CONSTRUCTAL THEORY BASED ON NON-LINEAR STABILITY ANALYSIS OF THIN PERFORATED PLATES

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ABSTRACT

Rectangular steel perforated sheets used in various engineering structures are subjected to uniaxial compressive loading, to find their critical buckling stress. Perforations of various shapes namely circle, ellipse, rectangle and diamond are considered in this study, aspect ratio of all these shapes are varied to study their effect on the critical buckling stress. The effect of the volume fraction (φ) of these perforations on the buckling behaviour of the considered plates is studied.

Constructal theory[1] based analysis is carried out in this buckling analysis by way of non-dimensionalising all the geometric parameters, constraints and choosing the objective function as maximization of critical buckling load. The results of this study are expressed in terms of the variation of non-dimensionalised critical buckling stress ($\sigma_{cr,ND}$) as a function of volume fraction (φ), aspect ratio of perforation shape (H^1,L^1). Further these results are analysed to find the optimal values of volume fraction (φopt), aspect ratio [(H^1, L^1)opt] that maximises the non-dimensional critical buckling stress.

Analytical expressions [2] for the buckling strength of plates without perforations exist but not for the ones with perforations. Also recently a study [3] has been performed to find the buckling behaviour of perforated plates with different shapes of perforations using constructal theory under the eigen buckling behaviour. Present study aims at studying the non-linear buckling behaviour of plates using constructal theory.

1 INTRODUCTION

1.1. Perforated plate

It is a sheet metal that is manually or mechanically stamped or punched to create a pattern of holes, slots, or decorative shapes, for weight reduction or for functional purposes.

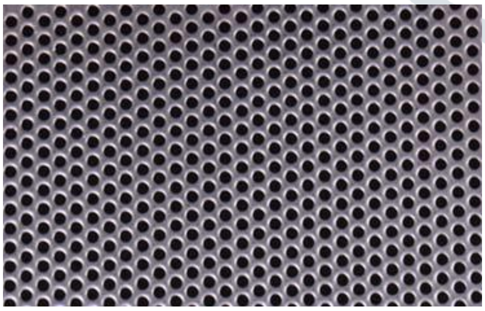
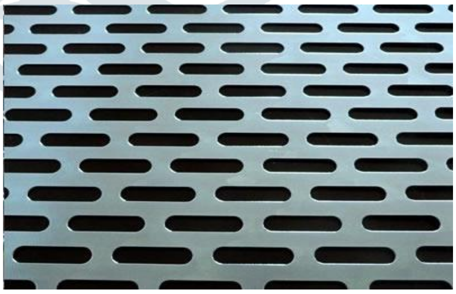



Figure.1 Perforated plate

1.2 Constructal design

Constructal design was proposed by Dr.AdrianBejan based on the constructal law which states that all processes in the universe tends to flow in the path of minimal resistance nature and engineering are united. This law has been employed for several applications in all the domains of design generation and evolution, from biology and physics to social organization, technology evolution, sustainability and engineering.

In this sense, constructal theory has been employed to explain deterministically how configurations in nature have been spontaneously generated, from inanimate rivers to animate designs, such as vascular tissues and social organization. Chief examples of unifying designs are vascular tree-shaped flow architectures, which serve as bases for many rules of animal design and river basin design. This same principle is used also to yield new designs for electronics, fuel cells, and tree networks for transport of people, goods and information. The applicability of this law to the physics of engineered flow systems has been widely discussed in recent literature.

2 Literature review

In the following, a literature review on the bucking of rectangular plates is presented to provide the background information for the present investigation.

2.1 Elastic buckling analysis of the perforated plates

Slender structural elements subjected to axial compressive loads can fail suddenly due to a phenomenon known as buckling instability [1]. The plates are thin structural elements commonly used in buildings, bridges, ships, planes and automobiles. In many cases, it is almost in- evitable to have holes in these plate elements for inspection, maintenance, service, and weight reduction purposes.

The presence of holes causes a redistribution of the membrane stresses accompanied by a change in the mechanical behavior of the plate [2]. When these structural components are subjected to axial compression the buckling phenomenon can occur. At a certain given critical load the plate will very sudden large deflections in the out-of-plane transverse direction [3]. An important characteristic of the buckling phenomenon is that this instability may occur at a stress level that is substantially lower than the material yield strength. The buckling behavior of perforated plates has been the object of a large number of researches in the last decade.

