Anti-Fuzzy Sub algebras and Anti Fuzzy K-ideals In INK-Algebras

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Abstract:

The aim of this paper is introduce the notion of anti-fuzzy sub algebras and anti-fuzzy K-ideals in INK-algebras, several theorems, properties are stated and proved. The fuzzy relations on INK-algebras are also studied.

Keywords:

INK- algebras, Anti homomorphism, Anti fuzzy K- ideal, anti-fuzzy subgroup, anti-fuzzy sub algebra, Cartesian product, level subset, conditions stated.

1. Introduction

K. Iseki and Tanaka introduced two classes of abstract algebras BCI-algebras and BCK-algebras in[2]. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebra. Q.P. Hu and X. Li introduced a wide class of abstract algebra namely BCH- algebras. L.A. Zadeh ,(1965), introduced the notion of fuzzy sets in [7]. This idea has been applied to several mathematical branches. Xi applied this concept to BCK- algebra.W.A. Dudek (1992) fuzzified the ideals in BCC-algebras. Y.B.Jun (2009) contributed a lot to develop the theory of fuzzy sets. M.Kaviyarasu et.all(2017), introduced a new notion called INK-algebra in[11], which is a generalization of TM/Q/ BCK/ BCI/ BCH-algebra and investigated some properties. In this study, we introduce the concepts of anti-fuzzy sub algebras and anti-fuzzy K-ideals in INK-algebra and investigate some of their properties.

2 Preliminaries

In this section we first review some elementary aspects which are useful in the sequel.

Definition: 2.1[2]

A BCK-algebra is algebra (X,*, 0) of type (2, 0) satisfying the following conditions:

i) ((x * y) * z = x * (z * (0* y)) ,

ii) (x *(x * y)) * y = 0,

iii) x * x = 0,

iv) 0 * x = 0,

v) x * y = 0and y * x = 0 imply x = y for all x, y, z ∈ X .We can

Define a partial ordering ≤on X by x ≤ y is defined by x * y = 0.

Definition 2.2.[11]

A INK-algebra (X,*,0) is a non-empty set X with a constant '0' and a binary operation * satisfying the following axioms :

i) x * x = 0 ,

ii) x * 0 = x, for x ∈X.

iii) 0 * x = x,

iv) (x * y) * z = x * (z * (0 * y))

In X we can define a binary relation ≤ by x ≤ y if and only if x * y = 0.

Example: 2.3.[11]

Consider the set X = {0, 1, 2, 3} with the following table

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Then we easily check that (X,*, 0) is a INK-algebra since Then we easily can check that $(X,*,0)$ is a k-algebra, since we have $x* x = 0$, $x * 0 = 0$ and

$(x * y) * z = x * (z * (0 * y))$, for all $x, y, z \in X$ .But $(X,*,0)$ is not a INK-algebra,

Since $0 *1 \neq 0$

x=0,y=1,z=3

(0*1)*3=0*(3*(0*1))

(1*3) = (3*1)

2=2 hence verified,

3. Anti-Fuzzy sub algebras of k-ideals

Definition:3.1.

A fuzzy set μ of X *is* called a anti-*fuzzy sub algebra* of K if $\mu (x * y) \leq max \{ \mu(x), \mu(y)\}$, for all $x, y \in X$`

Definition: 3.2.

A fuzzy set μ of X *is* called a anti fuzzy *ideals* of *K* if it satisfies:

(i) $\mu(0) \leq \mu(x)$,

(ii) $\mu(x) \leq max\{ \mu(x * y), \mu(y * (y * x)) \}$ for all x , y ϵX

Clearly x = 0 gives μ is a anti-fuzzy ideal of X.

Definition: 3.3.

Let G be a group. A fuzzy subset μ of a group G is called a anti fuzzy subgroup of the group if

(i) μ (xy) ≤ max{μ(x) , μ(y)} for every x,y Є G and

(ii) $\mu(x^{-1}) = \mu(x)$ for every x Є G

Example:3.4.

Consider the set X = {0, 1, 2, and 3} with the following *cayley's* table:

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Define a fuzzy set μ in X as by $\mu(0) = 0.5$,

$\mu(1) = 0.6$, $\mu(2) = 0.7$ $\mu(3)=0.9$.

. It is easy to verify that μ is a anti fuzzy K-ideal of X.

Theorem: 3.5.

If μ is a anti fuzzy sub algebra of a INK –algebra X, then μ (0) ≤μ(x), for any x ∈X.

