EFFECTS OR SURFACE TENSION ON THE STABILITY OF TWO SUPERPOSED VISCO-ELASTIC FLUIDS IN A MAGNETIC FIELD

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Abstract

For the viscoelastic fluid, a considerable amount of work has been done to study the stability of a shear flow past a rigid surface, to investigate the nature of the eigenspectrum for the growth rate. In most studies, the viscoelastic fluid is described either by the upper convected Maxwell UCM model or the Oldroyd-B model, wherein the polymer chains are treated as the elastic dumbbells. Viscoelastic fluid is a fluid which has characteristics both of viscous and elastic. Because of its special characteristics, many researchers conduct their research to observe this fluid.

Introduction

Consider the motion of an incompressible, infmitely conducting Newtonian and Rivlin-Ericksen viscoelastic fluids in porous medium in the presence of a uniform horizontal magnetic $\vec{H}(H,0,0)$. Let $\vec{h}(h_x,h_y,h_z)$ denote the perturbation in the magnetic field, then the linearized perturbation equations are

$$\frac{\rho}{\epsilon}\frac{\partial\vec{v}}{\partial t} = -\nabla\delta p + \vec{g}\delta\rho + \frac{\mu_e}{4\pi}\left(\nabla\times\vec{h}\right)\times\vec{H} - \frac{\rho}{k_1}\left(\upsilon+\upsilon'\frac{\partial}{\partial t}\right)\vec{v},$$

$$\nabla\cdot\vec{h} = 0,$$

$$\epsilon\frac{\partial\vec{h}}{\partial t} = \nabla\times\left(\vec{v}\times\vec{H}\right),$$

$$\left[1+\frac{\epsilon}{k_1}\upsilon_1'\alpha_1\right]n^2 + \frac{\epsilon}{k_1}\left[\upsilon_2\alpha_2+\upsilon_1\alpha_1\right]n + \left[2k_x^2V_A^2 - gk(\alpha_2-\alpha_1)\right] = 0,$$

The majority of previous studies for fluid flow behaviour and heat transfer in curved tubes/ channels are confined to Newtonian fluids. Very few attempts have been made to understand the flow behaviour of non-Newtonian fluids in curved tubes, in spite of the important applications of these non-Newtonian fluids in the field of polymers, biochemical and biomedical areas.

TECHNIQUES OF STABILITY THEORY

The concept of stability theory is so familiar to fluid dynamicists that any introduction may seem superfluous. Yet, for a better understanding of what has been done in the present work, some discussion of the basic ideas related to the theory of stability appears necessary before proceeding to the problems, investigated in the present work. The subject of stability theory is of considerable importance because of its relevance to physical situations such as the convective instability in stars, heating of a solar corona, stability of stellar interiors in magnetic stars, stability of highly ionized plasma surrounded by a slightly ionized cold gas, stability of the system where air in blown over mercury, thermal convective instability in stellar atmosphere, stability of tangential discontinuities in the solar wind, stability of streams of charged particles emanating from the sun and excitation of water waves by the wind. Thus, the subject of fluid mechanics in general and the stability theory in particular have received a great importance during the last few decades. Many physical phenomena are solved successfully with the help of this topic. The more we try to make our mathematical model physically realistic, the more the problem becomes mathematically difficult to solve. To be specific, the compressibility of air, its thermally conducting nature and some other factors introduce complications in the study of mathematically model for any atmosphere study. One is bound to assume certain approximations and assumptions regarding the nature of the fluid and the flow boundaries. These approximations and assumptions should be such that not only they simplify mathematical formulation of the problem, but should also agree considerable with the physical requirement of the problem and in this way they help us in providing the best representation to the physical phenomena under investigation. With the help of stability theory the question raised above can successfully be studied. While solving the problems on stability theory we always keep in mind that

(1) The magnitude of the disturbances should be as small as we please, it should never be zero.

(2) The result can be interpreted in terms of some realizable experiments if at all the theory in intended to account for the fluctuations which do always occur in nature.

Prior to proceeding to the stability investigations related to a physical system, we must have an idea as to an equilibrium configuration (not necessarily static) behaves with respect to small disturbances, a system is subjected to. The system is said to be stable with respect to a particular disturbance if the disturbance dies down gradually. But, if the disturbance grows in amplitude with time and the system never returns to its original position it is stable with respect to all perturbations to which it could be subjected. Even if the system is unstable for one special mode of disturbance, it is said to be unstable.

