DESIGNING DOUBLE SAMPLING PLANS UNDER THE CONDITIONS OF ZERO-INFLATED POISSON DISTRIBUTION

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Abstract: In a well monitored manufacturing environment, non-conformities occur infrequently so that the number of non-conformities will have more number of zeros. Under such circumstances, the appropriate probability distribution to the number of non-conformities is a zero-inflated Poisson distribution. In this paper designing of double sampling plans by attributes is considered when the number of non-conformities per product is distributed according to the zero-inflated Poisson distribution. Parameters of the sampling plans are obtained for some sets of values of (p_1, α, p_2, β). Numerical examples are given to illustrate the selection of sampling plans.

Keywords: Sampling inspection by attributes, double sampling plan, Zero-inflated Poisson distribution, operating characteristic function, Average sample number.

I Introduction

In the current world of continually increasing global competition, the main objectives of the manufacturing and service organizations include improving the quality of their products. Acceptance sampling is an important tool of statistical quality control (SQC) used in industries to take decision regarding the disposition of the lots of manufactured products. When the decision is made on the inspection of only one sample, the sampling plan is known as single sampling (SSP). Under certain circumstances, a second sample is required before the lot can be sentenced, such sampling plan is known as double sampling plan (DSP). A double sampling plan is defined by four parameters. n_1, sample size of the first sample, c_1, acceptance number of the first sample, n_2, sample size of the second sample, c_2, acceptance number of the second sample.

Modern developments which are taking place in technology help to improve the quality of products and their production. Production processes are now-a-days designed well in such a way that the products manufactured are in nearly perfect state so that the number of nonconforming items in the sample or the number of nonconformities in the sampled products will be nearly zero in those cases. However, random fluctuations in the production process may lead some products to an imperfect state. The probability model which is appropriate to study such situation is a zero-inflated model. Zero-inflated Poisson (ZIP) distribution can be used as the appropriate probability distribution to over dispersed data consisting of many zeros. ZIP distribution has been used as a probability distribution in diversified fields such as agriculture epidemiology, econometrics, public health, process control, medicine, manufacturing, etc., some of the applications of ZIP distribution can be found in (Singh(1963)), Lambert(1992), Bohning et. al., (1999), Naya et. al., (2008). Construction of control charts using ZIP distribution are discussed in Xie et. al., (2001), He et. al., (2003), Chen et. al., (2008), Sim and Lim (2008), He et. al., (2012), Katemee and Mayureesawan (2012a, 2012b, 2013), He et. al., (2014), Choi and Lee (2014), Katemee (2016). Some theoretical aspects of ZIP distributions are mentioned in McLachlan and Peel (2000) and Nanjundan and Pasha (2015). Loganathan and Shalini (2014a, 2014b) and Shalini et al., (2014) discussed the determination of SSPs and Bayesian SSPs under the conditions of ZIP distribution.

The objective of this paper is designing DSPs by attributes under the conditions of ZIP distribution. The operating characteristic (OC) function and the average sample number (ASN) of the plan are derived in Section 2. Determination of the plan parameters for the given two points on the OC curve are described in Section 3. Illustrations are given in Section 4 to describe the selection of sampling plans for specified strength. The results are summarized in Section 5.

II OC Function of DSP under the conditions of ZIP Distribution

A DSP by attributes is specified by four parameters N, n_1, c_1, n_2, c_2. The operating procedure of a DSP can be described as follows

(i) Take a sample of size n_1 from the lot of size N.
(ii) Inspect all the items included in the sample. Let x_1 be the number of nonconforming units (nonconformities) in the sample.
(iii) If $x_1 \leq c_1$, the lot is accepted.
(iv) If $x_1 > c_2$ the lot is rejected.
(v) If $c_1+1 < x_1 \leq c_2$, take a second sample of size n_2 and observe the number of nonconforming units (nonconformities) x_2.
(vi) Cumulate x_1 and x_2 and let $D = x_1+x_2$.
(vii) If $D \leq c_2$, accept the lot. If $D > c_2$ reject the lot.

