Effect of Oversampling on Timinb Jitter of OFDM Systems

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***Abstract*— This document gives the impairments caused by timing jitter which are a significant limiting factor in the performance of very high data rate OFDM systems. In this we show that oversampling can reduce the noise caused by timing jitter. Both fractional oversampling achieved by leaving some band-edge OFDM subcarriers unused and integral oversampling are considered. The theoretical outcomes are compared with Matlab results for the case of white timing jitter.**

***Keywords*—Timing jitter, OFDM, Oversampling, band-edge, subcarriers.**

I. INTRODUCTION

This Orthogonal Frequency Division Multiplexing (OFDM) is used in many wirelesses broadband communication systems because it is a simple and scalable solution to Inter symbol interference(ISI) caused by a multipath channel. Very recently the use of OFDM in optical systems has been attracted increasing interest. Data rates in optical fiber systems are typically very much higher compare to RF wireless systems. At these high data rates, timing jitter is appearing as an important limitation to the performance of OFDM. The sampling clock in the very high speed analog-to-digital converters (ADCs) which are required in these systems is the main source of timing jitter. Timing jitter is also emerging as a problem in high frequency band pass sampling OFDM radios. The effect of timing jitter has been analyzed in the literature focus on the older low pass timing jitter which is typical of systems using phase lock loops (PLL). They consider only integral oversampling. Oversampling can be achieved by taking some band-edge subcarriers unused. In this project work we investigate both fractional and integral oversampling. We extend the timing jitter matrix to analyze the detail of the inter carrier interference in an oversampled system. Very high speed ADCs typically uses parallel pipeline architecture not a PLL and for these the white jitter which is the focus of this paper is a more appropriate model.

II. OFDM(ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING)

Origin for OFDM were made in the 60s and the 70s. It has taken a quarter of a century for this technology to change from the research domain to the industry. The idea of OFDM is quite simple but the implementation has many complexities. So, it is a fully software project.

OFDM depends on Orthogonality principle. Orthogonality means, it allows the sub carriers, which are orthogonal to each other, meaning that cross talk between co-channels is eliminated and inter-carrier guard bands are not required. This simplifies the design of both the transmitter and receiver.

Orthogonal Frequency Division Multiplexing (OFDM) is a digital multi carrier modulation technique, which utilizes a large number of closely spaced orthogonal sub-carriers. A single stream of data is split into parallel streams each of which is coded and modulated on to a subcarrier, a term commonly used in OFDM systems. Each sub-carrier is modulated with a conventional modulation technique at a low symbol rate. Thus the high bit rates on a single carrier is reduced to lower bit rates in the subcarrier. OFDM signals are generated and detected by using the Fast Fourier Transform algorithm. OFDM is a popular scheme for wideband digital communication, wireless as well as wired.

III. INTRODUCTION TO TIMING JITTER

The different techniques for measuring jitter in serial digital video signals lead to different measurement results. We must be focuses on video jitter measurement techniques typically found in video-specific instruments, e.g., wave form monitors and video measurement sets. General purpose measuring instruments are also used to measure jitter in serial digital video signals. These instruments offer more capabilities to analyse jitter, based on sophisticated signal processing. We will briefly touch on some very basic aspects of video jitter measurement using general-purpose instruments , specifically related to comparing results with measurements made on video-specific instruments. We will not discuss the range of jitter measurement capabilities of sampling or real-time oscilloscopes, or other general-purpose instruments.

Timing variation in serial digital signals and the measure-meant of these timing variations are complex technical topics. To discuss how and why jitter measurements differ, this guide gives a overview of jitter measurement techniques and also several key concepts.

We focuses on describing common reasons for differences in measuring jitter in serial digital video signals. In particular, we examines differences associated with the jitter frequencies in the video signal and with the duration of the peak-to-peak amplitude measurements used to characterize jitter in video systems.

IV. OVERSAMPLING

Oversampling is for processing signals at a higher sample rate than the original material. This is an important technique that can be used to remove aliasing from a non-linear process such as modulation.

