \left\{v_1, e_{\lceil n/2 \rceil}\right\}$, $D_i(G^{---}) = \{\vartheta_1, \vartheta_2, e_1\}$ in general $D_i(G^{---}) = \{\vartheta_i, \vartheta_{i+1}, e_i\}$ is an independent dominating set therefore, $\gamma_i(G^{---}) = 3$

Hence, $\gamma(G^{---}) \leq 3$, In G^{---} , $N(v_1) = V(G^{---}) - \{v_2, v_n, e_1, e_n\}$,

$N(v_n) = V(G^{---}) - \{v_1, v_{n-1}, e_{n-1}, e_n\}$,

$N(v_i) = V(G^{---}) - \{v_{i-1}, v_{i+1}, e_{i-1}, e_i\} \, where \ 2 \leq i \leq n-1$

$N(e_i) = V(G^{---}) - \{v_1, v_2, e_1, e_n\}$, $N(e_n) = v(G^{---}) - \{v_1, v_n, e_1, e_{n-1}\}$

$N(e_i) = v(G^{---}) - \{v_1, v_{i+1}, e_{i-1}, e_{i+1}\}, 2 \leq i \leq n-1$ for all $v_i, e_i, \in V(G^{---})$

$D(G^{---}) = D_C(G^{---}) = D_t(G^{---}) = \{v_i, e_{i+2}\}_{\overline{1}} \leq i \leq n-2$; $D_i(G^{---}) = \{v_1, e_{1i}, v_{i+1}\}$ for all $1 \leq i \leq n-1$ are the independent dominating set of G^{---} from figure(3), $e_{i,} = v_{i,}v_{i+1}$ for all $1 \leq i \leq n-1$ that is $v_{i,}$and v_{i+1} are the end vertices of e_i in G. Therefore $e_{i,}v_i$ and v_{i+1} are independent in G^{---}. All the vertices and edge of G other than $\{v_i, e_i, v_{i+1}\}$ $I \leq i \leq n-1$ which are independent to either v_i or v_{i+1}or both. That is all the vertices of G^{---}other than e_i are adjacent with either v_i or v_{i+1} in G^{---} Hence, $\{v_i, v_{i+1}, e_i\}$ for all $1 \leq i \leq n-1$ are the dominating sets of G^{---}. Also $\{v_i, e_{i+1}, e_i\}$are independent in G^{---}.

Hence, $D_i(G^{---}) = \{v_i, v_{i+1}, e_i\}$ for all $1 \leq i \leq n-1$ therefore, $r_i(G^{---}) = 3$, similarly, $\{e_i, e_{i+1}, v_{i+1}\}$ for all $1 \leq i \leq n-1$ are also independent dominating sets of G^{---}

Hence, the different independent dominating sets of G^{---} are

$$D_i\,(G^{---}) = \begin{cases} \{v_i, v_{i+1}, e_i\}, \{v_1, v_n, e_n\} / \ 1 \leq i \leq n-1 \\ \{e_i, e_{i+1}, v_{i+1}\}, \{e_n, e_1, e_n\} / \ 1 \leq i \leq n-1 \end{cases}$$

Hence, $|D_i(G^{---})| = 2n$, Also every independent dominating sets $\{v_i, v_{i+1}, e_i\}$

of G^{---} , $\{e_j, v_{j+1}, e_{j+1}\}, j > i$ or $\{v_j, e_j, v_{j+1}\}, j > i+1$ are the inverse independent dominating sets of G^{---}. Hence, we can form $2n-5$ independent inverse dominating sets for each dominating set of G^{---}.

Theorem: 2.6

Let $G = K_{1,n}$ be any star graph with $n+1$ vertices then $\gamma(G^{---}) = \gamma_i(G^{---}) = 3$

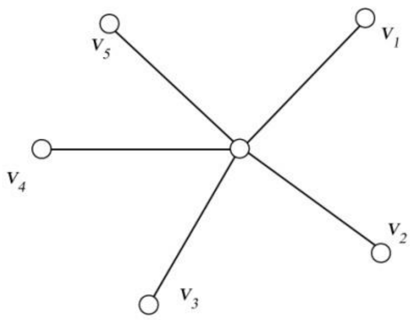
Proof: Let $G = K_{1,n}$ be any star graph let $V(G) = \{v_i / \ 1 \leq i \leq n+1\}$ and

$E(G) = \{e_i / \ 1 \leq i \leq n\}$ with $e_i = v_i v_{n+1}$ $d(v_i) = 1$ for all $i = 1,2,\dots,n$ and $d(v_{n+1}) = n$

Let $V(G^{---})$ be the vertex set of G^{---} then $V(G^{---}) = \{v_i, e_i / \ 1 \leq i \leq n+1; 1 \leq j < n\}$

And $|V(G^{---})| = 2n+1$.