According to Stephens (2001) there are atleast three advantages of using double sampling in contrast to single sampling. Double sampling (i) can involve a somewhat smaller average amount of inspection than single sampling for a given quality protection, particularly when the incoming product is very good are very bad; (ii) can provide a good feeling in terms of a psychological advantage of a second chance to the concerned parties and (iii) always provides an OC curve with an inflection point, hence better discrimination, even when the first acceptance number is zero (0).

The performance of a sampling plan can be assessed using its OC function. The OC function of a DSP is defined as

$$P_a(p) = P(x_1 \le c_1 \mid n_1) + \sum_{x_1=c_1+1}^{c_2} P(x_1 \mid n_1)\, P(x_2 \le c_2 - x_1 \mid n_2)$$

Since double sampling involves either one or two samples to reach as accept reject decision on a lot, the amount of sampling inspection per lot is a variable. Hence, the amount of sampling inspection per lot can be measured by weighted average referred as average sample number (ASN). The ASN can be computed as

$$ASN = n_1 + n_2\left[P\left(c_1 < x_1 < c_2 + 1\right)\right]$$

As pointed by Loganathan and Shalini (2014a, 2014b) in many industrial applications production processes are monitored well such that the occurrence of zero non-conformities would be more. Under such circumstances, the suitable probability distribution associated with the number of nonconformities is a ZIP distribution. The probability mass function of the ZIP distribution is defined as

$$P(X = x \mid \varphi, \lambda) = \begin{cases} \varphi + (1-\varphi)\, e^{-\lambda} & , \text{ when } x = 0 \\ (1-\varphi)\dfrac{e^{-\lambda}\lambda^{x}}{x!} & , \text{ when } x = 1, 2, \ldots, \quad 0 < \varphi < 1,\ \lambda > 0. \end{cases}$$

The OC function of the DSP under the conditions of ZIP (φ, λ) distribution can be defined as

$$P_a(p) = \varphi + (1-\varphi)e^{-np} + (1-\varphi)\sum_{x_1=1}^{c_1}\frac{e^{-np}(np)^{x_1}}{x_1!} + (1-\varphi)\sum_{x_1=c_1+1}^{c_2}\frac{e^{-np}(np)^{x_1}}{x_1!}\left(\varphi + (1-\varphi)e^{-np} + (1-\varphi)\sum_{x_2=1}^{c_2-x_1}\frac{e^{-np}(np)^{x_2}}{x_2!}\right)$$

The ASN function can be expressed as

$$ASN = n_1 + n_2\left[\varphi + (1-\varphi)e^{-np} + (1-\varphi)\sum_{x_1=c_1+1}^{c_2}\frac{e^{-np}(np)^{x_1}}{x_1!}\right]$$

III Designing Double Sampling Plans under the conditions of ZIP Distribution

Sampling Plans are constructed in such a way the protection to both producer and consumer is ensured. For given φ and (p_1, α, p_2, β) a DSP is determined satisfying the conditions

$P_a(p_1) = 1 - \alpha$ (1)

$P_a(p_2) = \beta$ (2)

where p_1, α, p_2 and β denote respectively acceptable quality level, producer's risk, limiting quality level and consumer's risk. The plan parameters N, n_1, c_1, n_2 and c_2 can be obtained for each set of values of φ, p_1, α, p_2 and β applying unity values approach proposed by Schilling and Neubauer (2009). Duncan (1986) has provided a compilation of unity values and operating ratios for double and multiple sampling as developed by the U.S. Army Chemical Corps Engineering Agency (1953). The operating ratio (R) defined as the ratio of p_2 to p_1 ie., $R = p_2/p_1$ is often used as the measure of discrimination. The values np_1 and np_2 satisfying respectively (1) and (2) are termed as unity values. The unity values are computed for various combinations of (φ, c_1, c_2, $P_a(p)$). The values taken for φ are 0.01, 0.04, 0.07, 0.1 and for $P_a(p)$ 0.95, 0.50, 0.10. The operating ratio values calculated corresponding to (α = 0.05, β = 0.10) are listed in Table 1 through Table 4.