A. DOWN SAMPLING

Down sampling also known as decimating. in the fig1, the process of down sampling requires us to low pass filter the processed signal to Fs/2 with a nice, sharp filter. Then we just throw away the samples we don' t want. In the case of 2x down sampling, this means we throw away 1 out of every 2 samples.

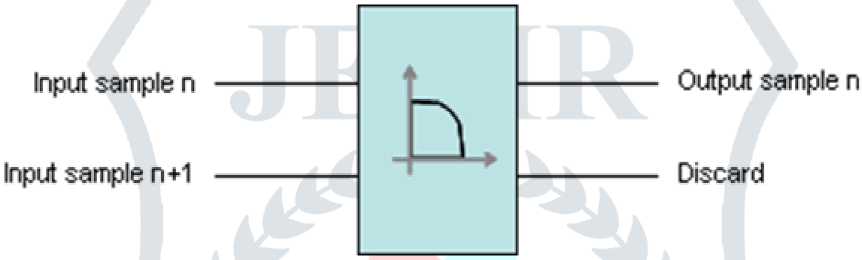


Fig:1 down sampling

B. UP SAMPLING

If we' re going to operate at a higher sample rate, where do we get the extra samples from? It might be a linear interpolation to estimate the in-between samples. The problem is that this process will introduces its own aliases. Better results can be obtained by polynomial interpolators, but these become computationally expensive when trying to obtain sufficient quality.

The most practical approach is to use the same filter as used for down sampling. In the case of a 2x filter, you feed it two inputs. In the fig 2,the first input is your original signal, while the second is simply set to zero which is called as " zero stuffing" and it results in aliasing. The filter preserves the original spectrum by removing the aliases. What we end up is the original waveform with increased sampling rate.

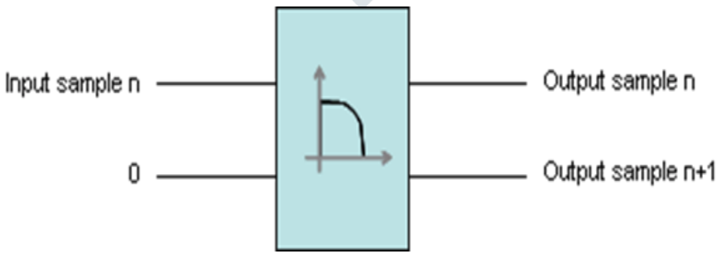


Fig2 up sampling

C. Processing At a Higher Sampling Rate

Consider the case of 2x oversampling. If we construct our up sampling filter so that it outputs two consecutive samples for each one that is input, then we have the two samples we need and they will arrive at our processor at the same time. If the processor is stateless and non-recursive (i.e. for any given input sample it will always give the same output regardless of what happens before or after), then we can simply duplicate our processing element and run a separate copy of it on the second output of the up sampler. Otherwise, we need to get a bit clever and feed the output of the first processing element into the second one, along with the second output from the up sampler (for an example of this, you can look at the implementations of the up/down samplers themselves).

V. EFFCT OF OVERSAMPLING ON JITTER NOISE POWER

In the general case, where both integral and fractional over sampling are applied, the signal samples after the ADC in the receiver are given by

$$y_{n_M} = y\left(\frac{n_M T}{NM}\right)$$
$$= \frac{1}{\sqrt{N}} \sum_{k=-N_L}^{N_u} H_k X_k e^{\left(\frac{j2\pi k}{T} \times \frac{n_M T}{NM}\right)} + \eta\left(\frac{n_M T}{NM}\right) \quad (1)$$

Where n_M is the oversampled discrete time index and η is the AWGN. The N point FFT at receiver is replaced by an ‘ oversized’ NM-point FFT. The elements of output FFT are.

$$Y_{l_M} = \frac{1}{\sqrt{M}} \frac{1}{\sqrt{NM}} \sum_{n_M=-NM/2+1}^{NM/2} y_{n_M} e^{\left(\frac{-j2\pi n_M l_M}{NM}\right)} \quad (2)$$

