THE GENERALIZED FRACTIONAL INTEGRAL OPERATORS AND THE IMAGE FORMULAS OF THE GENERALIZED K-FUNCTION

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Abstract
In this paper we will study the generalized fractional integral operators given by Saigo and Maeda and will establish the image formulas associated with the generalized K-function. The results produced from the integrals are in the form of the Wright generalized hyper-geometric function. We will also employ certain integral transforms on the results obtained from the integrals and present some more image formulas.

Keywords: **Appell function, Fox Wright function, Generalized K-Function, Saego-Maeda fractional integral operators.**

1. Introduction

The fractional calculus is that special branch of the applied mathematics, deals with the investigations of integrals and derivatives of arbitrary orders. This branch was not so popular in its initial phase mainly due to lack of applications. Recently it has gained a lot of significance and has drawn the attention of large number of mathematicians in view of its tremendous applicability and importance in almost all fields of science and engineering. This branch of applied mathematics is rapidly developing with large number of applications in the real world. Many authors such as Baleanu et al. [1, 2], Kilbas [3], K.Sharma [4], Mittag-Leffler [5], Saigo M.[6], Srivastava [7], P. Agarwal [8] and so on have studied, in depth, certain properties, applications, and different extensions of various hypergeometric operators of fractional integration.

Due to the large number of applications of fractional calculus in various fields (see [9],[10],[11],[12],[13],[14],[15],[16],[17]), we will establish various image formulas for the generalized K-function involving the fractional integral calculus operators of Saigo-Meada [6]

2. Mathematical Preliminaries

The generalization of the hypergeometric integrals has been given by Marichev [18] which was later studied and extended by Saigo and Maeda [19] involving the Appell function, or Horn's F_3 –function introduced by Appell and Kampe de Feriet [20] in the kernel

$$F_3(\alpha,\alpha',\beta,\beta';\ \gamma;x;y) = \sum_{m,n=0}^{\infty} \frac{(\alpha)_m(\alpha')_n(\beta)_m(\beta')_n}{(\gamma)_{m+n}} \frac{x^m}{m!}\frac{y^n}{n!} \quad (\max\{\ |x|,\ |y|\} < 1) \tag{2.1}$$

The Appell function F_3 has the property that it reduces to the Gauss hypergeometric function ${}_2F_1$ and also satisfies the two linear partial differential equations of the second order as follows [21]

$$F_3(\alpha,\gamma-\alpha,\beta,\gamma-\beta;\ \gamma;x;y) = {}_2F_1(\alpha,\beta;\gamma;x+y-xy) \tag{2.2}$$

also

$$F_3(\alpha,0,\beta,\beta';\ \gamma;x,y) = {}_2F_1(\alpha,\beta;\gamma;x) \tag{2.3}$$

and

$$F_3(0,\alpha',\beta,\beta';\ \gamma;x,y) = {}_2F_1(\alpha',\beta';\gamma;y) \tag{2.4}$$

In this paper we will establish and study the image formulas involving the generalized K-function using the generalized fractional calculus integral operators. We use the generalized Marichev Saigo-Maeda fractional integral operators involving the Appell function, or Horn's F_3 –function introduced by Appell and Kampe de Feriet [20] defined as follows

$$\left(I_{0+}^{\alpha,\alpha\prime,\beta,\beta\prime,\gamma}\right)f(x) = \frac{x}{\Gamma(\gamma)}^{-\alpha}\int_0^x (x-t)^{\gamma-1}t^{-\alpha\prime}F_3\left(\alpha,\alpha',\beta,\beta';\ \gamma;1-\frac{t}{x},1-\frac{x}{t}\right)f(t)dt \tag{2.5}$$

$$(\Re(\gamma) > 0)$$

$$\left(I_{0+}^{\alpha,\alpha\prime,\beta,\beta\prime,\gamma}\right)f(x) = \left(\frac{d}{dx}\right)^{k}\left(I_{0+}^{\alpha,\alpha\prime,\beta+k,\beta\prime,\gamma+k} f\right)(x), (\Re(\gamma)\ \leq\ 0\ );\ k = [\,-\Re(\gamma)+1\,] \tag{2.6}$$

$$\left(I_{0-}^{\alpha,\alpha\prime,\beta,\beta\prime,\gamma} f\right)(x) = \frac{x}{\Gamma(\gamma)}^{-\alpha'}\int_x^{\infty}(t-x)^{\gamma-1}t^{-\alpha}F_3\left(\alpha,\alpha',\beta,\beta';\ \gamma;1-\frac{x}{t},1-\frac{t}{x}\right)f(t)dt \tag{2.7}$$

$$(\Re(\gamma) > 0)$$

$$\left(I_{0-}^{\alpha,\alpha\prime,\beta,\beta\prime,\gamma} f\right)(x) = \left(-\frac{d}{dx}\right)^{k}\left(I_{0-}^{\alpha,\alpha\prime,\beta,\beta\prime+k,\gamma+k} f\right)(x), (\Re(\gamma)\ \leq\ 0\ );\ k = [-\Re(\gamma)+1] \tag{2.8}$$

where $\alpha,\alpha',\beta,\beta',\gamma\ \epsilon\ \mathbb{C}$ and $x\ >\ 0$

The image formulas for a power function induced by Saigo et al. [19, 22] under operators (2.5) and (2.7), are given by:

$$\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\,x^{\rho-1}\right)(x)=x^{\rho-\alpha-\alpha'+\gamma-1}\left[\frac{\Gamma(\rho)\Gamma(\rho+\gamma-\alpha-\alpha'-\beta)\Gamma(\rho+\beta'-\alpha')}{\Gamma(\rho+\beta')\Gamma(\rho+\gamma-\alpha-\alpha')\Gamma(\rho+\gamma-\alpha'-\beta)}\right] \quad (2.9)$$

where $\Re(\rho) > max\ \{0,\Re(\alpha+\alpha'+\beta-\gamma),\Re(\alpha'-\beta')\}\ and\ \Re(\gamma) > 0.$

$$\left(I_{0-}^{\alpha,\alpha',\beta,\beta',\gamma}\,x^{\rho-1}\right)(x)=x^{\rho+\gamma-\alpha-\alpha'-1}\left[\frac{\Gamma(1-\rho-\beta)\Gamma(1-\rho-\gamma+\alpha+\alpha')\Gamma(1-\rho+\beta'+\alpha-\gamma)}{\Gamma(1-\rho)\Gamma(1-\rho+\beta'-\gamma+\alpha+\alpha')\Gamma(1-\rho+\alpha-\beta)}\right] \quad (2.10)$$

where $\Re(\rho) < 1 + min\ \{\Re(-\beta),\Re(\alpha+\alpha'-\gamma),\Re(\alpha+\beta'-\gamma)\}\ and\ \Re(\gamma) > 0$

The K-function was introduced by Sharma [4] and he defined it as follows:

$$ {}^{\mu,\xi;\upsilon}_{p}K_q\left(a_1,\dots,a_p;b_1,\dots,b_q;x\right)= {}^{\mu,\xi;\upsilon}_{p}K_q(x)=\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\dots,(a_p)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\dots,(b_q)_n\Gamma(\mu n+\xi)}\frac{x^n}{n!} \quad (2.11)$$

where $\mu,\xi,\upsilon\epsilon\,\mathbb{C},\Re(\mu) > 0$ and $(a_i)_n\ (i=1,2,\dots p)$ and $(b_j)_n\ (j=1,2,\dots q)$ are the pochammer symbols.

The generalized K-function given by Sharma [23] is defined as

$$ {}^{\mu,\xi;\upsilon,\zeta}_{r}K_s\left(a_1,\dots,a_r;b_1,\dots,b_s;x\right)= {}^{\mu,\xi;\upsilon,\zeta}_{r}K_s(x)=\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\dots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\dots,(b_s)_n\Gamma(\mu n+\xi)}\frac{x^n}{(\zeta)_n} \quad (2.12)$$

where $\mu,\xi,\upsilon,\zeta\epsilon\,\mathbb{C},\Re(\mu) > 0,\Re(\xi)>0,\Re(\upsilon)>0,\Re(\zeta)>0$ and $(a_i)_n\ (i=1,2,\dots r)$ and $(b_j)_n\ (j=1,2,\dots s)$ are the Pochhammer symbols.