For example, $G = K_{1,5}$ and $G^{---}$is represented in figure:3 as below



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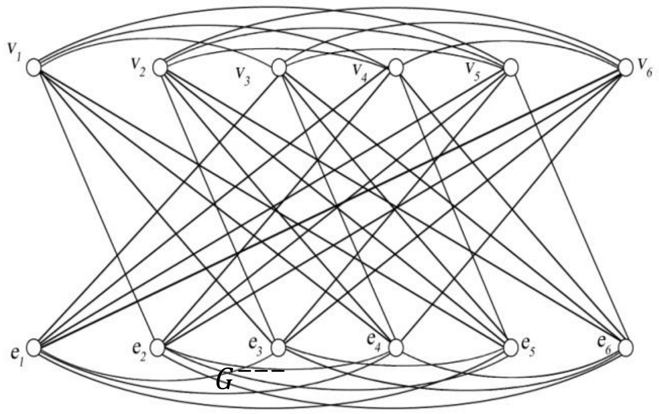

Figure: 3

In G^{---} $N(v_i) = V - \{e_i, v_{n+1}\}$

$$N(v_i) = V(G^{---}) - \{e_i, v_{n+1}\};\ 1 \le i \le n$$

$$N(e_i\ ) = V(G^{---}) - \{v_i, v_{n+1}\};\ 1 \le i \le n$$

Therefore, $d(v_i) = 2\,n - 1,\ \ d(e_i) = n - 1$

$d(v_{n+1}) = 0$ for all $1 \le i \le n$ and $v_i, e_i \in V(G^{---})$

That is every vertex $v_i \in V(G^{---})$ is adjacent with all the vertices of G^{---} other then its corresponding e_i and v_{n+1}. Since $d(v_{n+1}) = 0$ each set (v_i, e_i, v_{i+1}) $1 \le i \le n$ is a dominating set of G^{---}. Since v_i and e_i are independent in G^{---} and $d(v_{n+1}) = 0$, $\{v_i, e_i, v_{n+1}\}$, $1 \le i \le n$ is an independent dominating set of G^{---}. Hence, each set is of the from $\{v_i, e_i, v_{n+1}\}$; $1 \le i \le n$ is an independent dominating set of G^{---}. Therefore, $\gamma(G^{---}) = \gamma_i(G^{---}) = 3$

Theorem: 2.7

Let G be any tree with diam $(G) > 2, G^{---}$ is the transformation of G then $\gamma(G^{---}) = 2$

Proof:

Let G be any tree with diam $(G) > 2_j$then G is not $K_{1,n}$ and $0(G) > 3$ then G has altest four vertices. Since G is a tree, G has alteast two vertices of degree one. Without loss of generality we assume. that u and v are the peredent vertices of G such that $d(u,v) > 2$. Choose u' and v' are the neighbors of adjacent with u and v respectively. Then the distance between $u\ v'$ and distance between $u'v$ are at least two. Therefore, the vertex u is adjacent with all vertices of G^{---} other than u' and uu' (is a vertex in G^{---}). Since v adjacent with all vertices including u' and vu'other than v' and vv' in G^{---}. That is all the vertices of $G^{---}$are adjacent with u, v in G^{---}hence $\{u, v\}$ is a dominating set of G^{---}which implies $\gamma(G^{---}) = 2$.

Result: Let u and v are the pendent vertices of a graph G, $d(u,v) > 2$ then $\gamma(G^{---}) = 2$

Result: $\gamma(C_n) = \lceil {}^{n}/_{3} \rceil$ and $\gamma(G^{---}) = 2$

Theorem: 2.8

Let $G_1 = C_n$ and $G_2 = P_n$ for all $n > 2$ then $\gamma({G_1}^{---} + {G_2}^{---}) = 2$