The analysis methods adopted in the published articles can be divided into two categories, i.e., linear elastic buckling and nonlinear elasto-plastic buckling. Among the elastic buckling studies category, El-Sawy and Nazmy [4] using a numerical approach investigated the effect of aspect ratio on the elastic buckling critical loads of uniaxially loaded rectangular plates with eccentric circular and rectangular (with rounded corners) holes. El-Sawy and Martini [5] used the finite element method to determine the elastic buckling stresses of biaxially loaded perforated rectangular plates with longitudinal axis located circular holes.

Alternatively, Moen and Schafer [6] developed, validated and summarized analytical expressions for estimating the influence of single or multiple holes on the elastic buckling critical stress of plates in bending or compression. In the group of studies dedicated to the problem of elasto-plastic buckling, El-Sawy et al. investigated the elasto-plastic buckling of uniaxially loaded square and rectangular plates with circular cutouts by the use of the finite element method, including some recommendations about hole size and location for the perforated plates of different aspect ratios and slenderness ratios. Afterwards, Paik studied the ultimate strength characteristics of perforated plates under edge shear loading, axial compressive loading and the combined biaxial compression and edge shear loads, and proposed closed-form empirical formulae for predicting the ultimate strength of perforated plates based on the regression analysis of the nonlinear finite element analyses results.

Maiorana et al. focused on the linear and nonlinear finite element analyses of perforated plates subjected to localized symmetrical load. In summary, most of the previous researches focused on the changes in buckling behavior of perforated plates due to the presence of cutouts. The optimal hole shape and hole size to the plate buckling performance has not yet been systematically investigated. This task can be appropriately performed by relying in Constructal theory and has a practical relevance as in the self-weight reduction of structures.The essence of constructal law can be stated as the natural tendency of flowing withbetter and better configuration, "optimal distribution of imperfections.

The method applies the objective and constraints principle in such a way that the best architecture can emerge deterministically. The applicability of this method to engineered flow systems has been widely discussed in recent literature, e.g. in designing cavities and assembly of fins. Lorente et al. have shown that Constructal theory can also be applied to mechanical strength. Therefore, the main objective of this study is to show the application of the Constructal Design to the determination of the optimal hole shape and size to the elastic buckling performance of perforated plates (with elliptical, rectangular and diamond holes) under uniaxial compression loading in non- linear buckling analysis.

2.2 Elasto-plastic buckling analysis of the perforated plates

In the analysis of the mechanical behavior of slender members, equilibrium equations, constitutive laws and compatibility conditions are used in order to find the internal forces and deformations. In the simplest cases, the structural safety is evaluated by confirming that the maximum values computed for the stresses are lower than the allowable stress defined for the material the structure is made of. This is a necessary condition for structural safety, but it may not be sufficient, either because the deformations must be limited for some reason, or because there is the risk that the equilibrium configuration of the structure is not stable, i.e., that buckling may occur. In fact, while tensile forces may only cause deformation or failure of the structure, in the case of compression there is a third possibility, namely, buckling, which consists of a lateral deflection of the structural component, in relation to direction of actuation of the compressive forces. In accordance with these considerations, the stability of a structure may be analyzed by computing its critical load, i.e., the load corresponding to the situation in which a perturbation of the deformed state does not disturb the equilibrium between the external and internal forces.

In this context, it is well known that steel plate elements constitute very important structural components in many structures, such as ship grillages and hulls, dock gates, plate and box girders of bridges, platforms of offshore structures, and structures used in the aerospace industry. In many cases, these plates are subjected to axial compressive forces, which make them prone to instability or buckling. If the plate is slender, the buckling is elastic. However, if the plate is sturdy, it buckles in the plastic range causing the so-called inelastic (or elasto-plastic) buckling.

In addition, in several practical situations cutouts are provided in plate structures for the purposes of access, service, weight reduction, and even aesthetics. The presence of these holes results in a redistribution of the membrane stresses accompanied by a change in mechanical behavior of the plate.