The mathematical formulation of the stability theory proceeds from the nonlinear partial differential equations. The unknown quantities are functions of three space coordinates and time, and are subject to some boundary conditions. Certain special permanent type solutions of such general problems, called the basic solutions, are usually of particular interest. The simplest among these permanent type solutions are the stationary solutions, we superimpose infinitesimally small perturbations on the basic solutions so that the perturbed state is defined as the basic state plus the perturbations. If this perturbed solution goes on departing from the basic solution, the system is said to be unstable and on the other hand, if the perturbed solution approaches the theoretical solution, as the time passes, the system is said to be stable. We shell then pass from stable state to unstable state when the particular parameter takes a certain critical value. We say that instability sets in at this value of the chosen parameter. This value of the chosen parameter defines the marginal state. Thus the marginal state is the locus, which separates stable and unstable states and occurs when there exist some perturbations whose amplitude remains constant with the time, while the amplitude of others tend to zero in course of time, i.e. the marginal stability is the state of neutral stability if at the onset of instability a stationary pattern of motions prevails, then one says that the Principle of Exchange of Stabilities (PES) is valid at the marginal state and if at the onset of instability oscillatory motions prevails then we have a case of over stability.

Viscoelastic fluid flow in cross-slots

A combination of experimental investigations and numerical simulations have been carried out by Schoonen et al in order to study the extensional rheology of polymeric solutions in the (10mm × 20mm) cross-slot geometry with (20mm radius) rounded corners. In the experiments, Laser Doppler anemometry (LDA) and flow induced birefringence (FIB) were utilised for measuring velocity field and axial integrated stresses, respectively. The numerical simulations were separated into two parts: the 3D velocity field was obtained from finite element calculations with the viscous Carreau model and the viscoelastic stresses were determined with a four mode Giesekus and Phan-Thien-Tanner (PTT) model using a streamline integration technique. The local velocities and stresses obtained from numerical simulations illustrate a good agreement with those from experimental measurements. This was followed by several experimental studies and numerical simulations to study the appearance of a purely-elastic instability of extensional flow through the cross-slot geometry.

The steady two-dimensional boundary layer equations governing the flow are

$$u_x + v_y = 0 \tag{1.1}$$

$$uu_x + vu_y = \nu u_{yy} \quad \lambda_1 (uu_{yy})_x + u_y v_{yy} + vu_{yyy} \tag{1.2}$$

$$\rho C_p (uT_x + vT_y) = kT_{yy} + Q (T \quad T_¥) : \tag{1.3}$$

The boundary conditions for the problem when x 0 are:

$$u== Ax(A > 0);\ v = v_w;\ T = T_w \text{ or } \quad kT_y = q_w \text{ at } y = 0;$$

$$u\ !\ 0;\ T\ !\ T_¥ \text{ as } y\ !\ ¥: \tag{1.4}$$

Since the problem is parabolic, the velocity u and the temperature T have to be pre-scribed at certain value of $x = x_0(x_0 < 0)$ which are given by

$$u(x_0;\ y) = 0;\ T(x_0;\ y) = T_¥: \tag{1.5}$$

Here x and y are distances along and perpendicular to the surface, respectively, u and v are the components of the velocity along the x-and y-directions, respectively, T is the temperature, Q is the source or sink parameter, k is the thermal conductivity, λ is the viscoelastic parameter, ρ is the density, ν is the kinematic viscosity, C_p is the speci c heat at a constant pressure, q_w is the surface heat ux, the subscripts w and ¥ denote conditions at the surface and in the free stream, respectively.

The partial differential equations (1:1)-(1:3) under conditions (1:4) and (1.5) admit similarity solutions when the surface velocity $u_w = Ax(A > 0)$ and using the following similarity transformations

$$\eta = \frac{}{\nu} \quad y;\ u = Ax f^{0}(\eta);\ v = (A\nu)^{1=2} f(\eta);\ T \quad T_¥ = (T_w \quad T_¥)g(\eta);$$

$$T_w \quad T_¥ = (T_{w0} \quad T_¥) \frac{x}{L}^{b} \frac{}{A};\ T \quad T_¥ = \frac{q_w}{k} \frac{x}{L}^{b} \frac{\nu}{A}^{1=2} G(\eta);$$

$$\frac{x}{}^{b} \quad \frac{}{sC_p} \quad \frac{}{Q\nu} \quad \frac{}{(\nu)_w}$$

$$\lambda = \lambda_1 \; \nu \; ; \; q_w = q_{w0} \; L \quad ; Pr = \; k \quad ; s = kA \; ; \; \alpha = \quad (A\nu)^{1=2} \quad ; \tag{1.6}$$