Illustrations

The procedure for selecting DSPs under the conditions of ZIP distribution for specified strength is discussed in this section with numerical illustrations.

Illustration 1

Suppose that φ = 0.01 and the strength of the plan is specified as p_1 = 0.009, α = 0.05, p_2 = 0.04 and β = 0.10. The operating ratio corresponding to these specification is calculated as 4.44 that comes closest to matching this R value is plan number 2 in Table 1 with $n_1 = n_2$. This is the double sampling plan with c_1 = 1, c_2 = 3 with R value of 4.49. From Table 1 np_1 is 1.01 for this plan. Hence, $n_1 = n_2 = np_1/p_1$ =1.01/0.009 ≈ 112. The double sampling plan is n_1 = 112, c_1 =1, n_2 =112 and c_2 = 3.

Illustration 2

For specified φ = 0.01 and the strength of the plan is specified as (p_1 = 0.009, α = 0.05, p_2 = 0.04 and β = 0.10), the optimum ZIP SSP is (220, 4) (Loganathan and Shalini (2014a)) and optimum ZIP DSP is (112, 1, 112, 3). The average sample number of DSP under ZIP distribution is 174. This indicates that the sample size and acceptance numbers of double sampling plan are much lesser than that of single sampling plan under the conditions of ZIP distribution. Therefore, the application of DSP under ZIP distribution reduces the risk of both producer and consumer.

Conclusions

In a well-equipped production process, most of the products will meet the specified quality standards. Occurrence of zero-nonconformities per product would be more frequent in the sampling inspection. A zero-inflated model is the appropriate probability distribution to the number of non-conformities per product manufactured in such production process. In this paper, the OC function and ASN function of the DSP under the conditions of ZIP distribution is derived. The procedures for designing and selecting DSPs under the conditions of ZIP distribution are presented. Tables providing the DSPs under ZIP distribution are presented for some specified strength. The DSPs designed under the conditions of ZIP distribution are much lesser than the ZIP SSPs. Protection to both producer and consumer is ensured when ZIP DSPs are adopted.

Table 1: Double Sampling Plans under ZIP Distribution when $n_1 = n_2$

Plan Number	R	φ	Acceptance Numbers		Approximate Values of p^*n_1 for			Approximate (ASN)/n_1 for 0.95 Point
			c_1	c_2	P_a = 0.95	0.5	0.1	
1	6.07	0.01	1	2	0.69	2.07	4.19	1.631
2	4.49	0.01	1	3	1.01	2.49	4.53	1.552
3	4.01	0.01	2	4	1.42	3.22	5.7	1.365
4	3.46	0.01	2	5	1.73	3.59	5.99	1.401
5	3.12	0.01	2	6	2.05	4.01	6.4	1.454
6	3.29	0.01	3	6	2.16	4.31	7.11	1.272
7	2.99	0.01	3	7	2.47	4.66	7.38	1.312
8	2.90	0.01	4	8	2.92	5.38	8.47	1.221
9	2.69	0.01	4	9	3.24	5.72	8.72	1.267
10	2.54	0.01	4	10	3.57	6.11	9.06	1.321
11	2.42	0.01	5	11	4.15	6.78	10.03	1.258
12	2.37	0.01	5	12	4.37	7.16	10.35	1.295
13	2.28	0.01	5	13	4.72	7.58	10.74	1.348
14	2.25	0.01	6	14	5.17	8.2	11.62	1.274
15	2.17	0.01	6	15	5.53	8.61	11.99	1.327
15	2.17	0.01	6	15	5.53	8.61	11.99	1.327