Concretely, a significant decrease in elasto-plastic ultimate strength, when compared to solid plate (i.e., non-perforated plate), has always been found in perforated plates notwithstanding the occasional occurrence of an increase in the elastic buckling critical load as reported in literature [3]. In addition, the location, format and dimensions of the perforation have a direct influence in the occurrence of a linear elastic or nonlinear elasto-plastic buckling.

Among the elastic buckling studies category, El-Sawy and Nazmy [4] investigated the effect of aspect ratio on the elastic buckling critical loads of uniaxially loaded rectangular plates with eccentric circular and rectangular (with curved corners) holes. El-Sawy and Martini [5] used the finite element method to determine the elastic buckling stresses of biaxially loaded perforated rectangular plates with the longitudinal axis located circular holes. Alternatively, Moen and Schafer [6] developed, validated and summarized analytical expressions for estimating the influence of single or multiple holes on the elastic buckling critical stress of plates in bending or compression. In Rocha et al. [7], Isoldi et al. [8], and Rocha et al. [9] the constructal design method was employed to determine the best shape and size of centered cutout in a plate, aiming to maximize the critical buckling load. In the group of studies dedicated to the problem of elasto-plastic buckling, El-Sawy et al. [4] investigated the elasto-plastic buckling of uniaxially loaded square and rectangular plates with circular cutouts by the use of the finite element method, including some recommendations about hole size and location for the perforated plates of different aspect ratios and slenderness ratios. Afterwards, Paik[10–12] studied the ultimate strength characteristics of perforated plates under edge shear loading, axial compressive loading, combined biaxial compression and edge shear loads. Moreover, there were proposed closed-form empirical formulas for predicting the ultimate strength of perforated plates based on the regression analysis of the nonlinear finite element analysis results. Maiorana et al. [13–14] focused on the linear and nonlinear finite element analysis of perforated plates subjected to localized symmetrical load. The improvement of systems performance has been one of the major goals in engineering. In this sense, the study of geometric configurations for achievement of this purpose is also an important subject. In the past, the scientific and technical knowledge combined with practice and intuition has guided engineers in the design of man-made systems for specific purposes. Soon after, the advent of the computational tools has permitted to simulate and evaluate flow architectures with many degrees of freedom. However, while system performance was analyzed and evaluated on a scientific basis, system design was kept at the level of art [15].

3 PARAMETERS CONSIDERED IN THIS STUDY

In this study we consider A-36 Steel [17]Young's modulus & poison ratio is 210 Gpa& 0.3. chemical composition of A-36 steel is as follows:

- Carbon 0.260 %
- Copper 0.20 %
- Iron 99.0 %
- Manganese 0.75 %
- Phosphorous ≤ 0.040 %
- Sulphur ≤ 0.050 %

- Yield Strength : 345 Mpa
- Ultimate Strength : 500-550 Mpa

4 METHOD OF ANALYSIS

Many of the researchers as used the FEM andthe CLDM for determining buckling loads (or stresses) ofperforated plates. Both methods are based on solving aneigenvalue problem that describes the behavior of the plateat buckling. The lowest eigenvalue corresponds to thecritical load while the eigenvector defines the deformedshape at buckling.A general-purpose finite-element program, ABAQUS, hasbeen utilized in this investigation to determine the elasticbuckling loads (or stresses) of the perforated plates studied.

The main objective of this study is Optimizing the hole:

- Changing the hole shape
- Changing the hole orientation

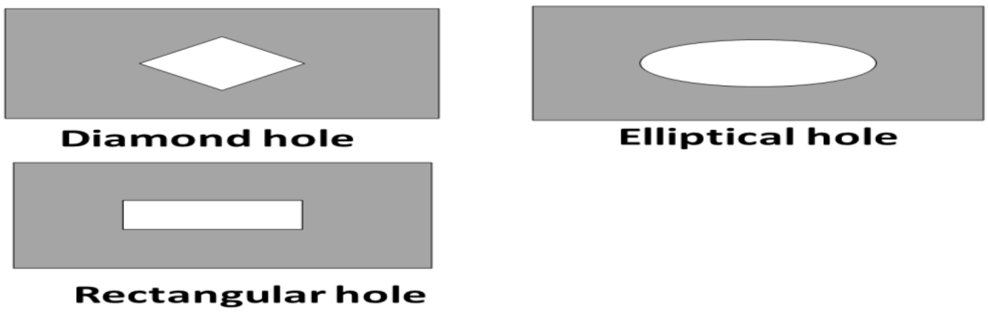


Fig. 2 Various shaped perforated plates considered in this study

4.1 Analysis procedure:

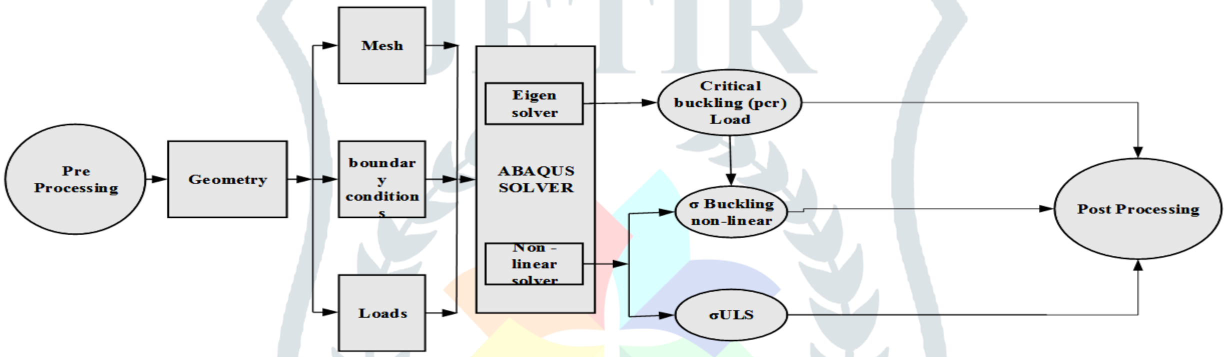
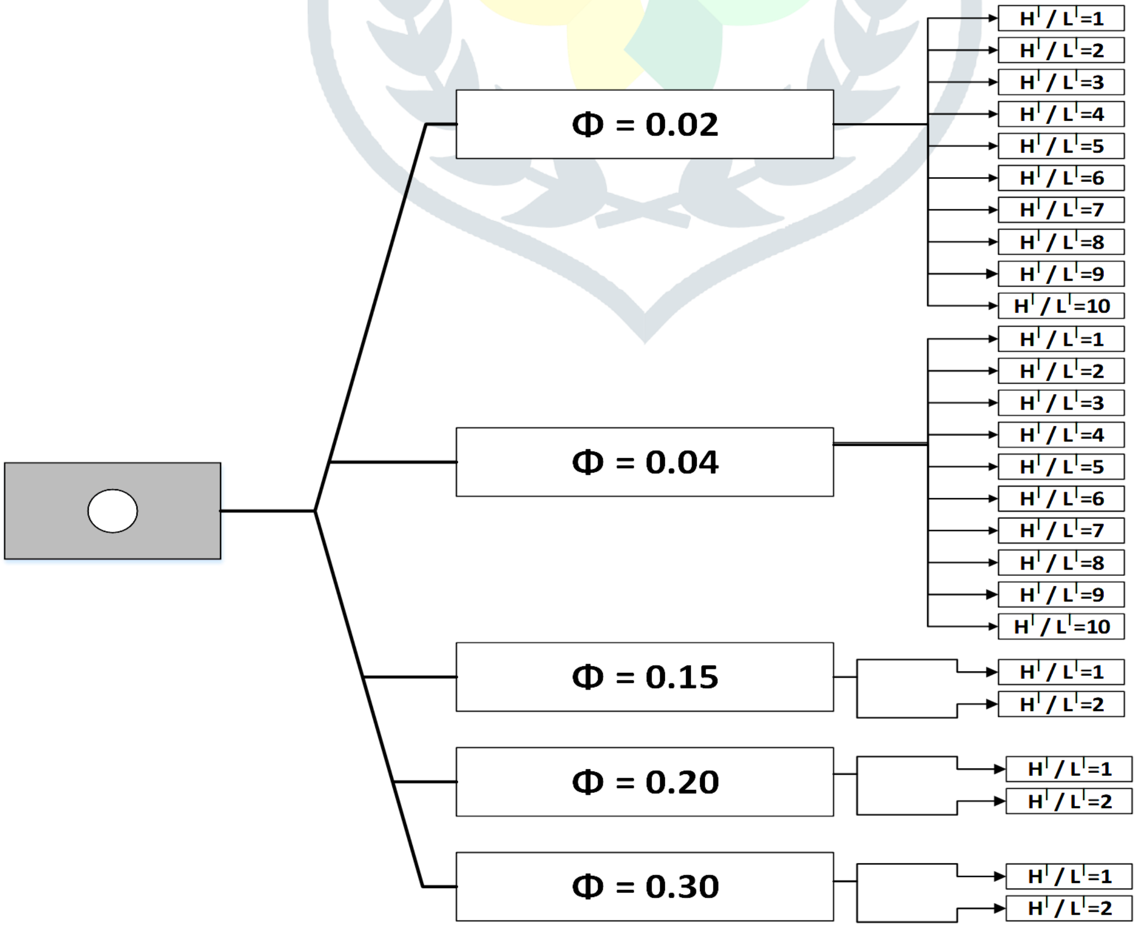