To apply the method of homotopy analysis, we choose the initial guesses and auxiliary linear operators as follows

$$f_0(\eta) = \alpha + 1 \quad e^{h} \; ; \; g_0(\eta) = e^{h} \; ; \; G_0(\eta) = e^{h} :$$

$$L_f = \frac{d^3 f}{d\eta^3} \quad \frac{d f}{d\eta} ; \; L_g = \frac{d^2 g}{d\eta^2} \; g ; \; L_G = \quad \frac{d^2 G}{d\eta^2} \; G ;$$

so that its solutions are simpler to evaluate analytically. They are given as

$$L_f \; C_1 + C_2 e^{h} + C_3 e^{h} \quad = 0 ; \; L_g \; C_4 e^{h} \; + C_5 e^{h} \quad = 0 ;$$

$$L_G \; C_6 e \; + C_7 e \quad = 0 ;$$

where C_i are arbitrary constants and can be obtained using (1:7)-(1:9).

To obtain the HAM solution for the eqs. (1:7)-(1:9), let $\gamma 2 [0; 1]$ be an embedding parameter and c_f, c_g and c_G are the basic convergence control parameters. Then the zeroth order deformation equation and the non-linear operators become,

$$(1 \quad \gamma) L_X \, [X\,(\eta; \gamma) \quad X_0(\eta)] = \gamma c_X \, \mathbf{N}_X \, [X\,(\eta; \gamma)]; \tag{1.7}$$

$$\begin{aligned} \mathcal{N}_g[g(\eta,\gamma), f(\eta,\gamma)] &= \frac{\partial^2 g(\eta,\gamma)}{\partial \eta^2} + Pr\left(f(\eta,\gamma)\frac{\partial g(\eta,\gamma)}{\partial \eta}\right) \\ &- g(\eta,\gamma)\left(Pr\beta\frac{\partial f(\eta,\gamma)}{\partial \eta} - s(\eta,\gamma)\right) \end{aligned} \tag{1.8}$$

$$\begin{aligned} \mathcal{N}_G[G(\eta,\gamma), f(\eta,\gamma)] &= \frac{\partial^2 G(\eta,\gamma)}{\partial \eta^2} + Pr\left(f(\eta,\gamma)\frac{\partial G(\eta,\gamma)}{\partial \eta}\right) \\ &- G(\eta,\gamma)\left(Pr\beta\frac{\partial f(\eta,\gamma)}{\partial \eta} - s(\eta,\gamma)\right), \end{aligned} \tag{1.9}$$

with appropriate boundary conditions rewritten from (1:10)-(1:12) as,

$$f(\eta;\gamma)\big|_{\eta=0}=\alpha,\ \left.\frac{\partial f(\eta;\gamma)}{\partial\eta}\right|_{\eta=0}=1,\ \left.\frac{\partial f(\eta;\gamma)}{\partial\eta}\right|_{\eta=\infty}=0 \qquad (1.10)$$

$$g(\eta;\gamma)\big|_{\eta=0}=1, g(\eta;\gamma)\big|_{\eta=\infty}=0, \qquad \text{(PST case)} \qquad (1.11)$$

$$G(\eta;\gamma)\big|_{\eta=\infty}=0,\ \left.\frac{\partial G(\eta;\gamma)}{\partial\eta}\right|_{\eta=0}=-1 \quad \text{(PHF case).} \qquad (1.12)$$

An optimal value of the convergence control parameter cx, integrated in the whole region $[0;\infty)$ have the exact squared residual errors at l^{th}-order of approximations, we get where $X = f$, g and G.

Conclusion

The ambient fluid condition is stagnant, but the flow is induced near the material surface being extruded due to entrainment of the fluid caused by stretching of a moving surface. In these processes, the quality of the final product depends largely on the skin friction and heat transfer rates at the surface. Hence it is important to control the flow and heat transfer rate in order to obtain the final product of desired characteristics. Viscoelastic fluid flow and heat transfer due to a stretching sheet with an applied magnetic field is considered for analysis. Industrial processes such as the extrusion of polymer sheets, the cooling of a metallic plate in a cooling bath, drawing of plastic films have viscoelastic fluids in the molten form before solidification. Hence the present study of heat transfer effects due to a stretching surface of a viscoelastic fluid is significant.

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