Proof. Subsequently x * x = 0, for any x ∈X, then:

$\mu(0) = \mu(x * x) \leq \max \{\mu(x), \mu(x)\}$

$\Rightarrow \mu(0) = \mu(x)$.

Theorem: 3.6.

If μ is a anti fuzzy sub algebra of a INK –algebra X, then μ(0) ≤μ(x),for any x ϵX.

Proof:

Subsequently x * x = 0, for any x ϵX, then:

$$\mu(0) = \mu(x * x) \leq \max\{\mu(x), \mu(x)\}$$

$$\Rightarrow \mu(0) = \mu(x).$$

Theorem: 3.7.

A fuzzy set μ of an INK-algebra X is a anti fuzzy sub algebra if and only if for every t∈[0,1] μ_t is either empty of a anti fuzzy sub algebra of X.

Proof.

Assume that μ is a anti fuzzy sub algebra of X. and ≠ ∅

Then for any x, y$\in \mu_t$

we have, μ(x * y) ≤ max {μ(x), μ(y)} ≤ t.

Therefore x,y$\in \mu_t$

μ_t. Hence μ_t is anti-fuzzy sub algebra of X. Conversely, μ_t is

a anti fuzzy sub algebra of X. Let x, y ∈ X.

Take t = max {μ(x), μ(y)}. Then by assumption μ_t is anti-fuzzy sub algebra of X implies:

x *y ∈ μ_t. Therefore μ(x *y) ≤ t = max {μ(x), μ(y)}

.Hence μ is a anti fuzzy sub algebra of X.

Definition: 3.8.

Let G be a group .A fuzzy subset μ of a group G is called a anti fuzzy subgroup of the group if

(i)$\mu(xy) \leq \max\{\mu(x), \mu(y)\}$), for every x,yϵG

(ii)$\mu(x^{-1}) = \mu(x)$, for every x,yϵG

Theorem: 3.9.

Anti-fuzzy sub algebra is a anti-fuzzy subgroup.

Proof:

Consider a anti-fuzzy sub algebra on the algebra A then there exists a

Fuzzy set μ Є K^A then for

(i) nullary operation

$\mu(f(x_1\ldots\ldots\ldots x_n)) \leq \mu(x_1)*\ldots\ldots\ldots\ldots\ldots*\mu(x_n)$ for all $x_1\ldots\ldots\ldots x_n$ Є A------------ (1)

From (1) let $x_2 = y$

μ(f(xy)) ≤ μ(x)*μ(y) ---------(2)

since μ(x*y) ≤ max{μ(x) , μ(y) } ---------(3)

from (2) and (3)

μ(x)*μ(y) = μ(x*y) ≤ max{μ(x) , μ(y)}-------(4)

Let μ: X⟶ [0, 1]

Therefore μ: X^{-1} ⟶ [0, 1].

⇒ $\mu(x^{-1}) = \mu(x)$ --------------(5)

(ii) nullary operation (for any constant)

If there exist any constant μ(c) then

μ(c) *μ(x) = μ(c * x) ≤max{μ(c) , μ(x)}= μ(x)------------(6)

μ(c) ≤μ(x), for all x Є A.

∵ all the element of μ(x) lies between 0 & 1. Also μ(c) is any constant and ≤ 1. Using the results (4), (5) and (6), it is clear that anti fuzzy sub algebra is a anti fuzzy subgroup.

Theorem: 3.10.

A anti fuzzy sub algebra of a group G is a anti-fuzzy subgroup of the group G iff

$\mu(xy^{-1}) \leq \max\{ \mu(x) , \mu(y) \}$ for every x,y Є G.

Proof:

Real part:

Consider a anti-fuzzy sub algebra of a group G is a anti fuzzy subgroup on the algebra A then we have to prove that

$$\mu(xy^{-1}) \leq \max \{ \mu(x) , \mu(y) \}$$

For anti-fuzzy sub algebra there exist a fuzzy set μ ЄK_A, then

(i) For n – ary operation

$$\mu(f(x_1\ldots\ldots\ldots.x_n)) \leq \mu(x_1)*\ldots\ldots\ldots\ldots\ldots*\mu(x_n) \text{ for all } x_1\ldots\ldots\ldots.x\ \ n\text{Є}A$$

Let x_1, x_2 Є k where k Є G .Consider $x_1 = x$ and $x_2 = y^{-1}$

Since it is a anti fuzzy subgroup using theorem (1)