Where l_M is the index of the NM point FFT. The modified weighting coefficients for the oversampling case,

$$w_{l_M,k} = \frac{1}{NM} \sum_{n_M=-NM/2+1}^{NM/2} e^{j2\pi k \frac{\tau_n M}{T}} e^{j\frac{2\pi}{NM}(k-l_M)n_M} \quad (3)$$

By using the approximation $e^{j\theta} = 1+j\theta$ for small θ

$$w_{l_M,k} = \frac{1}{NM} \sum_{n_M=-NM/2+1}^{NM/2} \left(1+\frac{j2\pi k \tau_{n_M}}{T}\right) e^{j2\Pi(k-l_M)n_M/NM} \quad (4)$$

So for $k \neq l_M$ the variance of the weighting coefficients is given by

$$\mathrm{E}\{|w_{l_M,k}|^2\} \approx \left(\frac{2\pi k}{MNT}\right)^2 \mathrm{E}\{\tau_{n_M} \tau_{p_M}\} e^{j\frac{2\pi}{MN}(k-l)d_M} \quad (5)$$

Where n_M-p_M=d_M. When the timing jitter is white, then $\{\tau_{n_M} \tau_{n_M} - d_M\}$ =0 for $d_M \neq 0$ so

$$\mathrm{E}\{|w_{l_M,k}|^2\} \approx \frac{1}{NM}\left(\frac{2\pi k}{T}\right)^2 \mathrm{E}\{\tau^2{}_{nM}\} \qquad k \neq l_M \quad (6)$$

From (6) it can be seen that white timing jitter $\mathrm{E}\{|w_{l_M,k}|^2\}$ is inversely proportional to M so increasing the integer over sampling factor reduces the inter carrier interference (ICI) due to timing jitter. Also that $\mathrm{E}\{|w_{l_M,k}|^2\}$ depends on k^2 but not on l_M, so higher frequency sub carriers cause more ICI, but the ICI affects all sub carriers equally.

A. AVERAGE JITTER NOISE POWER FOR EACH SUBCARRIER

$$Y_{l_M} = H_{l_M} X_{l_M} + \sum_{k=-N_L}^{N_U} \left(w_{l,k} - I_{l_M,k} \right) H_k X_k + N(l) \tag{7}$$

where the second term represents the jitter noise. We consider a flat channel with, $H_k = 1$ and assume that the transmitted signal power is distributed equally across the used subcarriers so that for each used subcarrier $E\{X_k^2\} = \sigma_s^2$. Then the average jitter noise power, $P_j(l)$ to received signal power of l^{th} subcarrier is given by

$$\frac{Pj(l)}{\sigma_s^2} = \frac{E\left\{ \left| \sum_{k=-N_l}^{N_v} \left(w_{l,k} - I_{l,k} \right) X_k \right|^2 \right\}}{\sigma_s^2} = \sum_{k=-N_L}^{N_u} E\left\{ \left| w_{l,k} - I_{l,k} \right|^2 \right\} \tag{8}$$

Rearranging the terms in (8) equation

$$\frac{Pj(l)}{\sigma_s^2} = \frac{\pi^2}{3M} \left(\frac{N_v N}{T_N^2} \right) E\{\tau_n^2\} \tag{9}$$

If there is no integral over sampling or fractional over sampling, M=1 and N_V=N,

$$\frac{Pj(l)}{\sigma_s^2} = \frac{\pi^2}{3} \left(\frac{N^2}{T_N^2} \right) E\{\tau_n^2\} \tag{10}$$

Comparing (9) and (10) it can be seen that the combination of integral oversampling and fractional over sampling reduces the jitter noise power by a factor of Nv/NM.

B. OVERSAMPLING TYPES:

In signal processing, Oversampling is the most important process in which a signal with a sampling frequency significantly higher than twice the bandwidth. Oversampling helps to eliminate aliasing, increases resolution and reduces noise. An oversampling factor of β, defined as

$$ß = f_s/2B \tag{11}$$

where f_s - the sampling frequency, B -the bandwidth or highest frequency of the signal; the Nyquist rate is 2B**.**

The impact of both fractional as well as integral over sampling in OFDM can be used to reduce the fading caused by timing jitter. To achieve integral over sampling, the received signal is sampled at a rate of MN/T, where M is an integer.