The series (2.12) is defined when none of the parameters $(b_j)_n, j=1,2,\dots,s$, is a negative integer or zero. If any numerator parameter $(a_i)_n$ is a negative integer or zero, then the series terminates to a polynomial in x. It is evident from ratio test that the series is convergent for all x if $r > s+1$. When $r = s+1$, the series can converge in some cases. Let $\Delta=\sum_{i=1}^{r}a_i-\sum_{j=1}^{s}b_j$.It can be shown that when $r = s+1$ the series is absolutely convergent for $|x|=1$ if $\Re(\Delta)<0$, conditionally convergent for $x=-1$ if $0 \leq \Re(\Delta) < 1$ and divergent for $|x|=1\ if\ 1\leq\Re(\Delta)$.

The generalized Fox- Wright function ${}_p\psi_q$ was introduced by Wright [24] and has been given by the series

$$ {}_p\psi_q = {}_p\psi_q\left\{\begin{matrix}(a_1,A_1),\dots(a_p,A_p);\\(b_1,B_1),\dots(b_q,B_q);\end{matrix}\ x\right\}=\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma(a_i+nA_i)}{\prod_{j=1}^{q}\Gamma(b_j+nB_j)}\frac{x^n}{n!} \quad (2.13)$$

where $\Gamma(x)$ is the Euler gamma function.

where $x,a_i,b_j\,\epsilon\,\mathbb{C};\ A_i,B_j\,\epsilon\,\mathbb{R};\ A_i\neq 0,B_j\neq 0;i=1,\dots,p;\ \ j=1,\dots,q$

This function is known as generalized Wright function for all values of x, the conditions for its existence are as follows:

$$1+\left(\sum_{j=1}^{q}B_j\right)-\left(\sum_{i=1}^{p}A_i\right)\geq 0 \quad (2.14)$$

The generalized hypergeometric function ${}_pF_q$ is defined as follows [25]

$$ {}_pF_q\left\{\begin{matrix}a_1,\dots,a_p;\\b_1,\dots,b_q;\end{matrix}\ x\right\}=\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}(a_i)_n}{\prod_{j=1}^{q}(b_j)_n}\frac{x^n}{n!}=\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\dots,(a_p)_n}{(b_1)_n,(b_2)_n,\dots,(b_q)_n}\frac{x^n}{n!} \quad (2.15)$$

where $(\propto)_n$ is the Pochhammer symbol, which is defined (for $\propto\,\epsilon\,\mathbb{C}$) by:

$$(\propto)_n=\begin{cases}0 & n=0\\ \propto(\propto+1)(\propto+2)\dots(\propto+n-1) & n>0\end{cases}$$

$$(\propto)_n=\frac{\Gamma(\propto+n)}{\Gamma(\propto)}\left(\propto\ \epsilon\,\mathbb{C}\setminus Z_0^-\right) \quad (2.16)$$

Z_0^- denotes the set of non-positive integers.

The relation between the Wright generalized hypergeometric function and the generalized K-function is given by:

$$ {}^{\mu,\xi;\upsilon,\zeta}_{r}K_s\left((a_i)_1^r;(b_j)_1^s\right)=\frac{\Gamma\zeta\prod_{j=1}^{q}\Gamma(b_j)}{\Gamma\upsilon\prod_{i=1}^{p}\Gamma(a_i)}\,{}_{r+2}\psi_{s+2}\left\{\begin{matrix}(a_1,1),\dots(a_r,1),(1,1),(\upsilon,1);\\(b_1,1),\dots(b_s,1),(\xi,\mu),(\zeta,1);\end{matrix}\ x\right\} \quad (2.17)$$

The gamma function, Γx introduced by Leonard Euler [25] as generalization of the factorial function on the set $\mathbb{R}$ of all real numbers and $\mathbb{C}$ for all complex numbers is defined by

$$\Gamma x=\int_0^{\infty}t^{(x-1)}e^{-t}dt,\,x\epsilon\,\mathbb{R}^+ \quad (2.18)$$

$$\Gamma 1=1,\Gamma\tfrac{1}{2}=\sqrt{\pi}$$

The beta transform of the function $f(z)$ is defined by [26]

$$B\{f(z):\ l,m\}=\int_0^1 z^{l-1}(1-z)^{m-1}f(z)dz. \quad (2.19)$$

$$\Re(z)>0,\Re(l),\Re(m)>0$$

For the power function z^{n-1}, the beta transform is given by

$$B\{z^{n-1}:l,m\}=\int_0^1 z^{l+n-2}(1-z)^{m-1}f(z)dz=\frac{\Gamma(l+n-1)\Gamma m}{\Gamma(l+m+n-1)} \quad (2.20)$$

$$\Re(z)>0,\Re(l),\Re(m)>0$$

The P_δ- transform of a complex valued function $f(z)$ for real values of z is given by [27]

$$P_\delta[f(z);s]=F(s)=\int_0^{\infty}[1+(\delta-1)s]^{-\frac{z}{\delta-1}}f(z)dz,\qquad \delta>1 \quad (2.21)$$

Here $f(z)$is integrable over any finite interval (a,b), $0<a<z<b$; there exists a real number c such that

(i) If $b>0$ is arbitrary, then $\int_b^{\varsigma}e^{-cz}f(z)dz$ tends to a finite limit as $\varsigma\to\infty$

(ii) For arbitrary $a>0,\int_\omega^{a}[f(z)dz]$ tends to a finite limit as $\omega\to 0+$, then the P_δ- transform$P_\delta[f(z);s]$ exists for $\Re\left(\frac{\ln[1+(\delta-1)s]}{\delta-1}\right)>c$ for $s\epsilon\mathbb{C}$

The P_δ- transform of a power function $z^{\chi-1}$ is given by

$$P_\delta[z^{\chi-1};s]=\left\{\frac{\delta-1}{\ln[1+(\delta-1)s]}\right\}^{\chi}\Gamma\chi,\quad \chi\epsilon\mathbb{C},\ \Re(\chi)>0,\ \ \delta>1 \tag{2.22}$$

The P_δ- transform has found a lot of applications in the real world. Srivastava et al. [28] and [29] also used P_δ- transform and found many results involving the generalized hypergeometric function and the generalized incomplete gamma function.

If we take $\delta\to 1$ in (2.21), the P_δ- transform reduces to Laplace integral transform. [26]

$$L\{f(z):s\}=\int_0^\infty e^{-zs}f(z)dz,\qquad \Re(s)>0 \tag{2.23}$$

The relation between the P_δ- transform defined by (2.21) and the classical Laplace transform defined by (2.23) is given by

$$P_\delta[f(z);s]=L\left[f(z):\frac{\ln[1+(\delta-1)s]}{\delta-1}\right],\qquad \delta>1 \tag{2.24}$$

Or

$$L[f(z);s]=P_\delta\left[f(z):\frac{e^{(\delta-1)s}-1}{\delta-1}\right],\qquad \delta>1 \tag{2.25}$$

The integral formulas involving the Whittaker function [30] used for deriving the image formulas is given by

$$\int_0^\infty z^{\tau-1}e^{-\frac{z}{2}}W_{\sigma,\eta}(z)dz=\frac{\Gamma(\tau+\eta+\frac{1}{2})\Gamma(\tau-\eta+\frac{1}{2})}{\Gamma(\tau-\sigma+\frac{1}{2})} \tag{2.26}$$

$$(\sigma\in\mathbb{C},\ \Re(\tau\pm\eta)>-\tfrac{1}{2})$$

The Whittaker function [30] is defined by

$$W_{\sigma,\eta}(z)=\frac{\Gamma(-2\eta)}{\Gamma(\frac{1}{2}-\sigma-\eta}M_{\sigma,\eta}(z)+\frac{\Gamma(2\eta)}{\Gamma(\frac{1}{2}-\sigma+\eta}M_{\sigma,-\eta}(z) \tag{2.27}$$

$$=W_{\sigma,-\eta}(z)$$

$$(\sigma\in\mathbb{C},\ \Re\left(\tfrac{1}{2}+\eta\pm\delta\right)>0)$$

Where

$$M_{\sigma,\eta}(z)=z^{\eta+\frac{1}{2}}e^{-\frac{z}{2}}\,{}_1F_1(\tfrac{1}{2}-\sigma+\eta;2\eta+1;z), \tag{2.28}$$

$$\Re\left(\tfrac{1}{2}+\eta\pm\delta\right)>0,\quad |\arg z\ |<\pi$$

3. Main Results

Image formulas related with fractional operators

In this section, we will establish the image formulas for the generalized K-function involving the Saigo-Maeda fractional integral (2.5), (2.7) in terms of the generalized Wright function. The formulas are given by the following theorems:

Theorem 3.1. Let $x>0$, $\alpha,\alpha',\beta,\beta',\gamma,\rho,\mu,\xi,\upsilon,\zeta\in\mathbb{C},\Re(\mu)>0$ and $\lambda>0$. Further let the constants satisfy the conditions $a_i,b_j\in\mathbb{C},A_i,B_j\in\Re\ (A_i,B_j\neq 0,\ i=1,2,\dots,r,j=1,2,\dots,s.)$, and if condition (2.14) is satisfied , then the fractional integral $I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}$ of the generalized K-function exists under the conditions $\Re(\gamma)>0,\Re(\lambda)>0,\Re(\rho)>max\{0,\Re(\alpha+\alpha'+\beta-\gamma),\Re(\alpha'-\beta')\}$, and is given by

$$\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\,t^{\rho-1}\,{}^{\mu,\xi;\upsilon,\zeta}_{\ \ r}K_s\left(t^\lambda\right)\right)(x)=x^{\rho-\alpha-\alpha'+\gamma-1}\frac{\Gamma\zeta}{\Gamma\upsilon}\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{s}\Gamma(b_j)}{\prod_{i=1}^{r}\Gamma(a_i)}$$

$$\times\,{}_{r+5}\psi_{s+5}\left[\begin{matrix}(a_1,1),\dots,(a_r,1),(\upsilon,1),(1,1),(\rho,\lambda),(\rho+\gamma-\alpha-\alpha'-\beta,\lambda),(\rho+\beta'-\alpha',\lambda)\\(b_1,1),\dots,(b_s,1),(\xi,\mu),(\zeta,1),(\rho+\beta',\lambda),(\rho+\gamma-\alpha-\alpha',\lambda),(\rho+\gamma-\beta-\alpha',\lambda)\end{matrix};x^\lambda\right] \tag{3.1}$$

Proof. Taking the LHS of (3.1) as $\mathcal{I}$, and using (2.12) we get

$$\mathcal{I}=\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\,t^{\rho-1}\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\dots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\dots,(b_s)_n\Gamma(\mu n+\xi)}\frac{(t^\lambda)^n}{(\zeta)_n}\right)(x)$$

$$\mathcal{I}=\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\dots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\dots,(b_s)_n\Gamma(\mu n+\xi)}\frac{1}{(\zeta)_n}\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho+\lambda n-1}\right)\right)(x)$$

Applying (2.9) we get

$$\mathcal{I}=x^{\rho-\alpha-\alpha'+\gamma-1}\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\dots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\dots,(b_s)_n\Gamma(\mu n+\xi)}\frac{(x^\lambda)^n}{(\zeta)_n}$$

$$\times\left[\frac{\Gamma(\rho+\lambda n)\ \Gamma(\rho+\gamma-\alpha-\alpha'-\beta+\lambda n)\ \Gamma(\rho+\beta'-\alpha'+\lambda n)}{\Gamma(\rho+\beta'+\lambda n)\ \Gamma(\rho+\gamma-\alpha-\alpha'+\lambda n)\ \Gamma(\rho+\gamma-\beta-\alpha'+\lambda n)}\right] \qquad (3.2)$$

Using (2.17) in (3.2) and interpreting the right hand side of (3.2) we arrive at the required result.

Theorem 3.2. Let $x>0,\alpha,\alpha',\beta,\beta',\gamma,\rho,\mu,\xi,\upsilon,\zeta\in\mathbb{C},\Re(\mu)>0$ and $\lambda>0.$ Further let the constants satisfy the conditions $a_i,b_j\in\mathbb{C},\ A_i,B_j\in\Re\ (A_i,B_j\neq 0, i=1,2,\dots,r, j=1,2,\dots,s.)$ and if condition (2.14) is satisfied, then the fractional integral $I_{0-}^{\alpha,\alpha',\beta,\beta',\gamma}$ of the generalized K-function exists under the conditions $\Re(\gamma)>0,\Re(\lambda)>0,\Re(1-\gamma-\rho)<1+min\{\Re(-\beta),\Re(\alpha+\alpha'-\gamma),\Re(\alpha+\beta'-\gamma)\}$,and is given by

$$\left(I_{0-}^{\alpha,\alpha',\beta,\beta',\gamma}\ t^{-\gamma-\rho}\ {}^{\mu,\xi;\upsilon,\zeta}_{r}K_s\left(t^{-\lambda}\right)\right)(x)=x^{-\rho-\alpha-\alpha'}\frac{\Gamma\zeta}{\Gamma\upsilon}\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{s}\Gamma(b_j)}{\prod_{i=1}^{r}\Gamma(a_i)}$$

$$\times\ {}_{r+5}\psi_{s+5}\left[{(a_1,1),\dots,(a_r,1),(\upsilon,1),(1,1),(\rho+\alpha+\beta',\lambda),(\rho+\alpha+\alpha',\lambda),(\rho-\beta+\gamma,\lambda) \atop (b_1,1),\dots,(b_s,1),(\xi,\mu),(\zeta,1),(\rho+\alpha+\alpha'+\beta',\lambda),(\rho+\gamma,\lambda),(\rho+\alpha-\beta+\gamma,\lambda)};x^{-\lambda}\right] \qquad (3.3)$$

Proof. Taking the LHS of (3.3) as $\mathcal{I}$, and using (2.12) we get

$$\mathcal{I}=\left(I_{0-}^{\alpha,\alpha',\beta,\beta',\gamma}\ t^{-\gamma-\rho}\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\dots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\dots,(b_s)_n\Gamma(\mu n+\xi)}\frac{(t^{-\lambda})^n}{(\zeta)_n}\right)(x)$$

$$\mathcal{I}=\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\dots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\dots,(b_s)_n\Gamma(\mu n+\xi)}\frac{1}{(\zeta)_n}\left(I_{0-}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{-\gamma-\rho-\lambda n}\right)\right)(x)$$

Applying (2.10),we get

$$\mathcal{I}=x^{-\rho-\alpha-\alpha'}\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\dots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\dots,(b_s)_n\Gamma(\mu n+\xi)}\frac{(x^{-\lambda})^n}{(\zeta)_n}$$

$$\times\left[\frac{\Gamma(\rho+\alpha+\alpha'+\lambda n)\ \Gamma(\rho+\alpha+\beta'+\lambda n)\ \Gamma(\rho-\beta+\gamma+\lambda n)}{\Gamma(\rho+\alpha+\alpha'+\beta'+\lambda n)\ \Gamma(\rho+\gamma+\lambda n)\ \Gamma(\rho+\alpha-\beta+\gamma+\lambda n)}\right] \qquad (3.4)$$

Using (2.17) in (3.4) and interpreting the right hand side of (3.4) we arrive at the required result.

Image formulas related with integral transforms

Theorem 3.3. Let $x>0,\alpha,\alpha',\beta,\beta',\gamma,\rho,\mu,\xi,\upsilon,\zeta\in\mathbb{C},\Re(\mu)>0$ and $\lambda>0.$ Further let the constants satisfy the conditions $a_i,b_j\in\mathbb{C},\ A_i,B_j\in\Re\ (A_i,B_j\neq 0, i=1,2,\dots,r, j=1,2,\dots,s.)$, and if condition (2.14) is satisfied, such that $\Re(\gamma)>0,\Re(\lambda)>0,\Re(\rho)>max\{0,\Re(\alpha+\alpha'+\beta-\gamma),\ \Re(\alpha'-\beta')\}$.Then

$$B\left\{\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\ t^{\rho-1}\ {}^{\mu,\xi;\upsilon,\zeta}_{r}K_s\left(zt^{\lambda}\right)\right)(x):l,m\right\}=B(l,m)x^{\rho-\alpha-\alpha'+\gamma-1}\frac{\Gamma\zeta}{\Gamma l\,\Gamma\upsilon}\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{s}\Gamma(b_j)}{\prod_{i=2}^{r}\Gamma(a_i)}$$

$$\times\ {}_{r+5}\psi_{s+5}\left[{(l,1),(a_2,1),\dots(a_r,1),(\upsilon,1),(1,1),(\rho,\lambda),(\rho+\gamma-\alpha-\alpha'-\beta,\lambda),(\rho+\beta'-\alpha',\lambda) \atop (b_1,1),\dots,(b_s,1),(\xi,\mu),(\zeta,1),(\rho+\beta',\lambda),(\rho+\gamma-\alpha-\alpha',\lambda),(\rho+\gamma-\beta-\alpha',\lambda)};x^{\lambda}\right] \qquad (3.5)$$