Therefore $\mu(f(x_1\ x_2)) = \mu(f(x\ y^{-1}))$

$$\leq \mu(x) * \mu(y^{-1})$$

$$\mu(x) * \mu(y^{-1}) = \mu(x * y^{-1})$$

since anti fuzzy algebra satisfies fuzzy relation

so $\mu(x * y^{-1}) \leq \mu(x) \vee \mu(y^{-1})$ using equation (5)

$$\leq \mu(x) \vee \mu(y)$$

$$\leq \max \{\mu(x), \mu(y)\}$$

i) For nullary operation

$$\mu(c) \leq \mu(x) \text{ for all } x \text{ Є } A.$$

Using theorem (1), it is a subgroup

∴ Let $\mu(e)$ be an identity element ,

We know that $\mu(c) \leq \mu(x)$

$\mu(c) = \mu(c) * \mu(e^{-1}) \leq \mu(c) * \mu(e)$ using equation (5)

$\leq \mu(c) \vee \mu(e)$

$\leq \max\{\mu(c), \mu(e)\} = \mu(e)$----------(7)

C is any constant it may be $\leq$ identity element

$\mu(x) = \mu(x) * \mu(e^{-1}) \leq \mu(x) * \mu(e)$ using equation (5)

$\leq \mu(x) \vee \mu(e)$

$\leq \max\{\mu(x), \mu(e)\} = \mu(x)$----------(8)

Converse part:

If $\mu(xy^{-1}) \leq \max\{\mu(x), \mu(y)\}$ for every x, y Є k and k Є G then a fuzzy partially ordered subset μ of a group is a anti fuzzy subgroup of the group G.

Proof:

$\mu(xy^{-1}) \leq \max\{\mu(x), \mu(y^{-1})\}$

$\leq \max\{\mu(x), \mu(y)\}$ using (5)

$\leq \mu(x) \vee \mu(y)$

$\mu(xy^{-1}) = \mu(x * y^{-1})$ using fuzzy algebra relation.

$\mu(x * y^{-1}) = \mu(x) * \mu(y^{-1}) = \mu(x) * \mu(y)$

$\leq \mu(f(x\ y))$ (n- ary operation)

$\Rightarrow \mu(f(x\ y)) \leq \mu(x) * \mu(y)$-----------(9)

Let μ(c) and μ(e) be a any constant& identity element, then

Assume that $\mu(c) = \mu(x)$

$\mu(c) * \mu(e^{-1}) = \mu(x) * \mu(e^{-1})$

Using (5) $\mu(c) * \mu(e^{-1}) = \mu(x) * \mu(e^{-1})$

$\mu(c) * \mu(e) \leq \max\{ \mu(c), \mu(e) \} = \mu(e)$

$\mu(x) * \mu(e) \leq \max\{ \mu(x), \mu(e) \} = \mu(x)$

C is constant it may be ≤ identity element and x

Therefore, $\mu(c) \leq \mu(x)$ --------- (10)

From (9) and (10) we get, If $\mu(xy^{-1}) \leq \max\{ \mu(x), \mu(y) \}$

Hence anti fuzzy sub algebra is a anti fuzzy subgroup

Result: 3.11.

A fuzzy subset μ of a group G is a anti fuzzy subgroup G⟺

$\mu(xy^{-1}) \leq \max\{ \mu(x), \mu(y) \}$ for every x , y Є G

4. Anti-Fuzzy k-ideals in INK-algebra:

Definition: 4.1.

A fuzzy subset μ in a INK-algebra X is called a anti fuzzy ideal of X, if:

i) $\mu(0) \leq \mu(x)$

ii) $\mu(x) \leq \max\{\mu(x * y), \mu(y)\}$,for all x, y, z Є X.

Definition:4.2.

A fuzzy subset μ in a INK-algebra X is called a anti fuzzy K- ideal of X, if:

(i)$\mu(0) \leq \mu(x)$

(ii)$\mu(x) \leq \max\{\mu(z * x) * (z * y), \mu(y)\}$, for all x, y, z Є X.

Clearly z = 0 gives μ is a anti fuzzy K- ideal of X.

Example: 4.3.

Consider INK-algebra X = {0, 1,2,3} with the following cayley table.

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Define $\mu: X\to[0,1]$ *by* $\mu(0) = 0.2$, $\mu(1)= 0.3$, $\mu(2) = \mu(3)=0.4$

$\mu(0) \leq max\{\mu(3 * 0), \mu(3*1), \mu(1)\}$

$= \max\{ \mu(3 * 1), \mu(1)\}$

$= max\{\mu(2), \mu(1)$

$0.2 \;\; = max\{0.4,0.3\}$

$\therefore 0.2 \leq 0.4$

Then it is easy to verify that μ is anti fuzzy K- ideal.

