MHD Free convection flow on a cooled vertical non-conducting porous plate

[1]Shashi Prakash Rai and [2]B.K Mishra
1.Mansa Pandey Bag, Ara- 802301,Bihar, India
2.Shreenath Niketan, Hazaribagh-825301,Jharkhand,India

***Abstract* :** A Study of free convective flow past an infinite vertical porous plate in an incompressible electrically conducting fluid has been considered, The equations of motion are integrated by perturbation technique. Expressions for the temperature distribution, velocity profile and local Nusselt number have been derived and numerically computed for various values of magnetic parameter and source /sink parameter.

1. Introduction

Free convective flows are of great interest in a number of industrial applications such as fiber and granulur insulation and geothermal system. Although, the free convection flow on a cooled vertical porous plate has been investigated by several authors (1-10) .

In the present paper the steady laminar free convection flow of viscous incompressible fluid of small electrical conductivity along a thin permeable cooled vertical plate in the presence of heat /sink is investigated. The Partial differential equations governing the flow are solved. By numerical computation,several important results are obtained .

2. Formulation of the problem

We consider the flow of viscous incompressible fluid of small electrical conductivity over an infinitely long non-coducting cooled vertical thin porous plate. The x'-axis is taken along the plate in the vertical direction and the y'- axis normal to the plate. A uniform transverse magnetic field is applied in the direction of y'- axis.

As the length of the plate is very large and motion is two-dimenstonal, all the physical quantities are independent of x'.Therefore, for steady laminar free convection flow of viscous incompressible fluid of an electrically conducting fluid along the cooled non- conducting porous vertical plate in the presence of heat source /sink, the governing equations are as follows

Equation of motion are

$$\rho v' \frac{du'}{dy'} = \mu \frac{d^2u'}{dy'^2} + \rho g \beta (T - T_\infty) - \sigma B_0^2 u' \quad (1)$$

And,

$$\frac{dp'}{dy'} = 0 \quad (2)$$

Equation of continuity is

$$\frac{dv'}{dy'} = 0 \quad (3)$$

Equation of energy is

$$c_p \qquad v'\frac{dT'}{dy'} = k\frac{d^2T'}{dy'^2} + \mu\left(\frac{du'}{dy'}\right)^2 + s(T - T_\infty) \qquad (4)$$

where ρ is density, μ is co- efficient of viscosity , β is co-efficient of volume expansion , g is acceleration due to gravity , B_0 is component of electromagnetic induction , Cp is sp. heat at constant pressure , k is consistency, s is couple stress parameter . p' is pressure of the fluid , T' is the temperature in the bourdary layer, u' is the component of velocity parallal to the plate in x'- direction, v' is velocity component normal to the plate in y'–direction , T is temperature distribution inside the boundary layer and T_∞ is the temperature of the free stream.

The boundary conditions are

$$\left.\begin{array}{l} y'=0 : u'=0,\ T'=T_w \\ y' \to \infty;\ u'=0,\ T'=T_\infty \end{array}\right| \qquad (5)$$

From equatiou (3), we get v'= constant= - v_0 (say), and from equatiou (2), p'=constant.

Equatiou(1) and (4) are transformed respectively into dimensionless form as

$$\frac{d^2u}{dy^2} + \frac{du}{dy} - M^2u = -Gr\,\theta \qquad (6)$$

$$\text{And } \frac{d^2\theta}{dy^2} + P_r\frac{d\theta}{dy} + sP_r\theta + P_rEc\left(\frac{du}{dy}\right)^2 = 0 \qquad (7)$$

Where P_r is Prandtl number , s is couple stress parameter,Ec is Eckert number,θ = (T-T_∞) / (Tw-T_∞) , M is Hartmann number ,G_r is Groshoff number and u is velocity component of fluid in x-direction.

The boundary conditions takes the from

$$\left.\begin{array}{l} y=0: u=0, \theta = 1 \\ y \to \infty: u = 0, \theta = 0 \end{array}\right| \qquad (8)$$

3. Solution by Perturbation Technique

As Eckert number Ec is very small for flow of incompressible fluid, we express u(y) and $\theta(y)$ in the following forms for the solution of equations (6) and(7)

$$u\,(y) = u_0(y) + E_c\ \ u_1\,(y) + 0(E_c^2) \qquad (9)$$

$$\theta(y) = \theta_0(y) + E_c\ \theta_1(y) + 0(E_c^2) \qquad (10)$$

Diffentiating equations (9) and (10) with respect to y and substituting in equation (6) and (7) respectively and equating the co-efficient of like prowers of Ec,weget

$$\frac{d^2u_0}{dy^2} + \frac{du_0}{dy} - M^2u_0 + Gr\,\theta_0 = 0 \qquad (11)$$

$$\frac{d^2u_1}{dy^2} + \frac{du_1}{dy} - M^2u_1 + Gr\theta_1 = 0 \qquad (12)$$

$$\frac{d^2\theta_0}{dy^2} + P_r\frac{d\theta_0}{dy} + sP_r\theta_0 = 0 \qquad (13)$$

$$\frac{d^2\theta_1}{dy^2} + P_r\frac{d\theta_1}{dy} + sP_r\theta_1 + P_r\left(\frac{du_0}{dy}\right)^2 = 0 \qquad (14)$$