Proof. Taking the LHS of (3.5) as $\mathcal{I}$ and using (2.19) we get

$$\mathcal{I}=\int_0^1 z^{l-1}(1-z)^{m-1}\left\{\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\ t^{\rho-1}\ {}^{\mu,\xi;\upsilon,\zeta}_{r}K_s\left(zt^{\lambda}\right)\right)(x)\right\}dz \qquad (3.6)$$

using (3.2) in (3.6) we get

$$\mathcal{I}=\int_0^1 z^{l-1}(1-z)^{m-1}\left\{x^{\rho-\alpha-\alpha'+\gamma-1}\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\dots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\dots,(b_s)_n\Gamma(\mu n+\xi)}\frac{(zx^{\lambda})^n}{(\zeta)_n}\right.$$

$$\left.\times\left[\frac{\Gamma(\rho+\lambda n)\ \Gamma(\rho+\gamma-\alpha-\alpha'-\beta+\lambda n)\ \Gamma(\rho+\beta'-\alpha'+\lambda n)}{\Gamma(\rho+\beta'+\lambda n)\ \Gamma(\rho+\gamma-\alpha-\alpha'+\lambda n)\ \Gamma(\rho+\gamma-\beta-\alpha'+\lambda n)}\right]\right\}dz \qquad (3.7)$$

Replacing a_1 by $(l+m)$ in (3.7) we get

$$\mathcal{I}=\int_0^1 z^{l-1}(1-z)^{m-1}\left\{x^{\rho-\alpha-\alpha'+\gamma-1}\sum_{n=0}^{\infty}\frac{(l+m)_n,(a_2)_n,\dots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\dots,(b_s)_n\Gamma(\mu n+\xi)}\frac{(zx^{\lambda})^n}{(\zeta)_n}\right.$$

$$\times\left[\frac{\Gamma(\rho+\lambda n)\ \Gamma(\rho+\gamma-\alpha-\alpha'-\beta+\lambda n)\,\Gamma(\rho+\beta'-\alpha'+\lambda n)}{\Gamma(\rho+\beta'+\lambda n)\ \Gamma(\rho+\gamma-\alpha-\alpha'+\lambda n)\ \Gamma(\rho+\gamma-\beta-\alpha'+\lambda n)}\right]\Bigg\}\,dz$$

Interchanging the order of integration and summation we get

$$\mathcal{I}=x^{\rho-\alpha-\alpha'+\gamma-1}\sum_{n=0}^{\infty}\frac{(l+m)_n,(a_2)_n,\ldots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\ldots,(b_s)_n\Gamma(\mu n+\xi)}\frac{(x^{\lambda})^n}{(\zeta)_n}$$
$$\times\left[\frac{\Gamma(\rho+\lambda n)\ \Gamma(\rho+\gamma-\alpha-\alpha'-\beta+\lambda n)\,\Gamma(\rho+\beta'-\alpha'+\lambda n)}{\Gamma(\rho+\beta'+\lambda n)\ \Gamma(\rho+\gamma-\alpha-\alpha'+\lambda n)\ \Gamma(\rho+\gamma-\beta-\alpha'+\lambda n)}\right]\int_0^1 z^{l+n-1}(1-z)^{m-1}\,dz$$

$$\mathcal{I}=x^{\rho-\alpha-\alpha'+\gamma-1}\sum_{n=0}^{\infty}\frac{(l+m)_n,(a_2)_n,\ldots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\ldots,(b_s)_n\Gamma(\mu n+\xi)}\frac{(x^{\lambda})^n}{(\zeta)_n}$$
$$\times\left[\frac{\Gamma(\rho+\lambda n)\ \Gamma(\rho+\gamma-\alpha-\alpha'-\beta+\lambda n)\,\Gamma(\rho+\beta'-\alpha'+\lambda n)}{\Gamma(\rho+\beta'+\lambda n)\ \Gamma(\rho+\gamma-\alpha-\alpha'+\lambda n)\ \Gamma(\rho+\gamma-\beta-\alpha'+\lambda n)}\right]\frac{\Gamma(l+n)\Gamma(m)}{\Gamma(l+m+n)}$$

$$\mathcal{I}=x^{\rho-\alpha-\alpha'+\gamma-1}\sum_{n=0}^{\infty}\frac{(l)_n,(a_2)_n,\ldots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\ldots,(b_s)_n\Gamma(\mu n+\xi)}\frac{(x^{\lambda})^n}{(\zeta)_n}$$
$$\times\left[\frac{\Gamma(\rho+\lambda n)\ \Gamma(\rho+\gamma-\alpha-\alpha'-\beta+\lambda n)\,\Gamma(\rho+\beta'-\alpha'+\lambda n)}{\Gamma(\rho+\beta'+\lambda n)\ \Gamma(\rho+\gamma-\alpha-\alpha'+\lambda n)\ \Gamma(\rho+\gamma-\beta-\alpha'+\lambda n)}\right]\frac{\Gamma(l)\Gamma(m)}{\Gamma(l+m)} \qquad (3.8)$$

Using (2.17) in (3.8), and interpreting the right hand side of (3.8), we arrive at the required result.

Theorem 3.4. Let $x>0,\alpha,\alpha',\beta,\beta',\gamma,\rho,\mu,\xi,\upsilon,\zeta\in\mathbb{C},\Re(\mu)>0$ and $\lambda>0$. Further let the constants satisfy the conditions $a_i,b_j\in\mathbb{C}$, $A_i,B_j\in\Re\,(A_i,B_j\neq 0, i=1,2,\ldots,r, j=1,2,\ldots,s.)$ and if condition (2.14) is satisfied such that $\Re(\gamma)>0,\Re(\lambda)>0,\Re(1-\gamma-\rho)<1\,min\{\Re(-\beta),\Re(\alpha+\alpha'-\gamma),\Re(\alpha+\beta'-\gamma)\}$.
then

$$B\left\{\left(I_{0-}^{\alpha,\alpha\prime,\beta,\beta\prime,\gamma}\,t^{-\gamma-\rho}\ {}_{r}^{\mu,\xi;\upsilon,\zeta}K_s\left(zt^{-\lambda}\right)\right)(x):l,m\right\}=B(l,m)\,x^{-\rho-\alpha-\alpha'}\frac{\Gamma\zeta}{\Gamma l\,\Gamma\upsilon}\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{s}\Gamma(b_j)}{\prod_{i=2}^{r}\Gamma(a_i)}$$
$$\times\,{}_{r+5}\psi_{s+5}\left[\begin{matrix}(l,1),(a_2,1),\ldots(a_r,1),(\upsilon,1),(1,1),(\rho+\alpha+\beta',\lambda),(\rho+\alpha+\alpha',\lambda),(\rho-\beta+\gamma,\lambda)\\(b_1,1),\ldots,(b_s,1),(\xi,\mu),(\zeta,1),(\rho+\alpha+\alpha'+\beta',\lambda),(\rho+\gamma,\lambda),(\rho+\alpha-\beta+\gamma,\lambda)\end{matrix};x^{-\lambda}\right] \qquad (3.9)$$

Proof. Taking the LHS of (3.9) as $\mathcal{I}$, and using (2.19) we get

$$\mathcal{I}=\int_0^1 z^{l-1}(1-z)^{m-1}\left\{\left(I_{0-}^{\alpha,\alpha\prime,\beta,\beta\prime,\gamma}\,t^{-\gamma-\rho}\ {}_{r}^{\mu,\xi;\upsilon,\zeta}K_s\left(zt^{-\lambda}\right)\right)(x)\right\}dz \qquad (3.10)$$

Using (3.4) in (3.10) we get

$$\mathcal{I}=\int_0^1 z^{l-1}(1-z)^{m-1}\Bigg\{x^{-\rho-\alpha-\alpha\prime}\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\ldots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\ldots,(b_s)_n\Gamma(\mu n+\xi)}\frac{(zx^{-\lambda})^n}{(\zeta)_n}$$
$$\times\left[\frac{\Gamma(\rho+\alpha+\alpha'+\lambda n)\ \Gamma(\rho+\alpha+\beta'+\lambda n)\,\Gamma(\rho-\beta+\gamma+\lambda n)}{\Gamma(\rho+\alpha+\alpha'+\beta'+\lambda n)\ \Gamma(\rho+\gamma+\lambda n)\ \Gamma(\rho+\alpha-\beta+\gamma+\lambda n)}\right]\Bigg\}\,dz \qquad (3.11)$$

