A thermodynamical study of supersymmetry at a fixed temperature

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Abstract

A thermodynamical approach can be used in detailed study of supersymmetry quantum mechanics at a given temperature. Such study are used in atomic physics, molecular physics, nuclear physics and particle physics. The fundamentals of thermodynamics can be used in this study. Such study associated with exactly solvable supersymmetric potential. The formalism and the techniques of the SUSY quantum mechanics is generalized to the cases where the superpotential is generated / defined by higher excited eigenstates.

Key words: Thermodynamic field, Exactly solvable potential and supersymmetry.

1. Introduction:

In recent years, there has been considerable interest in the study of fixed temperature effects [1,2] in supersymmetry quantum mechanics [3]. Supersymmetric quantum mechanics has been developed as an elegant analytical approach to one dimensional problems. As an analytical approach, the SUSY quantum mechanical has been utilized to study a number of quantum mechanics problems including the Morse oscillator [4] and the radial hydrogen atom equation [5]. In addition, SUSYQM has been applied to the discovery of new exactly solvable potentials, the development of a more accurate WKB approximation and the improvement of large N expansion and variational methods, [6, 7]. In particular, it has been shown that if supersymmetry is unbroken at an absolute temperature, T=0, then it is spontaneously broken at $T \neq 0$. However, in the case when SUSY is broken at T=0, it was conjectured that the fixed temperature effect could restore broken SUSY [8]. In this research paper, I shall examine whether fixed temperature effect can restore broken SUSY and the entire investigation will be carried out in the context of an exactly solvable SUSYQM model which exhibits broken SUSY at T=0. It should be mentioned at this point that the ideas involved behind the SUSY property and shape invariance were formulated first by Infeld and Hull (1951), where they were called the " factorization method", and the author refer further to the related ideas in works of Erwin Schroedinger (1940, 41). It is shown that the ground state for any bound quantum mechanical system minimizes the SUSY- displacement-standard momentum uncertainty product.

2. SUPERSYMMETRIC HAMILTONIAN AND SUPER CHARGES:

Let us consider the superpotential of the type [9] and is given by

$$\left.\begin{array}{ll} V(x) = kx, & x \neq 0 \\ \quad = 0\,, & x = 0 \end{array}\right\} \qquad \text{.........(1)}$$

Here $x = variable\ distance$

and $k = parametric\ constant$

The corresponding supercharges are

$$Q^{+} = \frac{1}{\sqrt{2}}(p + ikx)\Psi \qquad \text{.........(2)}$$

and $$Q = \frac{1}{\sqrt{2}}(p - ikx)\Psi^{+} \qquad \text{.........(3)}$$

Now, the supersymmetric Hamiltonian is

$$H = \frac{p^2}{2} + \frac{V^2}{2} + \frac{1}{2}V'(x)[\Psi^{+}, \Psi] \qquad \text{......(4)}$$

$$H = \frac{1}{2}p^2 + \frac{1}{2}k^2x^2 + \frac{1}{2}k[\Psi^{+}, \Psi] \qquad \text{.......(5)}$$

Hamiltonian in terms of supercharges can be written as

$$H = \{Q^{+}, Q\} \qquad \text{.......(6)}$$

The anti commutation relations are

$$[x, p] = i\ \{\Psi^{+}, \Psi\} = 1 \qquad \text{........(7)}$$

Consider the creation and annihilation operators as follows:

$$a^{+} = \frac{1}{\sqrt{2k}}\ (p + ikx) \qquad \text{.........(8)}$$

and $$a = \frac{1}{\sqrt{2k}}\ (p - ikx) \qquad \text{..........(9)}$$

provided $$b^{+} = \Psi^{+}\ abd\ b = \Psi \qquad \text{..........(10)}$$

On combining equation (7), (8), (9) & (10), we have

$$[a^+, a] = \{b^+, b\} = 1 \qquad \text{.........(11)}$$

and $$[a, a] = [a^+, a^+] = \{b, b\} = \{b^+, b^+\} = [a, b] = [a^+, b^+] = 0 \qquad \text{.........(12)}$$

It also gives $$H = k(a^+a + b^+b) \qquad \text{.........(13)}$$

If N_B *and* N_F be Bosonic and Fermionic number operators then

$$N_B = a^+a \ \ and \ \ N_F = b^+b \qquad \text{.........(14)}$$

Since it is the problem of harmonic oscillator with infinite potential barrier at origin such that the wave function vanish at origin.