Figure 3 Analysis procedure of Non-Linear Buckling of perforated plates



Result:

From the results the best shape for

- small volume fractions- diamond hole
- intermediate volume fractions – rectangular hole
- Large volume fractions – elliptical hole

The variation of von mises stress distribution for three different hole shapes at three different aspect ratios have been estimated using general purpose FEA tool ABAQUS tool and the results are shown in figure 5.

The stress (σ_{ux}) in X direction is normalized and plotted against the aspect ratio (H_o/L_o) for three different shaped holes i.e., elliptical, rectangular and diamond at four different volume fractions that are structurally critical.

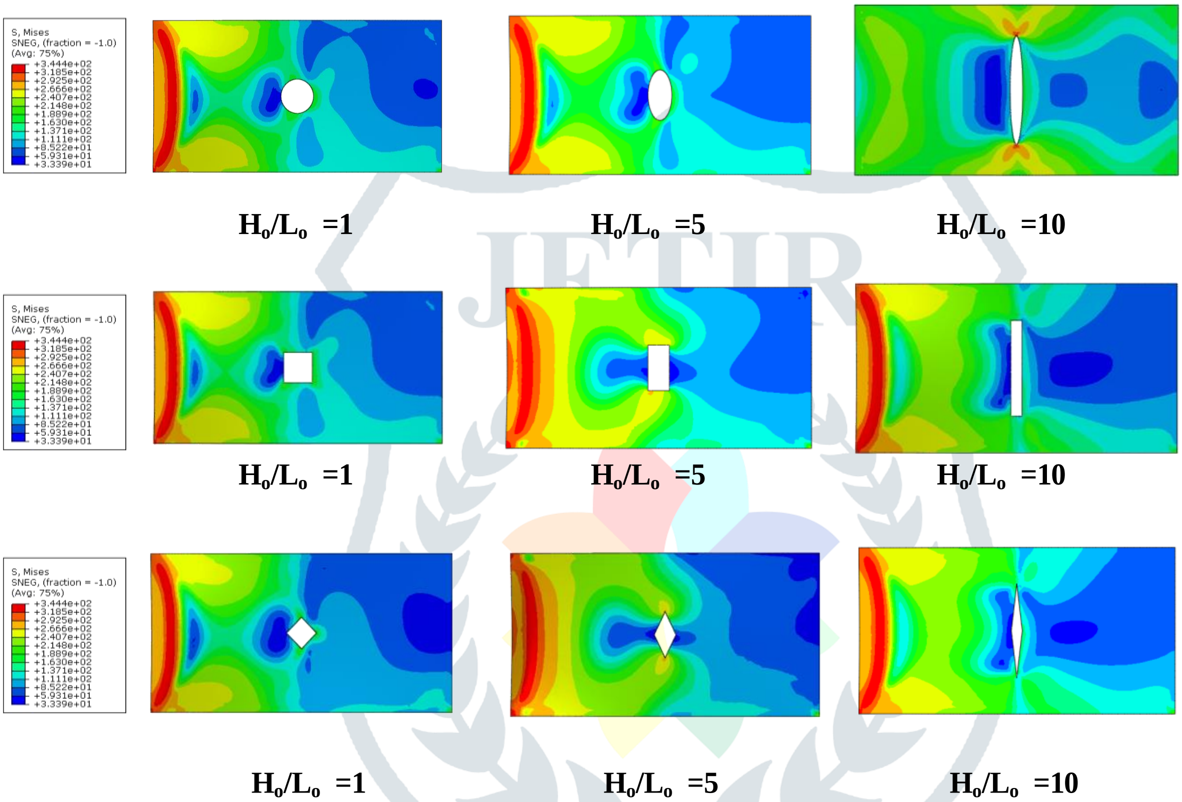
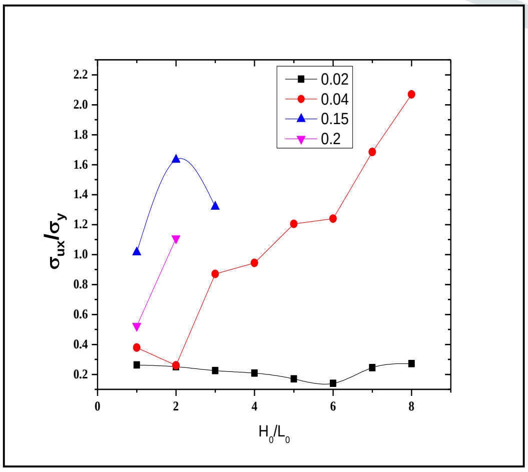
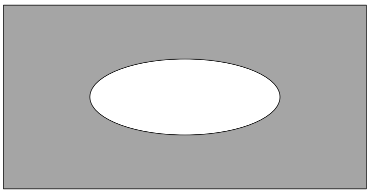
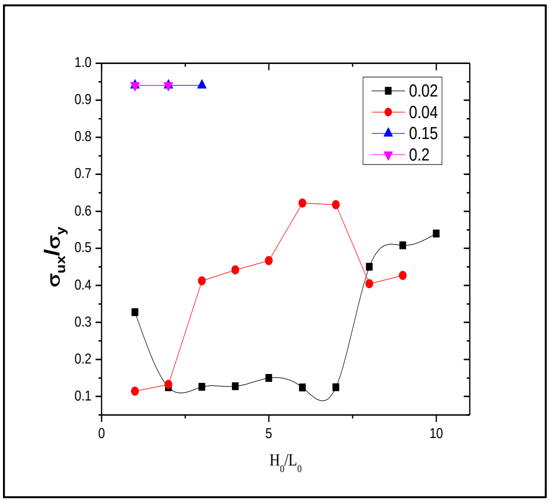


Figure 5 Limit state von mises stress for platewith various hole shapes







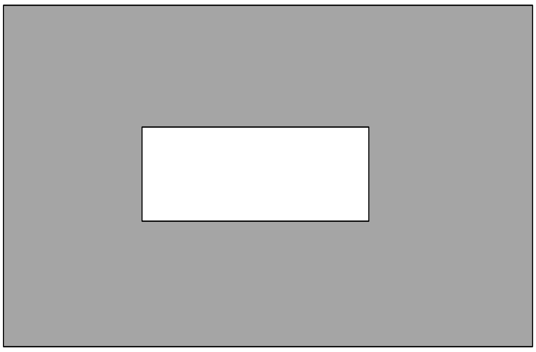
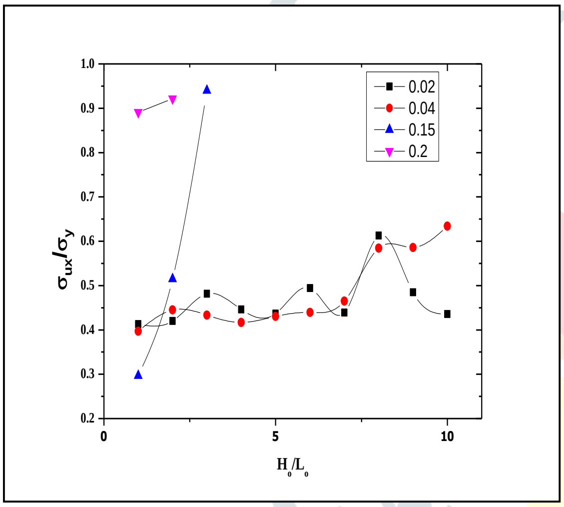