Replacing a_1 by $(l+m)$ in (3.11) we get

$$\mathcal{I}=\int_0^1 z^{l-1}(1-z)^{m-1}\Bigg\{x^{-\rho-\alpha-\alpha\prime}\sum_{n=0}^{\infty}\frac{(l+m)_n,(a_2)_n,\ldots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\ldots,(b_s)_n\Gamma(\mu n+\xi)}\frac{(zx^{-\lambda})^n}{(\zeta)_n}$$
$$\times\left[\frac{\Gamma(\rho+\alpha+\alpha'+\lambda n)\ \Gamma(\rho+\alpha+\beta'+\lambda n)\,\Gamma(\rho-\beta+\gamma+\lambda n)}{\Gamma(\rho+\alpha+\alpha'+\beta'+\lambda n)\ \Gamma(\rho+\gamma+\lambda n)\ \Gamma(\rho+\alpha-\beta+\gamma+\lambda n)}\right]\Bigg\}\,dz$$

Interchanging the order of integration and summation we get

$$\mathcal{I}=x^{-\rho-\alpha-\alpha\prime}\sum_{n=0}^{\infty}\frac{(l+m)_n,(a_2)_n,\ldots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\ldots,(b_s)_n\Gamma(\mu n+\xi)}\frac{(x^{-\lambda})^n}{(\zeta)_n}$$
$$\times\left[\frac{\Gamma(\rho+\alpha+\alpha'+\lambda n)\ \Gamma(\rho+\alpha+\beta'+\lambda n)\,\Gamma(\rho-\beta+\gamma+\lambda n)}{\Gamma(\rho+\alpha+\alpha'+\beta'+\lambda n)\ \Gamma(\rho+\gamma+\lambda n)\ \Gamma(\rho+\alpha-\beta+\gamma+\lambda n)}\right]\int_0^1 z^{l+n-1}(1-z)^{m-1}\,dz$$

$$\mathcal{I} = x^{-\rho-\alpha-\alpha'} \sum_{n=0}^{\infty} \frac{(l+m)_n, (a_2)_n, \ldots, (a_r)_n (\upsilon)_n}{(b_1)_n, (b_2)_n, \ldots, (b_s)_n \Gamma(\mu n + \xi)} \frac{(x^{-\lambda})^n}{(\zeta)_n}$$
$$\times \left[\frac{\Gamma(\rho+\alpha+\alpha'+\lambda n)\ \Gamma(\rho+\alpha+\beta'+\lambda n)\ \Gamma(\rho-\beta+\gamma+\lambda n)}{\Gamma(\rho+\alpha+\alpha'+\beta'+\lambda n)\ \Gamma(\rho+\gamma+\lambda n)\ \Gamma(\rho+\alpha-\beta+\gamma+\lambda n)} \right] \frac{\Gamma(l+n)\Gamma(m)}{\Gamma(l+m+n)}$$
$$\mathcal{I} = x^{-\rho-\alpha-\alpha'} \sum_{n=0}^{\infty} \frac{(l)_n, (a_2)_n, \ldots, (a_r)_n (\upsilon)_n}{(b_1)_n, (b_2)_n, \ldots, (b_s)_n \Gamma(\mu n + \xi)} \frac{(x^{-\lambda})^n}{(\zeta)_n}$$
$$\times \left[\frac{\Gamma(\rho+\alpha+\alpha'+\lambda n)\ \Gamma(\rho+\alpha+\beta'+\lambda n)\ \Gamma(\rho-\beta+\gamma+\lambda n)}{\Gamma(\rho+\alpha+\alpha'+\beta'+\lambda n)\ \Gamma(\rho+\gamma+\lambda n)\ \Gamma(\rho+\alpha-\beta+\gamma+\lambda n)} \right] \frac{\Gamma(l)\Gamma(m)}{\Gamma(l+m)} \tag{3.12}$$

Using (2.17) in (3.12) and interpreting the right hand side of (3.12), we arrive at the required result.

Theorem 3.5. Let $x > 0, \alpha, \alpha', \beta, \beta', \gamma, \rho, \mu, \xi, \upsilon, \zeta \in \mathbb{C}, \Re(\mu) > 0, \delta > 1$ and $\lambda > 0$. Further let the constants satisfy the conditions $a_i, b_j \in \mathbb{C}, A_i, B_j \in \Re(A_i, B_j \neq 0, , i = 1,2, \ldots, r, j = 1,2, \ldots, s.)$and if condition (2.14) is satisfied, such that $\Re(\gamma) > 0, \Re(\lambda) > 0, \Re(s) > 0, \Re(\rho) > \max\{0, \Re(\alpha + \alpha' + \beta - \gamma), \Re(\alpha' - \beta')\}$. Then

$$P_\delta \left\{ z^{l-1} \left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\ t^{\rho-1}\ {}^{\mu,\xi;\upsilon,\zeta}_{r}K_s\left(zt^{\lambda}\right) \right)(x) : s \right\} = \{\Lambda(\delta;s)\}^l\ x^{\rho-\alpha-\alpha'+\gamma-1} \frac{\Gamma\zeta \prod_{j=1}^{s}\Gamma(b_j)}{\Gamma\upsilon \prod_{i=1}^{r}\Gamma(a_i)}$$
$$\times {}_{r+6}\psi_{s+5}\left[\begin{matrix} (a_1,1),\ldots,(a_r,1),(\upsilon,1),(1,1),(\rho,\lambda),(\rho+\gamma-\alpha-\alpha'-\beta,\lambda),(\rho+\beta'-\alpha',\lambda),(l,1) \\ (b_1,1),\ldots,(b_s,1),(\xi,\mu),(\zeta,1),(\rho+\beta',\lambda),(\rho+\gamma-\alpha-\alpha',\lambda),(\rho+\gamma-\beta-\alpha',\lambda) \end{matrix} ; \Lambda(\delta;s)x^{\lambda} \right] \tag{3.13}$$

where $\Lambda(\delta; s) = \frac{\delta-1}{\ln[1+(\delta-1)s]}$

Proof. Taking the LHS of (3.13) as $\mathcal{I}$, and using (2.12) and (2.21) we get

$$\mathcal{I} = \int_0^\infty [1+(\delta-1)s]^{-\frac{z}{\delta-1}}\ z^{l-1} \left\{ \left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\ t^{\rho-1} \sum_{n=0}^{\infty} \frac{(a_1)_n, (a_2)_n, \ldots, (a_r)_n (\upsilon)_n}{(b_1)_n, (b_2)_n, \ldots, (b_s)_n \Gamma(\mu n+\xi)} \frac{(zt^{\lambda})^n}{(\zeta)_n} \right)(x) \right\} dz$$

Applying (3.2) and interchanging the order of summation and integration, we get

$$\mathcal{I} = x^{\rho-\alpha-\alpha'+\gamma-1} \sum_{n=0}^{\infty} \frac{(a_1)_n, (a_2)_n, \ldots, (a_r)_n (\upsilon)_n}{(b_1)_n, (b_2)_n, \ldots, (b_s)_n \Gamma(\mu n + \xi)} \frac{(x^{\lambda})^n}{(\zeta)_n}$$
$$\left[\frac{\Gamma(\rho+\lambda n)\ \Gamma(\rho+\gamma-\alpha-\alpha'-\beta+\lambda n)\ \Gamma(\rho+\beta'-\alpha'+\lambda n)}{\Gamma(\rho+\beta'+\lambda n)\ \Gamma(\rho+\gamma-\alpha-\alpha'+\lambda n)\ \Gamma(\rho+\gamma-\beta-\alpha'+\lambda n)} \right] \int_0^\infty [1+(\delta-1)s]^{-\frac{z}{\delta-1}}\ z^{l+n-1} dz$$