Table 2: Double Sampling Plans under ZIP Distribution when $n_1 = n_2$

Plan Number	R	φ	Acceptance Numbers		Approximate Values of p^*n_1 for			Approximate (ASN)/n_1 for 0.95 Point
			c_1	c_2	P_a = 0.95	0.5	0.1	
1	6.67	0.04	1	2	0.7	2.13	4.67	1.656
2	4.90	0.04	1	3	1.03	2.57	5.05	1.575
3	4.38	0.04	2	4	1.44	3.31	6.31	1.391
4	3.80	0.04	2	5	1.75	3.69	6.65	1.430
5	3.41	0.04	2	6	2.08	4.12	7.1	1.485
6	3.61	0.04	3	6	2.18	4.42	7.87	1.302

7	3.27	0.04	3	7	2.5	4.78	8.18	1.349
8	3.18	0.04	4	8	2.95	5.51	9.37	1.257
9	2.96	0.04	4	9	3.27	5.86	9.67	1.299
10	2.78	0.04	4	10	3.61	6.26	10.05	1.355
11	2.74	0.04	5	11	4.06	6.93	11.13	1.276
12	2.61	0.04	5	12	4.41	7.32	11.49	1.329
13	2.50	0.04	5	13	4.77	7.74	11.92	1.387
14	2.47	0.04	6	14	5.22	8.38	12.91	1.312
15	2.39	0.04	6	15	5.58	8.79	13.31	1.366

Table 3: Double Sampling Plans under ZIP Distribution when $n_1 = n_2$

Plan Number	R	φ	Acceptance Numbers		Approximate Values of p^*n_1 for			Approximate (ASN)/n_1 for 0.95 Point
			c_1	c_2	$P_a = 0.95$	0.5	0.1	
1	7.65	0.07	1	2	0.72	2.2	5.51	1.677
2	5.72	0.07	1	3	1.04	2.65	5.95	1.615
3	5.04	0.07	2	4	1.46	3.41	7.36	1.417
4	4.41	0.07	2	5	1.77	3.8	7.81	1.466
5	3.98	0.07	2	6	2.11	4.23	8.39	1.516
6	4.14	0.07	3	6	2.21	4.55	9.15	1.337
7	3.81	0.07	3	7	2.53	4.92	9.63	1.380
8	3.65	0.07	4	8	2.99	5.66	10.92	1.288
9	3.46	0.07	4	9	3.31	6.02	11.44	1.337
10	3.30	0.07	4	10	3.65	6.42	12.05	1.394
11	3.22	0.07	5	11	4.11	7.11	13.24	1.314
12	3.12	0.07	5	12	4.45	7.5	13.89	1.363
13	3.04	0.07	5	13	4.81	7.92	14.63	1.422
14	2.99	0.07	6	14	5.27	8.57	15.75	1.351
15	2.94	0.07	6	15	5.63	8.98	16.53	1.406

Table 4: Double Sampling Plans under ZIP Distribution when $n_1 = n_2$

Plan Number	R	φ	Acceptance Numbers		Approximate Values of p^*n_1 for			Approximate (ASN)/n_1 for 0.95 Point
			c_1	c_2	$P_a = 0.95$	0.5	0.1	
1	20.12	0.1	1	2	0.73	2.28	14.69	1.702132
2	15.09	0.1	1	3	1.06	2.74	16	1.638
3	12.15	0.1	2	4	1.48	3.51	17.98	1.444
4	10.84	0.1	2	5	1.8	3.92	19.51	1.494
5	9.94	0.1	2	6	2.13	4.36	21.17	1.554
6	9.50	0.1	3	6	2.24	4.69	21.27	1.366
7	8.93	0.1	3	7	2.56	5.06	22.85	1.411
8	8.09	0.1	4	8	3.03	5.83	24.5	1.325
9	7.78	0.1	4	9	3.35	6.19	26.05	1.370
10	7.48	0.1	4	10	3.69	6.6	27.61	1.428
11	7.02	0.1	5	11	4.15	7.3	29.15	1.348
12	6.82	0.1	5	12	4.5	7.7	30.67	1.403
13	6.62	0.1	5	13	4.86	8.12	32.17	1.462
14	6.33	0.1	6	14	5.32	8.79	33.65	1.386
15	6.18	0.1	6	15	5.68	9.2	35.11	1.441

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