Figure. 7Distribution of limit state stress with aspect ratio of a plate with rectangular hole



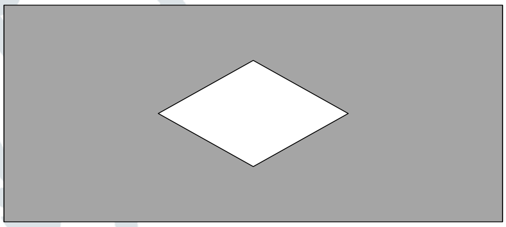


Figure. 8Distribution of limit state stress with aspect ratio of a plate with diamond hole

The three graphs shown above shows the variation of limit state stress against the aspect ratio of plate with three different holes at four different hole volume fractions. These four hole volumes fractions are chosen based on their structural importance based on the past experience.

For the plate with elliptical hole the normalized stress increases and then decreases linearlythe limit state stress is maximum with a volume fraction of $\varphi = 0.04$.

For the plate with rectangular hole the limit state stress values are almost same at hole volume fractions of $\varphi = 0.15$ & 0.2 at a lower aspect ratio of the plate which shows that the effect of volume fraction is of less importance at small aspect ratios.

For the plate with diamond shaped hole the limit state stress is maximum at a hole volume fraction of $\varphi = 0.15$ at an intermediate values of plate aspect ratios.
Figure 9 shows that the elliptical hole is optimal at higher volume fractions whereas the diamond & rectangular shapes are suitable for lower & intermediate volume fractions respectively.

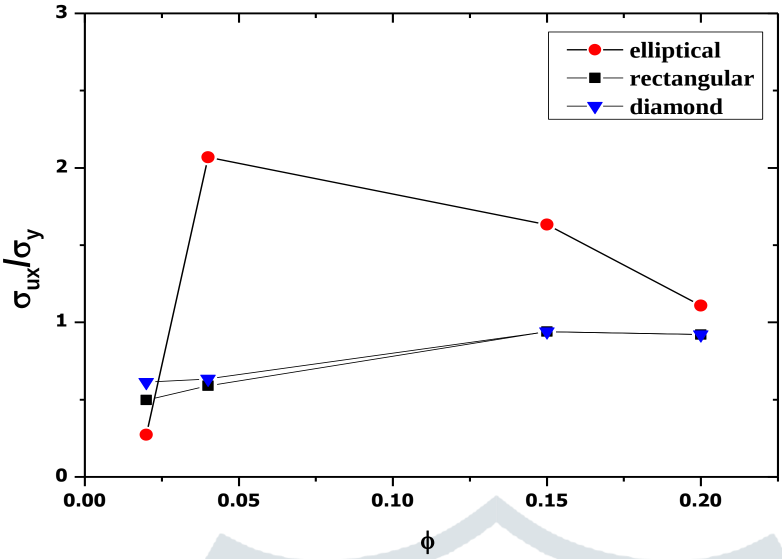


Figure 10 shows that among the three shapes large aspect ratios are suitable for lower hole volume fractions and vice a versa.

Conclusions:

Non-linear elastic and elasto-plastic buckling analysis is carried out on three different shaped perforation at differenthole volume fractions & aspect ratios. Constructal law is successfullyapplied while selecting the optimal hole volumes and non-linear buckling analysis is carried out in fem tool which reveals that for plates with large aspect ratios the holes with smaller volume fractions are optimalandthe elliptical hole is optimal at higher volume fractions whereas the diamond & rectangular shapes are suitable for lower & intermediate volume fractions respectively.

Future Scope: The present work can be extended by applying the non-linear buckling analysis to plates attached with stiffeners and other structural members with the help of Constructal theory.

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