Using (2.22), we get

$$\mathcal{I} = x^{\rho-\alpha-\alpha'+\gamma-1} \sum_{n=0}^{\infty} \frac{(a_1)_n, (a_2)_n, \ldots, (a_r)_n (\upsilon)_n}{(b_1)_n, (b_2)_n, \ldots, (b_s)_n \Gamma(\mu n + \xi)} \frac{(x^{\lambda})^n}{(\zeta)_n}$$
$$\left[\frac{\Gamma(\rho+\lambda n)\ \Gamma(\rho+\gamma-\alpha-\alpha'-\beta+\lambda n)\ \Gamma(\rho+\beta'-\alpha'+\lambda n)}{\Gamma(\rho+\beta'+\lambda n)\ \Gamma(\rho+\gamma-\alpha-\alpha'+\lambda n)\ \Gamma(\rho+\gamma-\beta-\alpha'+\lambda n)} \right] \{\Lambda(\delta;s)\}^{l+n}\Gamma(l+n)$$

where $\Lambda(\delta; s) = \frac{\delta-1}{\ln[1+(\delta-1)s]}$

$$\mathcal{I} = x^{\rho-\alpha-\alpha'+\gamma-1} \frac{\Gamma\zeta \prod_{j=1}^{s}\Gamma(b_j)}{\Gamma\upsilon \prod_{i=1}^{r}\Gamma(a_i)}\ {}_{r+2}\psi_{s+2} \left\{ \begin{matrix} (a_1,1),\ldots(a_r,1),(1,1),(\upsilon,1); \\ (b_1,1),\ldots(b_s,1),(\xi,\mu),(\zeta,1); \end{matrix}\ x^{\lambda} \right\}$$
$$\left[\frac{\Gamma(\rho+\lambda n)\ \Gamma(\rho+\gamma-\alpha-\alpha'-\beta+\lambda n)\ \Gamma(\rho+\beta'-\alpha'+\lambda n)}{\Gamma(\rho+\beta'+\lambda n)\ \Gamma(\rho+\gamma-\alpha-\alpha'+\lambda n)\ \Gamma(\rho+\gamma-\beta-\alpha'+\lambda n)} \right] \{\Lambda(\delta;s)\}^{l+n}\Gamma(l+n) \tag{3.14}$$

Interpreting the right hand side of (3.14), we arrive at the required result.

Theorem 3.6. Let $x > 0, \alpha, \alpha', \beta, \beta', \gamma, \rho, \mu, \xi, \upsilon, \zeta \in \mathbb{C}, \Re(\mu) > 0, \delta > 1$ and $\lambda > 0$.Further let the constants satisfy the conditions $a_i, b_j \in \mathbb{C}, A_i, B_j \in \Re\ (A_i, B_j \neq 0, i = 1,2, \ldots, r, j = 1,2, \ldots, s.)$ and if condition (2.14) is satisfied such that $\Re(\gamma) > 0, \Re(\lambda) > 0, \Re(s) > 0, \Re(1 - \gamma - \rho) < 1 + \min\{\Re(-\beta), \Re(\alpha + \alpha' - \gamma), \Re(\alpha + \beta' - \gamma)\}$.
Then

$$P_\delta \left\{ z^{l-1} \left(I_{0-}^{\alpha,\alpha',\beta,\beta',\gamma}\ t^{-\gamma-\rho}\ {}^{\mu,\xi;\upsilon,\zeta}_{r}K_s\left(zt^{-\lambda}\right) \right)(x) : s \right\} = \{\Lambda(\delta;s)\}^l\ x^{-\rho-\alpha-\alpha'} \frac{\Gamma\zeta \prod_{j=1}^{s}\Gamma(b_j)}{\Gamma\upsilon \prod_{i=1}^{r}\Gamma(a_i)}$$

$$\times\ {}_{r+6}\psi_{s+5}\left[\begin{matrix}(a_1,1),\dots,(a_r,1),(\upsilon,1),(1,1),(\rho+\alpha+\beta',\lambda),(\rho+\alpha+\alpha',\lambda),(\rho-\beta+\gamma,\lambda),(l,1)\\(b_1,1),\dots,(b_s,1),(\xi,\mu),(\zeta,1),(\rho+\alpha+\alpha'+\beta',\lambda),(\rho+\gamma,\lambda),(\rho+\alpha-\beta+\gamma,\lambda)\end{matrix};\Lambda(\delta;s)x^{-\lambda}\right] \quad (3.15)$$

where $\Lambda(\delta;s)=\frac{\delta-1}{\ln[1+(\delta-1)s]}$

Proof. Taking the LHS of (3.15) as $\mathcal{I}$, and using (2.12) and (2.21) we get

$$\mathcal{I}=\int_0^\infty[1+(\delta-1)s]^{-\frac{z}{\delta-1}}z^{l-1}\left\{\left(I_{0-}^{\alpha,\alpha',\beta,\beta',\gamma}t^{-\gamma-\rho}\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\dots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\dots,(b_s)_n\Gamma(\mu n+\xi)}\frac{(zt^{-\lambda})^n}{(\zeta)_n}\right)(x)\right\}dz$$

Applying (3.4) and interchanging the order of summation and integration, we get

$$\mathcal{I}=x^{-\rho-\alpha-\alpha'}\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\dots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\dots,(b_s)_n\Gamma(\mu n+\xi)}\frac{(x^{-\lambda})^n}{(\zeta)_n}$$

$$\times\left[\frac{\Gamma(\rho+\alpha+\alpha'+\lambda n)\ \Gamma(\rho+\alpha+\beta'+\lambda n)\ \Gamma(\rho-\beta+\gamma+\lambda n)}{\Gamma(\rho+\alpha+\alpha'+\beta'+\lambda n)\ \Gamma(\rho+\gamma+\lambda n)\ \Gamma(\rho+\alpha-\beta+\gamma+\lambda n)}\right]\int_0^\infty[1+(\delta-1)s]^{-\frac{z}{\delta-1}}z^{l+n-1}dz$$

Using (2.22), we get

$$\mathcal{I}=x^{-\rho-\alpha-\alpha'}\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\dots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\dots,(b_s)_n\Gamma(\mu n+\xi)}\frac{(x^{-\lambda})^n}{(\zeta)_n}$$

$$\times\left[\frac{\Gamma(\rho+\alpha+\alpha'+\lambda n)\ \Gamma(\rho+\alpha+\beta'+\lambda n)\ \Gamma(\rho-\beta+\gamma+\lambda n)}{\Gamma(\rho+\alpha+\alpha'+\beta'+\lambda n)\ \Gamma(\rho+\gamma+\lambda n)\ \Gamma(\rho+\alpha-\beta+\gamma+\lambda n)}\right]\{\Lambda(\delta;s)\}^{l+n}\Gamma(l+n)$$

where $\Lambda(\delta;s)=\frac{\delta-1}{\ln[1+(\delta-1)s]}$

$$\mathcal{I}=x^{-\rho-\alpha-\alpha'}\frac{\Gamma\zeta\ \prod_{j=1}^{s}\Gamma(b_j)}{\Gamma\upsilon\ \prod_{i=1}^{r}\Gamma(a_i)}\ {}_{r+2}\psi_{s+2}\left\{\begin{matrix}(a_1,1),\dots(a_r,1),(1,1),(\upsilon,1);\\(b_1,1),\dots(b_s,1),(\xi,\mu),(\zeta,1);\end{matrix}x^{-\lambda}\right\}$$

$$\times\left[\frac{\Gamma(\rho+\alpha+\alpha'+\lambda n)\ \Gamma(\rho+\alpha+\beta'+\lambda n)\Gamma(\rho-\beta+\gamma+\lambda n)}{\Gamma(\rho+\alpha+\alpha'+\beta'+\lambda n)\Gamma(\rho+\gamma+\lambda n)\Gamma(\rho+\alpha-\beta+\gamma+\lambda n)}\right]\{\Lambda(\delta;s)\}^{l+n}\Gamma(l+n) \quad (3.16)$$

Interpreting the right hand side of (3.16), we arrive at the required result.

Theorem 3.7. Let $x>0,\alpha,\alpha',\beta,\beta',\gamma,\rho,\mu,\xi,\upsilon,\zeta\in\mathbb{C},\Re(\mu)>0,$ and $\lambda>0$. Further let the constants satisfy the conditions $a_i,b_j\in\mathbb{C},A_i,B_j\in\Re\ (A_i,B_j\neq 0,i=1,2,\dots,r,j=1,2,\dots,s.)$ and if condition (2.14) is satisfied such that $\Re(\gamma)>0,\Re(\lambda)>0,\Re(s)>0,\Re(\rho)>\max\{0,\Re(\alpha+\alpha'+\beta-\gamma),\Re(\alpha'-\beta')\}$.then

$$L\left\{z^{l-1}\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}t^{\rho-1}\ {}_{r}^{\mu,\xi;\upsilon,\zeta}K_s\left(zt^{\lambda}\right)\right)(x)\right\}=\frac{x^{\rho-\alpha-\alpha'+\gamma-1}}{s^l}\frac{\Gamma\zeta}{\Gamma\upsilon}\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{s}\Gamma(b_j)}{\prod_{i=1}^{r}\Gamma(a_i)}$$

$$\times\ {}_{r+6}\psi_{s+5}\left[\begin{matrix}(a_1,1),\dots,(a_r,1),(\upsilon,1),(1,1),(l,1).(\rho,\lambda),(\rho+\gamma-\alpha-\alpha'-\beta,\lambda),(\rho+\beta'-\alpha',\lambda)\\(b_1,1),\dots,(b_s,1),(\xi,\mu),(\zeta,1),(\rho+\beta',\lambda),(\rho+\gamma-\alpha-\alpha',\lambda),(\rho+\gamma-\beta-\alpha',\lambda)\end{matrix};\frac{x^{\lambda}}{s}\right] \quad (3.17)$$