For fractional over sampling some band-edge sub carriers are unused in the transmitted signal. When all N sub carriers are modulated, the bandwidth of the baseband OFDM signal is N/2T, so sampling at intervals of T/N is Nyquist rate sampling. If instead, only the sub carriers with indices between $-N_L$ and $+N_U$ are non zero, the bandwidth of the signal is $(N_L+N_U)/2$. In this situation sampling is above the Nyquist rate. The degree of Oversampling is shown by $(N_L+N_U)/N$.

Advantages:

1.To analyze the detail of the inter carrier interference (ICI) in an oversampled system.

2.Very high speed ADCs used in a parallel pipeline architecture.

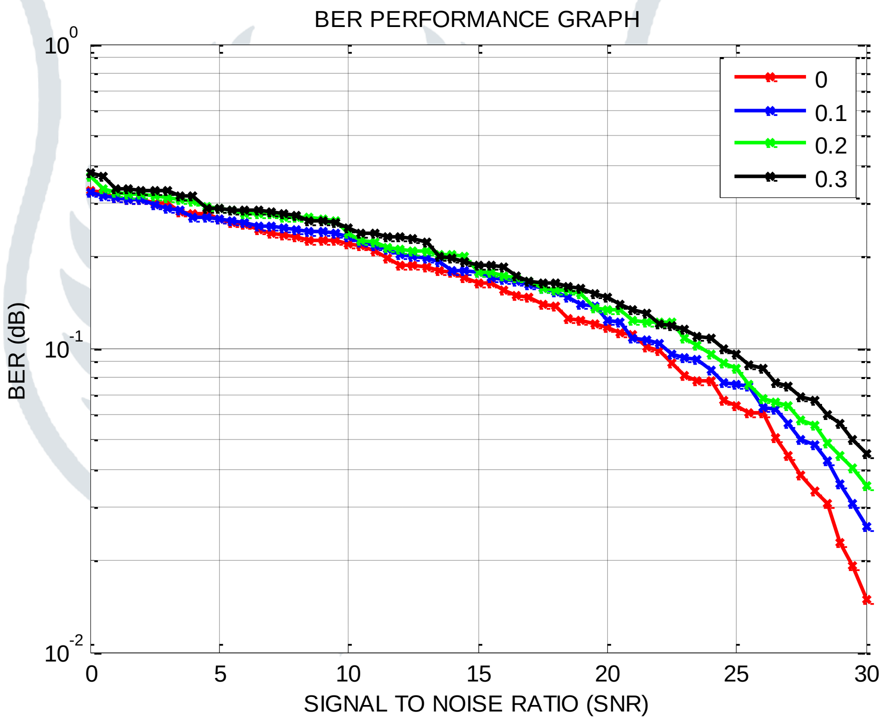
3.Oversampling can limit the degradation caused by timing jitter in OFDM systems.

Disadvantages:

1.High frequency subcarriers cause more ICI than lower frequency subcarriers, but that the resulting ICI is spread equally across all subcarriers.

VI. SIMULATIONS

We now present Matlab simulation results for 2000 OFDM symbols, N= 512 and jitter variance. Note that the jitter variance is not changed when oversampling is applied, so the jitter indicates a larger fraction of the sampling periods. Fig. 7 shows the variance of the noise due to jitter as a function of received subcarrier index when band-edge subcarriers are unused. According to the results power of the jitter noise is not a function of subcarrier index and the removing of band-edge subcarriers removes the noise equally across all subcarriers. Fig. 6 shows both the theoretical and Matlab simulation results for average jitter noise power with a function of the oversampling factor. There is close agreement between theory and simulation. Increasing the sampling factor gives a reduction of 10log10 in jitter noise power, so every doubling of the sampling rate reduces the jitter noise power by 3 db.