Theorem: 4.4.

Let μ be a anti fuzzy K- ideal of a INK-algebra X. Then the following are equivalent:

(K_1) μ is a anti fuzzy k-ideal of X,

(K_2) $\mu((x * y) * z) \leq \mu(x * (y * z))$ for all x, y, z ∈ X,

(K_3) $\mu(x * y) \leq \mu(x * (0 * y))$.

Proof.

(K_1)→ (K_2) since μ is a anti fuzzy K- ideal of X, we have

$\mu((x * y) * z) \leq \max \{\mu((x*y)*(0*z)),\mu(0)\}$

$= \mu((x*y)*(0*z))$. On the other hand

$(x*y)*(0*z) =(x * y) * ((y * z) * y)$

$\leq (x * (y * z)$,

Consequently μ((x ∗ y)∗ z)) ≤ μ((x * y) * (0 * z)).

$(K_2)\rightarrow(K_3)$ Letting y = 0 and z = y ,

$(K_3)\rightarrow(K_1)$ since (x * (0 * y)) * (x * (z * y)) ≤ (z ∗ y) ∗ (0 ∗ y) ≤ z,

we have μ(x ∗ (0 ∗ y)) ≤ max{μ(x * (z * y)), μ(z)}.

Hence by hypothesis

$$\mu(x * y) \leq \max\{\mu(x * (z * y)), \mu(z)\}.$$

Therefore μ is a anti fuzzy K-ideal of X.

Theorem: 4.5.

Every anti fuzzy K-ideal μ of a INK-algebra X is order reversing, that is if x ≥ ythen: μ(x) ≤ μ(y), for all x, y ∈ X.

Proof.

Let x, y ∈ X such that x ≥ y. Therefore x ∗ y = 0, Put z = 0,Now,

μ(x) = μ(0 ∗ x)

≤ max {μ((z ∗ x) ∗ (z ∗ y)), μ(y)}

= max {μ((0 ∗ x) ∗ (0 ∗ y)), μ(y)}

= max{μ(x ∗ y), μ(y)}

= max{μ(0), μ(y)}

μ(x) = μ(y). Hence complete the proof.

Theorem: 4.6.

Let μ be an anti-fuzzy ideal of INK-algebra X. If μ(x ∗ y) ≤ μ(x) for all x, y ∈ X, then μ is a K-ideal of X.

Proof:

Since μ is an anti-fuzzy ideal of X, by hypothesis we have

max{μ(x*(y *z)), μ(y)} ≤max{μ((x * z) * (y * z)), μ(y * z)}

≤ μ(x * z), ∀ x, y, z ∈ X.

Let X is an INK-algebra. A fuzzy set μ in X is called an anti-fuzzy sub algebra of X if $\mu(x * y) \leq \max\{\mu(x), \mu(y)\}$ for all x, y, z ∈ X

Theorem: 4.7.

A fuzzy set μ in a INK-algebra X is a anti fuzzy K-ideal if and only if it is a anti fuzzy ideal of X.

Proof: Let μ be a anti fuzzy K-ideal of X.

Then i) $\mu(0) \leq \mu(x)$ ii) $\mu(x)$

$\leq \max\{\mu(x * y), \mu(y)\}$ for all x, y, z ∈ X.

Putting z = 0 in (ii) we have, $\mu(x) \leq \max\{\mu(x * y), \mu(y)\}$.

Hence μ is a anti fuzzy ideal of X. Conversely, μ is a anti fuzzy ideal of X.

$$\mu(x) \leq \max\{\mu(x * y), \mu(y)\}$$

Then: $\mu(x) \leq \max\{\mu((0 * x) * (0 * y)), \mu(y)\}$.

if we replace z for 0,we have $\mu(x) \leq \max\{\mu((z * x) * (z * y)), \mu(y)\}$.

Hence a fuzzy set μ in a INK-algebra.

5 Cartesian products of anti-fuzzy K-ideals of INK-algebras

Definition: 5.1.

Let μ and ϑ be the fuzzy sets in a set X. The Cartesian product $\mu \times \vartheta : X \times X \rightarrow [0, 1]$ is defined by:

$\mu \times \vartheta(x, y) = \max\{\mu(x), \vartheta(y)\}$, for all x, y 2 X.

Theorem: 5.2.