Proof: Let $G_1 = C_n$ then ${G_1}^{---}$ is a conneeted graph with $2n$ vertices similarly $G_2 = P_n$ then ${G_2}^{---}$ is also a connected graph with $2n - 1$ vertices. Let u and v are the arbitrary vertices of ${G_1}^{---}$ and ${G_2}^{---}$ respectably then all vertices of ${G_2}^{---}$ is adjacent with u in ${G_1}^{---} + {G_2}^{---}$ similarly all vertices of ${G_1}^{---}$ is adjacent with v in ${G_1}^{---} + {G_2}^{---}$. That is all elements of ${G_1}^{---} + {G_2}^{---}$ is adjacent with u, v where $u \epsilon {G_1}^{---}$ and $v \epsilon {G_2}^{---}$ since u and v are arbitrary, any pair (u, v) is a dominating set of ${G_1}^{---} + {G_2}^{---}$. Hence, $\gamma({G_1}^{---} + {G_2}^{---}) = 2$

Corollary: 2.9

Let $G_1 = C_n$ and $G_2 = P_n$ then $\gamma_c(G_1{}^{---} + G_2{}^{---}) = \gamma_t(G_1{}^{---} + G_2{}^{---}) = 2$

Proof:

By above theorem every pair (u, v) where $u\epsilon G_1$ and $v\epsilon G_2$ is connected in $G_1{}^{---} + G_2{}^{---}$

Hence, $\gamma_c\ (G_1{}^{---} + G_2{}^{---}) = \gamma_t(G_1{}^{---} + G_2{}^{---}) = 2$

Theorem: 2.10

Let $G = P_m \times P_n,\ \ G^{---}$ is the transformation of G,

then $\gamma(G^{---}) = \gamma_t(G^{---}) = \gamma_c(G^{---}) = 2, m, n > ,2$

Proof:

Let $G = P_m \times P_n$

Let $V(P_m) = \{v_i / \ 1 \le i \le m\}, V(P_n) = \{v_j / \ 1 \le i \le n\}$

$P_4 \times P_5$ is given in figure as below

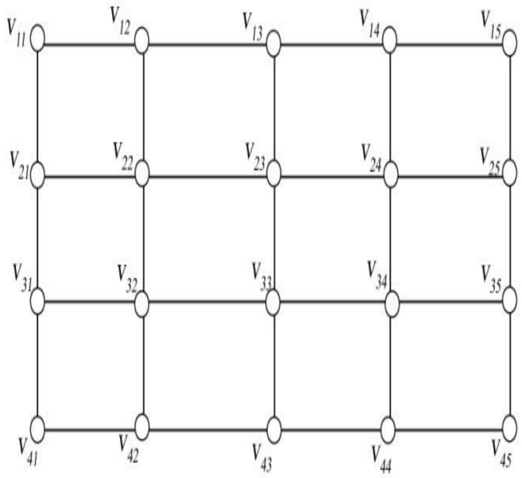

Figure : 5 $P_4 \times P_5$

Let $V(G^{---})$ be the vertices of G^{---}.

That is $V(G^{---}) = \{v_{ij} / \ 1 \le i \le m, 1 \le j \le n\}$

where, $d(v_{11}) = d(v_{1,n}) = d(v_{m,1}) = d(v_{m,n}) = 2$

$d(v_{i,j}) = 3$ if $i = 1$ or $j = 1$, $\ d(v_{i,j}) = 4$ for all $2 \le i \le m - 1$ and $\ 2 \le j \le n - 1$

Therefore, $|V(G^{---})| = 3mn\ \ - (m + n)$ and $\ |E(G^{---})| = 4mn - 2(m + n)$

In $G, N(v_{1,1}) = \{v_{1,2}, v_{2,1}\},\ \ N(v_{m,n}) = \{v_{m,n-1}, v_{m-1,n}\}$

Let $e_1 = v_{1,1}, v_{1,2}; e_2 = v_{1,1}, v_{2,1}$

$e_s = v_{m,n-1}\ v_{m,n}$ and $e_t = v_{m-1,n}\ v_{m,n}$ are the edges of G which are ineident with $v_{1,1}$ and $v_{m,n}$ respectively

In $G^{---}\ N(v_{1,1}) = V(G^{---}) - \{v_{1,2}, v_{2,1}; e_1, e_2\}, N(v_{m,n}) = V(G^{---}) - \{v_{m,n-1}, v_{n-1,m} e_s, e_t\}$

Hence, all the elements of G^{---} are connected with either $v_{1,1}$ or $v_{m,n}$

Since $v_{1,1}$ and $v_{m,n}$ are independent in G. Therefore, $v_{1,1}$ and $v_{m,n}$ connected in G^{---}.

Hence, $D(G^{---}) = D_t(G^{---}) = D_c(G^{---}) = \{v_{1,1}, v_{m,n}\}$

$\Rightarrow \gamma(G^{---}) = \gamma_1(G^{---}) = \gamma_c(G^{---}) = 2$

Hence the proof.

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