Proof. Taking the LHS of (3.17) as $\mathcal{I}$, and using (2.12) and (2.23), we get

$$\mathcal{I}=\int_0^\infty z^{l-1}e^{-sz}\left\{\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}t^{\rho-1}\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\dots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\dots,(b_s)_n\Gamma(\mu n+\xi)}\frac{(zt^{\lambda})^n}{(\zeta)_n}\right)(x)\right\}dz$$

Applying (3.2) and interchanging the order of integration and summation we get

$$\mathcal{I}=x^{\rho-\alpha-\alpha'+\gamma-1}\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\dots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\dots,(b_s)_n\Gamma(\mu n+\xi)}\frac{(x^{\lambda})^n}{(\zeta)_n}$$

$$\left[\frac{\Gamma(\rho+\lambda n)\ \Gamma(\rho+\gamma-\alpha-\alpha'-\beta+\lambda n)\ \Gamma(\rho+\beta'-\alpha'+\lambda n)}{\Gamma(\rho+\beta'+\lambda n)\ \Gamma(\rho+\gamma-\alpha-\alpha'+\lambda n)\ \Gamma(\rho+\gamma-\beta-\alpha'+\lambda n)}\right]\int_0^\infty z^{l+n-1}e^{-sz}\,dz$$

$$\mathcal{I}=\frac{x^{\rho-\alpha-\alpha'+\gamma-1}}{s^l}\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\dots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\dots,(b_s)_n\Gamma(\mu n+\xi)}\frac{(x^{\lambda})^n}{(\zeta)_n}$$

$$\times\left[\frac{\Gamma(\rho+\lambda n)\ \Gamma(\rho+\gamma-\alpha-\alpha'-\beta+\lambda n)\ \Gamma(\rho+\beta'-\alpha'+\lambda n)}{\Gamma(\rho+\beta'+\lambda n)\ \Gamma(\rho+\gamma-\alpha-\alpha'+\lambda n)\ \Gamma(\rho+\gamma-\beta-\alpha'+\lambda n)}\right]\frac{\Gamma(l+n)}{s^n} \quad (3.18)$$

Using (2.17) in (3.18), and interpreting the right hand side of (3.18), we arrive at the required result.

Theorem 3.8. Let $x > 0, \alpha, \alpha', \beta, \beta', \gamma, \rho, \mu, \xi, \upsilon, \zeta \in \mathbb{C}, \Re(\mu) > 0$, and $\lambda > 0$. Further let the constants satisfy the conditions $a_i, b_j \in \mathbb{C}, A_i, B_j \in \Re\ (A_i, B_j \neq 0, i = 1,2, \ldots, r, j = 1,2, \ldots, s)$and if condition (2.14) is satisfied such that $\Re(\gamma) > 0, \Re(\lambda) > 0, \Re(s) > 0, \Re(1 - \gamma - \rho) < 1 + min\{\Re(-\beta), \Re(\alpha + \alpha' - \gamma), \Re(\alpha + \beta' - \gamma)\}$.Then

$$L\left\{z^{l-1}\left(I_{0-}^{\alpha,\alpha\prime,\beta,\beta\prime,\gamma}\, t^{-\gamma-\rho}\ {}^{\mu,\xi;\upsilon,\zeta}{}_{r}K_s\left(zt^{-\lambda}\right)\right)(x)\right\} = \frac{x^{-\rho-\alpha-\alpha\prime}}{s^l}\frac{\Gamma\zeta}{\Gamma\upsilon}\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{s}\Gamma(b_j)}{\prod_{i=1}^{r}\Gamma(a_i)}$$

$$\times\ {}_{r+6}\psi_{s+5}\left[\begin{matrix}(a_1,1),\ldots,(a_r,1),(\upsilon,1),(1,1),(\rho+\alpha+\beta',\lambda),(\rho+\alpha+\alpha',\lambda),(\rho-\beta+\gamma,\lambda),(l,1)\\(b_1,1),\ldots,(b_s,1),(\xi,\mu),(\zeta,1),(\rho+\alpha+\alpha'+\beta',\lambda),(\rho+\gamma,\lambda),(\rho+\alpha-\beta+\gamma,\lambda)\end{matrix};\frac{x^{-\lambda}}{s}\right] \quad (3.19)$$

Proof. Taking the LHS of (3.19) as $\mathcal{I}$, and using (2.12) and (2.23), we get

$$\mathcal{I} = \int_0^{\infty} z^{l-1}e^{-sz}\left\{\left(I_{0-}^{\alpha,\alpha\prime,\beta,\beta\prime,\gamma}\, t^{-\gamma-\rho}\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\ldots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\ldots,(b_s)_n\Gamma(\mu n+\xi)}\frac{(zt^{-\lambda})^n}{(\zeta)_n}\right)(x)\right\}dz$$

Applying (3.4) and interchanging the order of integration and summation we get

$$\mathcal{I} = x^{-\rho-\alpha-\alpha\prime}\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\ldots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\ldots,(b_s)_n\Gamma(\mu n+\xi)}\frac{(x^{-\lambda})^n}{(\zeta)_n}$$

$$\times\left[\frac{\Gamma(\rho+\alpha+\alpha'+\lambda n)\ \Gamma(\rho+\alpha+\beta'+\lambda n)\,\Gamma(\rho-\beta+\gamma+\lambda n)}{\Gamma(\rho+\alpha+\alpha'+\beta'+\lambda n)\ \Gamma(\rho+\gamma+\lambda n)\ \Gamma(\rho+\alpha-\beta+\gamma+\lambda n)}\right]\int_0^{\infty} z^{l+n-1}e^{-sz}\,dz$$

$$\mathcal{I} = \frac{x^{-\rho-\alpha-\alpha\prime}}{s^l}\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\ldots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\ldots,(b_s)_n\Gamma(\mu n+\xi)}\frac{(x^{-\lambda})^n}{(\zeta)_n}$$

$$\times\left[\frac{\Gamma(\rho+\alpha+\alpha'+\lambda n)\ \Gamma(\rho+\alpha+\beta'+\lambda n)\,\Gamma(\rho-\beta+\gamma+\lambda n)}{\Gamma(\rho+\alpha+\alpha'+\beta'+\lambda n)\ \Gamma(\rho+\gamma+\lambda n)\ \Gamma(\rho+\alpha-\beta+\gamma+\lambda n)}\right]\frac{\Gamma(l+n)}{s^n} \quad (3.20)$$

Using (2.17) in (3.20), and interpreting the right hand side of (3.20), we arrive at the required result.

Theorem 3.9. Let $x > 0, \alpha, \alpha', \beta, \beta', \gamma, \rho, \mu, \xi, \upsilon, \zeta, \eta, \sigma \in \mathbb{C}, \Re(\mu) > 0$, and $\lambda > 0$. Further let the constants satisfy the conditions $a_i, b_j \in \mathbb{C}, A_i, B_j \in \Re\ (A_i, B_j \neq 0, i = 1,2, \ldots, r, j = 1,2, \ldots, s)$ and if condition (2.14) is satisfied such that $\Re(\gamma) > 0, \Re(\lambda) > 0, \Re(s) > 0, \Re(\rho) > \max\{0, \Re(\alpha + \alpha' + \beta - \gamma), \Re(\alpha' - \beta')\}$.Then the following Whittaker transform formula holds:

$$\int_0^{\infty} z^{\tau-1}e^{-\frac{z}{2}}\left[W_{\sigma,\eta}\left(I_{0+}^{\alpha,\alpha\prime,\beta,\beta\prime,\gamma}\, t^{\rho-1}\ {}^{\mu,\xi;\upsilon,\zeta}{}_{r}K_s\left(zt^{\lambda}\right)\right)(x)\right]dz = x^{\rho-\alpha-\alpha\prime+\gamma-1}\frac{\Gamma\zeta}{\Gamma\upsilon}\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{s}\Gamma(b_j)}{\prod_{i=1}^{r}\Gamma(a_i)}$$

$$\times\ {}_{r+7}\psi_{s+6}\left[\begin{matrix}(a_1,1),\ldots,(a_r,1),(\upsilon,1),(1,1),(\rho,\lambda),(\rho+\gamma-\alpha-\alpha'-\beta,\lambda),(\rho+\beta'-\alpha',\lambda),(\mathrm{A}+\eta,1),(\mathrm{A}-\eta,1)\\(b_1,1),\ldots,(b_s,1),(\xi,\mu),(\zeta,1),(\rho+\beta',\lambda),(\rho+\gamma-\alpha-\alpha',\lambda),(\rho+\gamma-\beta-\alpha',\lambda),(\mathrm{A}-\sigma,1)\end{matrix};x^{\lambda}\right] \quad (3.21)$$

where $\mathrm{A} = \tau + \frac{1}{2}$

Proof. Taking the LHS of (3.21) as $\mathcal{I}$, and using (2.12) and (2.26), we get

$$\mathcal{I} = \int_0^{\infty} z^{\tau-1}e^{-\frac{z}{2}}W_{\sigma,\eta}\left[\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\, t^{\rho-1}\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\ldots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\ldots,(b_s)_n\Gamma(\mu n+\xi)}\frac{(zt^{\lambda})^n}{(\zeta)_n}\right)(x)\right]dz$$