If μ and ϑ are anti-fuzzy K-ideals in a INK- algebra X, then $\mu \times \vartheta$ is a anti-fuzzy K-ideal in X × X.

Proof.

If any (x, y) ∈ X × X, we have:

i)$(\mu \times \vartheta)(0, 0) = \max\{\mu(0), \vartheta(0)\}$

$\leq \max\{\mu(x), \vartheta(y)\}$

$= (\mu \times \vartheta)(x, y)$.

ii) Let (x_1, x_2), (y_1, y_2) and $(z_1, z_2) \in X \times X$.

$(\mu \times \mu)(x_1, x_2) = (\mu \times \vartheta)(x_1 * x_2)$

$= \max\{\mu(x_1), \vartheta(x_2)\}$

$\leq \max \{\max \{((z_1 * x_1) * (z_1 * y_1)), \mu(y_1)\},$

$\max\{ \vartheta((z_2 * x_2) * (z_2 * y_2)), \vartheta(y_2)\}$

$= \max\{\max\{\mu((z_1 * x_1) * (z_1 * y_1)), \vartheta((z_2 * x_2) * (z_2 * x_2))\},$

$\max \{\mu(y_1), \vartheta(y_2)\}$

$= \max\{(\mu \times\vartheta)((z_1 * x_1) * (z_1 * y_1)), \vartheta((z_2 * x_2) * (z_2 * y_2)),$

$(\mu \times\vartheta)(y_1, y_2)\},$

$= \max\{(\mu \times\vartheta)((z_1, z_2) * (x_1, x_2)), \vartheta((z_1, z_2) * (y_1, y_2)), (\mu \times\vartheta)(y_1, y_2)\}.$

Hence $(\mu \times\vartheta)$ is a anti fuzzy K-ideal of a INK-algebra in $X \times X$.

Theorem: 5.3.

Let μ and ϑ be fuzzy sets in a INK-algebra X such that $\mu \times\vartheta$ is a anti fuzzy K-ideal of $X \times X$.

i) Either $\mu(0)\leq \mu(x)$ or $\vartheta(0) \geq \vartheta(x)$,for all x 2 X

ii) If $\mu(0) \leq \mu(x)$,for all $x \in X$, either $\vartheta(0) \leq\mu(x)$ or $\vartheta(0)\leq(x)$

iii) If $\vartheta(0) \leq \vartheta(x)$,for all $x \in X$, either $\mu(0) \leq\mu(x)$ or $\mu(0) \leq \vartheta(x)$

iv) Either μ or ϑ is a anti fuzzy K-ideal of X.

Proof.

$\mu\times\vartheta$ is a anti fuzzy K-ideal of $X\times X$. Therefore $(\mu\times\vartheta)(0, 0) \leq(\mu \times\vartheta)(x, y)$,for all $(x, y) \in X \in X$ .And $(\mu \times\vartheta)(x_1, x_2)$

$\leq \max \{(\mu \times\vartheta)((z_1, z_2) * (x_1, x_2)) * ((z_1, z_2) * (y_1, y_2)), (\mu \times\vartheta)(y_1, y_2)\}$,for all (x_1, x_2), (y_1, y_2) and (z_1, z_2) $\in X \times X$.

Suppose that $\mu(0) > \mu(x)$ and $\vartheta(0)> \vartheta(y)$, for some $x, y \leq X$.

Then: $(\mu \times\vartheta)(x, y) = \max \{\mu(x), \vartheta(y)\} < \max \{\mu(0), \vartheta(0)\} = (\mu \times\vartheta)(0, 0)$. a contradiction.

Therefore either $\mu(0) \leq \mu(x)$ or $\vartheta(0) \leq \vartheta(x)$,for all $x \in X$.

Assume that $\exists$ x, y $\in$ X such that:

$\vartheta(0) > \mu(x)$ and $\vartheta(0) < \vartheta(y)$. Then: $(\mu\times\vartheta)(0, 0) = \max \{\mu(0), \vartheta(0)\} = \vartheta(0)$

and hence $(\mu \times \vartheta)(x, y) = \max \{\mu(x), \vartheta(y)\}< \vartheta(0)$

$= (\mu\times\vartheta)(0, 0)$, a contradiction. Hence if $\mu(0) \leq\mu(x)$,for all $x \in X$,

then either: $\mu(0)\leq \mu(x)$ or $\mu(0) \leq \vartheta(x)$

Similarly we can prove that if$\vartheta(0) < \vartheta(x)$, for all $x \in X$,

then either $\mu(0) \leq \mu(x)$ or $\mu(0) \leq \vartheta(x)$.