A. **Simulation result-**

Fig 3 BER Performance graph

The figure8.1 shows The BER Performance varies with Signal to Noise Ratio for different timing deviations, like t_n=0,0.1,0.2,0.3 . The most timing deviations causes slowly BER Varies with the signal to noise ratio.

B. **Simulation result-2:**

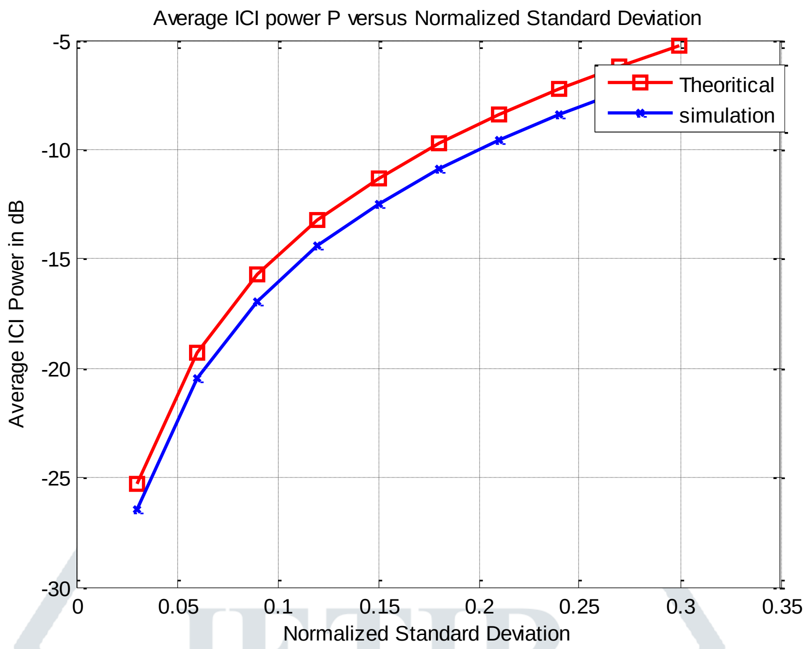


Fig 4 Average ICI power vs Normalized standard deviation

Fig 8.2 shows the graph between average inter carrier interference power and normalized standard deviation. The theoretical and simulation results are compared. While inter carrier interference power increases Exponentially with the Standard Deviation.

C. **Simulation result-3:**

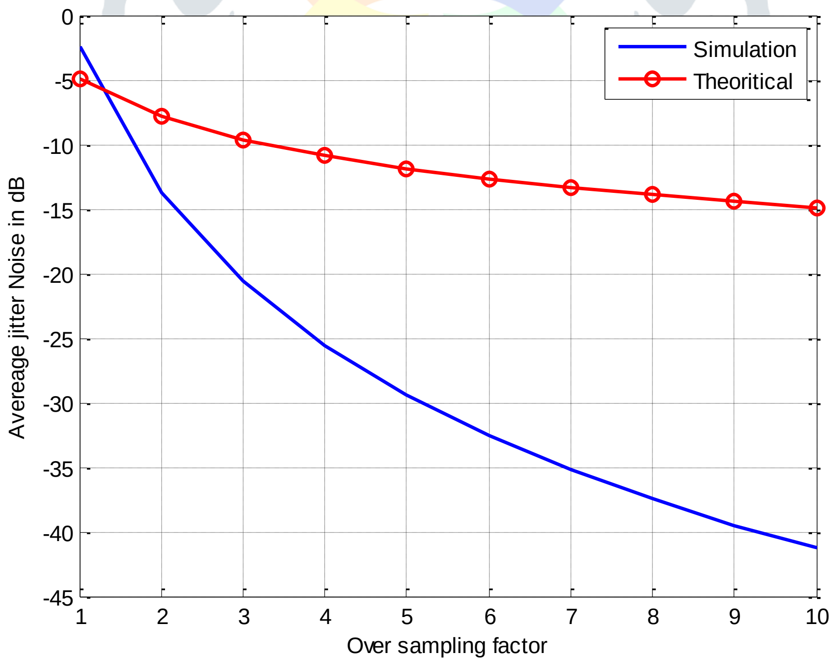


Fig 5: Reduction of Jitter Noise power

The figure 5 shows the graph between Average Jitter Noise Power and over sampling factor . The Jitter Noise Power does not depend on the Received index sub carrier, It only depends on the over sampling factor.