In this case, energy spectrum is

$$< n_B, n_F|H|n_B, n_F >= k(2n_B + n_F + 1) \qquad \text{.........(15)}$$

Here n_B *and* n_F denotes the eigenvalues of the number operators N_B *and* N_F respectively such that

$$n_B = 0, 1, 2, 3, \qquad \text{.........(16)}$$

and $$n_F = 0, 1 \qquad \text{.........(17)}$$

The first few eigenstates and their eigenvalues are

$$|n_B = 1, n_F = 0 >, E = k \qquad \text{.........(18)}$$

$$|n_B = 1, n_F = 1 >, E = 2k \qquad \text{.........(19)}$$

$$|n_B = 3, n_F = 0 >, E = 3k \qquad \text{.........(20)}$$

$$|n_B = 3, n_F = 1 >, E = 4k \qquad \text{.........(21)}$$

In this case, the Fock space becomes

$$\left|2n_B + 1, n_F >= \frac{(a^+)^{2n_B}(b^+)^{n_F}}{\sqrt{2n_B}}\right| n_B = 1, n_F = 0 > \qquad \text{.........(22)}$$

In ground state, Fock space gives

$$|\Omega >= |n_B = 1, n_F = 0 > \qquad \text{.........(23)}$$

and the ground state energy is

$$E_\Omega = k \text{ where } k \neq 0 \qquad \text{.........(24)}$$

It is clear that ground state energy which serves as an order parameter in supersymmetry is nonvanishing and supersymmetry is broken. Since ground state solves the time independent Schroedinger equation for the corresponding Hamiltonian, the system specific coherent states build in the dynamics of the system under investigation. This property leads to the expectation that these dynamically adopted and system specific coherent states will prove more rapidly convergent in calculations of the excited state energies and wave functions for quantum systems using variational methods.

In thermodynamical field, the Hillbert space [10,11] of states is doubled with the introduction of "tilde" states. Thus, a state $|n_B, n_F>$ would now be written as $|n_B, n_F> \otimes |\bar{n}_B, \bar{n}_F>$. There are creation and annihilation operators for the "tilde" space and represented as $\bar{a}^+, \bar{a}, \bar{b}^+$ and $\bar{b}$. These tilde operators have the same commutation relations as the ordinary ones and these operators commute with the nontilde operators.

Now, ground state would be

$$|\Omega>=|n_B=1, n_F=0>\otimes|\bar{n}_B=1, \bar{n}_F=0> \quad ....(25)$$

To study the effect of finite temperature on supersymmetric quantum mechanics, it is necessary to calculate the ground state energy at fixed temperature and for this we have to find creation and annihilation operators. In order to get new form of such operators, letus us " Bogoliubov" transformation [10,12], we have

$$a(\beta) = \exp(-iG_B)\, a \exp(iG_B) \quad ....(26)$$

This implies that

$$\Longrightarrow \quad a(\beta) = \mathrm{a} \cos\mathrm{h}\, \theta\,(\beta) - \bar{a}^+ Sin\ h\theta(\beta) \quad ....(27)$$

and $\bar{a}(\beta) = \exp\left(-iG_\beta\right) \bar{a} \exp\left(iG_\beta\right)$ ....(28)

This implies that

$$\bar{a}(\beta) = \bar{\mathrm{a}} \cosh \theta(\beta) - \overline{a^+}\, \mathrm{Sin\ h}\ \theta(\beta) \quad ....(29)$$

Where $iG_\beta = \theta(\beta)(\bar{a}a - a^+\bar{a}^+$ ....(30)

Here $\theta(\beta)\ defined\ as$

$$\mathrm{Cosh}\ \theta(\beta) = \left(1-\bar{e}^{\beta k}\right)^{-\frac{1}{2}} and\ Sin\ h\ \theta(\beta) = e^{-\frac{\beta k}{2}}\left(1-e^{-\beta k}\right)^{-\frac{1}{2}} \quad ....(31)$$

In this way, the Fermionic annihilation operators are

$$b(\beta) = b \cos\bar{\theta}(\beta) - \bar{b}^+ \sin\bar{\theta}\,(\beta) \quad ....(32)$$

and $\bar{b}(\beta) = \bar{b}\cos\bar{\theta}(\beta) + b^+ \sin\bar{\theta}\,(\beta)$ ....(33)

$$\text{Here } \cos\bar{\theta}(\beta) = \left(1+\bar{e}^{\beta k}\right)^{-\frac{1}{2}} and\ Sin\ \bar{\theta}(\beta) = e^{-\frac{\beta k}{2}}\left(1+e^{-\beta k}\right)^{-\frac{1}{2}} \quad ....(34)$$