First we prove that ϑ is a anti fuzzy k-ideal of X.

Since ,by (i),either $\mu(0) \leq\mu(x)$ or $\vartheta(0) \leq \vartheta(x)$, for all $x \in X$.

Assume that $\vartheta(0)\leq \vartheta(x)$ for all $x \in X$. It follows from (iii)

That either $\mu(0) \leq\mu(x)$ or $\mu(0) \leq \vartheta(x)$. If $\mu(0)\leq \vartheta(x)$, for any $x \in X$, then: $\vartheta(x) = \max\{\mu(0), \vartheta(x)\}$

$= (\mu \times\vartheta)(0, x).\ \vartheta(x)$

$= \max\{\mu(0), \vartheta(x)\} = (\mu \times\vartheta)(0, x)$

$\leq \max\ \{(\mu \times\vartheta)((0, z) * (0, x)) * ((0 * z) * (0, y)), (\mu \times\vartheta)(0, y)\}$

$= \max\{(\mu \times\vartheta)(((0 * 0), (z * x)) * ((0 * 0), (z * y))), (\mu \times\vartheta)(0, y)\}$

$= \min\ \{(\mu \times\vartheta)(((0 * 0) * (0 * 0)), ((z * x) * (z * y))), (\mu \times\vartheta)(0, y)\}$

$= \min\ \{(\mu \times\vartheta)(0, ((z * x) * (z * y))), (\mu \times\vartheta)(0, y)\}$

$\vartheta(x) = \max\{\vartheta(z * x) * (z*y), \vartheta(y)\}$.

Hence ϑ is a anti fuzzy K-ideal of X.

Now we will prove that μ is a anti fuzzy K-ideal of X. Let $\mu(0) \leq \mu(x)$.

By(ii) either $\vartheta(0) \leq (x)$ or $\vartheta(0) \leq \vartheta(x)$.Assume that $\vartheta(0) \leq \vartheta(x)$,

Then: $\mu(x) = \max\{\mu(x), \vartheta(0)\}$

$= (\mu \times\vartheta)(x, 0).$

$\mu(x) = \max\{\mu(x), \vartheta(0)\}$

$= (\mu \times\vartheta)(x, 0)$

$\leq \max\{(\mu \times\vartheta)(((z, 0) * (x, 0)) * ((z * 0) * (y, 0))), (\mu \times\vartheta)(y, 0)\}$

$= \max\{(\mu \times\vartheta)(((z * x), (0 * 0)) * ((z * y), (0 * 0))), (\mu \times\vartheta)(y, 0)\}$

$= \max\{(\mu \times\vartheta)(((z * x) * (z * y)), ((0 * 0) * (0 * 0))), (\mu \times\vartheta)(y, 0)\}$

$\mu(x) = \max\{\mu((z * y) * (y * z)), \mu(y)\}$.

Hence μ is a anti fuzzy K-ideal of X.

6. Anti-homomorphism of INK-algebras

Definition: 6.1.

Let X_1 and X_2 be K-algebras. A mapping

$g : X_1 \rightarrow X_2$ is said to be a anti homomorphism if it satisfies:

$f(x_1 * x_2) = f(x_2) * f(x_1)$, for all x, y ∈ X.

Definition:6.2.

Let $g : X^1 \rightarrow X^1$ be an endomorphism and μ

a fuzzy set in X^1. We define a new fuzzy set in X by μ_g in X by

$\mu_g(x) = \mu(g(x))$, for all x in X.

Theorem: 6.3.

Let f be an endomorphism of a INK- algebra

X^1. If μ is a anti fuzzy K-ideal of X^1, then so is μ_g.

Proof :

$\mu g(x) = \mu(g(x)) * \mu(0)$

$= \mu(g(0))$

$= \mu g(x)$. for all $x \in X^1$. Let $x, y, z \in X^1$.

Then: $\mu_g(x) = \mu(g(x))$

$\leq \max \{\mu((g(z) * g(x)) * (g(z) * g(y))), \mu(g(y))\}$

$= \max \{\mu((g(z * x)) * g(z * y)), \mu(g(y))\}$

$= \max \{\mu(g((z * x) * (z * y)), \mu(g(y))\}$

$= \max \{\mu(g((z * x) * (z * y)), \mu(g(y))$

$= \max \{\mu g((z * x) * (z * y)), \mu g(y))\}$.

Hence μ_g is a fuzzy k-ideal of X^1.

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