It can be seen that white timing jitter is inversely proportional to M so increasing the integer over sampling factor reduces the inter carrier interference (ICI) due to timing jitter. Also that White Timing Jitter depends on

Transmitted Index Sub Carrier but not on the received Index Sub Carrier , so higher frequency sub carriers cause more ICI, but the ICI affects all sub carriers equally.

D. **Simulation result-4:**

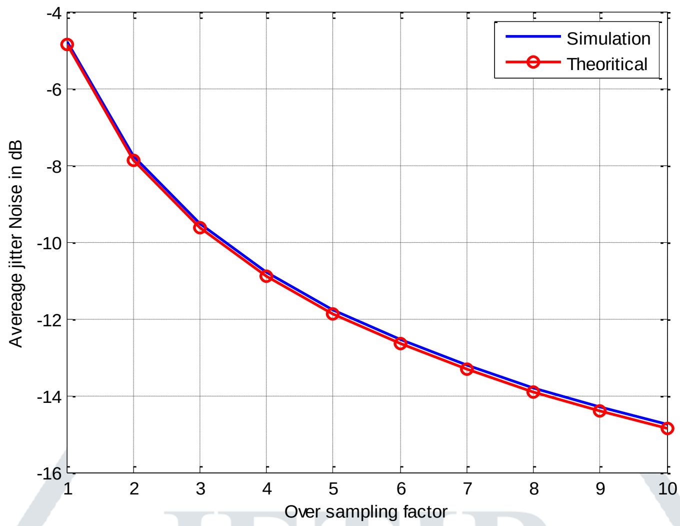


Fig 6: Reduction of Jitter Noise power

The fig 6 shows the graph of over sampling factor versus Average Jitter noise power in dB. It shows that due to over sampling whenever the sampling factor increases by two ,there was the decrease in jitter noise power by 3db .

E. Simulation result-5:

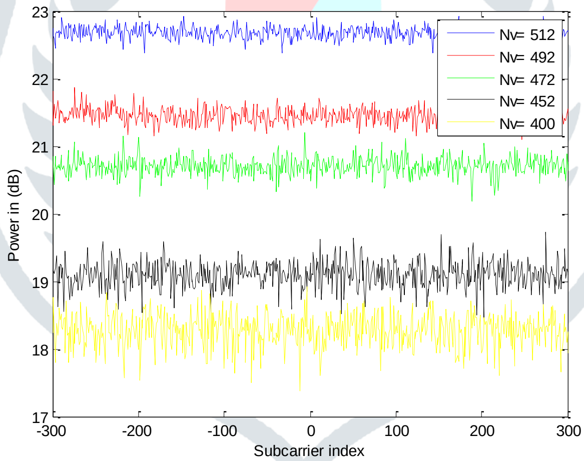


Fig 7: Variation of power for different sub carrier indices

The fig 7 shows that the variance of the noise due to jitter as a function of received sub carrier index when band-edge sub carriers are unused. The power of the jitter noise is not a function of sub carrier index and at the same time, removing the band-edge sub carriers reduces the noise equally across all sub carriers. The average jitter noise power is a function of the over sampling factor. There is close agreement between theory and simulation.

VI. CONCLUSIONS

It has been shown both theoretically and by simulation that oversampling can reduce the degradation caused by timing jitter in OFDM systems. There are two methods of oversampling.: fractional oversampling achielly leaving some of the band-edge subcarriers unused, and integral oversampling implemented by increasing the sampling rate at the receiver.

The jitter variance is not changed with oversampling, so the jitter represents a larger fraction of the sampling period for the oversampled systems. In thethe case of white timing jitter, both techniques result in a linear reduction of jitter noise power as a function of oversampling rate. Thus oversampling results a 3 dB reduction of jitter noise power for every doubling of sampling rate.

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