Applying (3.2), we get

$$\mathcal{I} = \int_0^{\infty} z^{\tau+n-1}e^{-\frac{z}{2}}W_{\sigma,\eta}\left[x^{\rho-\alpha-\alpha'+\gamma-1}\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\ldots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\ldots,(b_s)_n\Gamma(\mu n+\xi)}\frac{(x^{\lambda})^n}{(\zeta)_n}\right.$$

$$\left.\frac{\Gamma(\rho+\lambda n)\ \Gamma(\rho+\gamma-\alpha-\alpha'-\beta+\lambda n)\,\Gamma(\rho+\beta'-\alpha'+\lambda n)}{\Gamma(\rho+\beta'+\lambda n)\ \Gamma(\rho+\gamma-\alpha-\alpha'+\lambda n)\ \Gamma(\rho+\gamma-\beta-\alpha'+\lambda n)}\right]dz$$

$$\mathcal{I} = x^{\rho-\alpha-\alpha'+\gamma-1}\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\ldots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\ldots,(b_s)_n\Gamma(\mu n+\xi)}\frac{(x^{\lambda})^n}{(\zeta)_n}$$

$$\frac{\Gamma(\rho+\lambda n)\ \Gamma(\rho+\gamma-\alpha-\alpha'-\beta+\lambda n)\,\Gamma(\rho+\beta'-\alpha'+\lambda n)\Gamma(\mathrm{A}+\eta+n)\Gamma(\mathrm{A}-\eta+n)}{\Gamma(\rho+\beta'+\lambda n)\ \Gamma(\rho+\gamma-\alpha-\alpha'+\lambda n)\ \Gamma(\rho+\gamma-\beta-\alpha'+\lambda n)\Gamma(\mathrm{A}-\sigma+n)} \quad (3.22)$$

where $\mathrm{A} = \tau + \frac{1}{2}$

Using (2.17) in (3.22), and interpreting the right hand side of (3.22), we arrive at the required result.

Theorem 3.10. Let $x > 0, \alpha, \alpha', \beta, \beta', \gamma, \rho, \mu, \xi, \upsilon, \zeta \in \mathbb{C}, \Re(\mu) > 0$, and $\lambda > 0$. Further let the constants satisfy the conditions $a_i, b_j \in \mathbb{C}, A_i, B_j \in \Re\ (A_i, B_j \neq 0, i = 1,2, \ldots, r, j = 1,2, \ldots, s)$ and if condition(2.14) is satisfied such that $\Re(\gamma) > 0, \Re(\lambda) > 0, \Re(s) > 0, \Re(1 - \gamma - \rho) < 1 + min\{\Re(-\beta), \Re(\alpha + \alpha' - \gamma), \Re(\alpha + \beta' - \gamma)\}$.Then

$$\int_0^\infty z^{\tau-1}e^{-\frac{z}{2}}\left[W_{\sigma,\eta}\left(I_{0-}^{\alpha,\alpha',\beta,\beta',\gamma}\, t^{-\gamma-\rho}\ {}^{\mu,\xi;\upsilon,\zeta}_{\ r}K_s\left(zt^{-\lambda}\right)\right)(x)\right]dz = x^{-\rho-\alpha-\alpha'}\frac{\Gamma\zeta}{\Gamma\upsilon}\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{s}\Gamma(b_j)}{\prod_{i=1}^{r}\Gamma(a_i)}$$
$$\times\ {}_{r+7}\psi_{s+6}\left[\begin{matrix}(a_1,1),\dots,(a_r,1),(\upsilon,1),(1,1),(\rho+\alpha+\alpha',\lambda),(\rho+\alpha+\beta',\lambda),(\rho-\beta+\gamma,\lambda)\\(b_1,1),\dots,(b_s,1),(\xi,\mu),(\zeta,1),(\rho+\alpha+\alpha'+\beta',\lambda),(\rho+\gamma,\lambda),(\rho+\alpha-\beta+\gamma,\lambda)\end{matrix}\right.$$
$$\left.\begin{matrix}(\mathrm{A}+\eta,1),(\mathrm{A}-\eta,1)\\(\mathrm{A}-\sigma,1)\end{matrix};x^{\lambda}\right] \quad (3.23)$$

where $\mathrm{A}=\tau+\frac{1}{2}$

Proof. Taking the LHS of (3.23) as $\mathcal{I}$, and using (2.12) and (2.26), we get

$$\mathcal{I}=\int_0^\infty z^{\tau-1}e^{-\frac{z}{2}}W_{\sigma,\eta}\left[\left(I_{0-}^{\alpha,\alpha',\beta,\beta',\gamma}\, t^{-\gamma-\rho}\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\dots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\dots,(b_s)_n\Gamma(\mu n+\xi)}\frac{\left(zt^{-\lambda}\right)^n}{(\zeta)_n}\right)(x)\right]dz$$

Applying (3.4), we get

$$\mathcal{I}=\int_0^\infty z^{\tau+n-1}e^{-\frac{z}{2}}W_{\sigma,\eta}\left[x^{-\rho-\alpha-\alpha'}\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\dots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\dots,(b_s)_n\Gamma(\mu n+\xi)}\frac{(x^{-\lambda})^n}{(\zeta)_n}\right.$$
$$\left.\frac{\Gamma(\rho+\alpha+\alpha'+\lambda n)\ \Gamma(\rho+\alpha+\beta'+\lambda n)\,\Gamma(\rho-\beta+\gamma+\lambda n)}{\Gamma(\rho+\alpha+\alpha'+\beta'+\lambda n)\ \Gamma(\rho+\gamma+\lambda n)\ \Gamma(\rho+\alpha-\beta+\gamma+\lambda n)}\right]$$

$$\mathcal{I}=x^{-\rho-\alpha-\alpha'}\sum_{n=0}^{\infty}\frac{(a_1)_n,(a_2)_n,\dots,(a_r)_n(\upsilon)_n}{(b_1)_n,(b_2)_n,\dots,(b_s)_n\Gamma(\mu n+\xi)}\frac{(x^{-\lambda})^n}{(\zeta)_n}$$
$$\frac{\Gamma(\rho+\alpha+\alpha'+\lambda n)\ \Gamma(\rho+\alpha+\beta'+\lambda n)\,\Gamma(\rho-\beta+\gamma+\lambda n)\Gamma(\mathrm{A}+\eta+n)\Gamma(\mathrm{A}-\eta+n)}{\Gamma(\rho+\alpha+\alpha'+\beta'+\lambda n)\ \Gamma(\rho+\gamma+\lambda n)\ \Gamma(\rho+\alpha-\beta+\gamma+\lambda n)\Gamma(\mathrm{A}-\sigma+n)} \quad (3.24)$$

where $\mathrm{A}=\tau+\frac{1}{2}$

Using (2.17) in (3.24), and interpreting the right hand side of (3.24), we arrive at the required result.

4. CONCLUSION

In this paper we presented the different generalized theorems associated with the generalized integral operators given by Marichev-Saigo Maeda. The main fractional operators operated on the generalized K-function given in section III, are quite general in nature and can be specialized to yield the large number of simpler and different results. We also employed certain integral transforms on the results obtained from the integrals and presented some more image formulas. The main results may find the large number of applications in various fields of applicable mathematical analysis.

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