All these temperature dependent operators satisfy the following relations

$$[a(\beta), a^+(\beta)] = [\bar{a}(\beta), \bar{a}^+(\beta)] = \{b(\beta), b^+(\beta)\} = \{\bar{b}(\beta), \bar{b}^+(\beta)\} = 1 \quad ..(35)$$

and all other brackets vanishes

On using equations (28), (29), (32) and (33), we have

$$a^{+}(\beta) = a^{+}\cos h\,\theta\,(\beta) - \bar{a}\sin h\,\theta(\beta) \qquad \ldots\ldots(36)$$

$$\bar{a}^{+}(\beta) = \bar{a}^{+}\cos h\,\theta\,(\beta) - a\sin h\,\theta(\beta) \qquad \ldots\ldots(37)$$

$$b^{+}(\beta) = b^{+}\cos\,\bar{\theta}\,(\beta) - \bar{b}\sin\,\bar{\theta}(\beta) \qquad \ldots\ldots(38)$$

and $$\bar{b}^{+}(\beta) = \bar{b}^{+}\cos\,\bar{\theta}\,(\beta) - b\sin\,\bar{\theta}(\beta) \qquad \ldots\ldots(39)$$

On which simplifications, we have

$$a = a(\beta)\cos h\,\theta(\beta) + \bar{a}^{+}(\beta)\sin h\,\theta\,(\beta) \qquad \ldots\ldots(40)$$

and $$b = b(\beta)\cos\bar{\theta}(\beta) + \bar{b}^{+}(\beta)\sin\,\bar{\theta}\,(\beta) \qquad \ldots\ldots(41)$$

Using equations (40) and (41), it is possible to obtain Hamiltonian in terms of temperature dependent operators.

It is now necessary to calculate groundstate energy at a given temperature and let it denoted by $E_{\Omega}(\beta)$ such that

$$E_{\Omega}(\beta) =< \Omega(\beta)|H|\Omega(\beta) > \qquad \ldots\ldots(42)$$

Here $|\Omega(\beta) >$ denotes the thermal groundstate and is given by

$$|\Omega(\beta) >= |n_B(\beta) = 1, n_F(\beta) = 0 > \otimes |\bar{n}_B(\beta) = 1, \bar{n}_F(\beta) = 0 > \ldots(43)$$

Now from equation (42), we have

$$E_{\Omega}(\beta) = k[\cos h^2\theta(\beta) + 2\,Sin\,h^2\theta(\beta) + Sin^2\bar{\theta}(\beta)] \qquad \ldots\ldots(44)$$

$$\Rightarrow \quad E_{\Omega}(\beta) = k\left[\frac{1}{(1-e^{-\beta k})} + \frac{2e^{-\beta k}}{(1-e^{-\beta k})} + \frac{1}{(1+e^{-\beta k})}\right] \qquad \ldots\ldots(45)$$

Thus, the thermal ground state energy is nonvanishing.

3. Results and Discussion:

From equation (45), it is clear that thermal energy of particles in ground state is nonvanishing. Hence, supersymmetry remains broken at fixed temperature. It will be noted that thermal energy in ground state correctly reproduces the zero temperature groundstate energy as $\beta \to \infty$

$$\text{such that } E_{\Omega}(\beta) \to k \qquad \ldots\ldots(46)$$

It shows that the supercharges do not annihilate the thermal ground state. In other words, we can say that charge operators associated with Bosons and Fermion annihilates the ground state of quantum systems. The similarity between the lowering operator of harmonic oscillator and SUSY charge operator implies that the superpotential can be regarded as a system specific generalized displacement variable. Analogous to the ground state of harmonic oscillator which minimizes the Heisenberg uncertainty product, the ground state of any bound quantum system was identified as the minimizer of the SUSY Heisenberg uncertainty product. It is also applicable in study of system specific coherent states.

4. **Conclusion**:

In this research paper, I have shown here that if supersymmetry is broken at zero temperature, then the finite temperature effect does not restore it. It is in agreement with the result obtained previously [8,13]. Finally, finite temperature effect causing spontaneous breakdown of SUSY. Such result strongly correlate the study of system specific coherent states and SUSY Heisenberg uncertainty product. This observation suggests that the connection of the SUSY-QM with the Heisenberg minimum uncertainty ($\mu-$) wavelets should be explored [14-17]. The SUSY displacement with the SUSY Heisenberg uncertainty product can lead to the construction of the SUSY minimum uncertainty wavelets and the SUSY distributed approximating functionals. These new functions and their potential applications in mathematics and physics are currently under investigation. In addition, this study presents a practical computational approach for discretized system specific coherent states in calculations of excited states.

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