The corresponding boundary conditions are changed to

$$\left.\begin{array}{l} y=0: \quad u_0=0, u_1=0, \ \theta_0 = 1, \theta_1 = 0 \\ y \to \infty : \ u_0 \to 0, u_1 \to 0, \theta_0 \to 0, \theta_1 \to 0 \end{array}\right| \qquad (15)$$

Solving equations(11),(12),(13),and (14),we get

Skin friction co-efficient = $c_f = k_4(k_3-k_1) + E_c(k_3 k_{20} - k_1 k_{16} + k_8 k_{17} + k_9 k_{18} - k_{10} k_{19})$ (16)

Temperature distribution = $\theta(y) = e^{-k_1 y} + E_c(k_{12} e^{-k_1 y} - k_{13} e^{-k_8 y} - k_{14} e^{-k_9 y} + k_{15} e^{-k_{10} y})$ (17)

And, the rate of heat transfer at the plate is obtained by using the expression for Local Nusselt number given by

$$Nu = k_1 - E_c(-k_1 k_{12} + k_8 k_{13} + k_9 k_{14} - k_{10} k_{15}) \qquad (18)$$

4. Analysis of the result of numerical calculations and discussions:

Expressions for temperature distribution (eqn.17) the rate of heat transfer at the plate (eqn.18) and skin friction (eqn.16) are derived. But these expression contain k_r (r=1,2,3,4……..,20) and for flow on a cooled plate the Garashoff number Gr<0, Therefore firstly k_r are calculated for various values of different parameters.

Calculations for temperature profile θ are made for magnetic parameter m and heat source /sink parameter *s*, keeping Garashoff number Gr (=-2.0), Eckert number Ec(=0.02) and Prandtl number Pr (=0.0328).

The result of calculation for θ are entered in table (1).

Nusselt number Nu and skin friction co-efficient C_f at the plate are also calculated and the table (2) and table (3) contain these.The variation of temperature with y for various values of *s* and m have been shown in table (1) .

From the numerical computation and graphical representation ,it is observed that increase in the strength of magnetic field (or increase of heat parameter *s*) the thermal layer thickness decreases (table 1). The Nusselt number Nu increases with the increase of strength of magnetic field m (table 2) Also Nu increases with the heat source/sink parameter *s* for fixed m. The skin coefficient C_f decreases with the increase in the magnetic field parameter m for fixed *s*. Also c_f decreases with increase in *s* for a fixed m (table 3).

Table 1

y	θ			
	s=0.0	s=-0.5	s=-0.8	s=-1.0
0	1	1	1	1
10	0.7204056	0.2333962	0.1666112	0.1377351
20	0.5189619	0.544708	0.0277577	0.989698
30	0.3738454	0.0127125	0.0046248	0.0026123
40	0.2693067	0.0029669	0.0007706	0.0003599
50	0.1939997	0.0006924	0.0001284	0.0000498
60	0.1397508	0.0001623	0.00002138	0.0000068
70	0.1007779	0.00003840	0.00000356	0.00000094

Variation of θ against y when Gr = -2 , P_r= 0.0328 and Ec =0.02 for various values of s

Table 2

m	Nu			
	s=0.0	s=-0.5	s=-0.8	s=-1.0
2	0.0326010	0.1453862	0.1720323	0.1823411
4	0.0327864	0.1454880	0.1791960	0.1982293
6	0.0327935	0.1455023	0.1782098	0.1982429
8	0.0327973	0.1455058	0.1792131	0.1982462
10	0.0327986	0.1456083	0.1792160	0.1982475

Variation of Nu against m when Gr=-2, P_r= 0.328 and Ec = 0.02 for various values of s.

Table 3

M	C_f			
	s=0.0	s=-0.5	s=-0.8	s=-1.0
2	1.2544413	1.1716321	1.1489438	1.1365160
4	0.5611789	0.5439761	0.5390335	0.5362831
6	0.3601268	0.3529634	0.3508766	0.349705
8	0.2979456	0.2610598	0.2599159	0.2592751
10	0.02095272	0.2070826	0.2063620	0.2059575

Variation of skin friction C_f against m and Gr =-2, P_r=0.0328 and Ec =0.02 for various values of s.

References

1. Smith , A.G. & Spalding ,D.B. 1958. J.Roy. Aero- Soc ; -62:60-64
2. Skopet, M.B. 1959. Sowet. Phys. 4: 411-499
3. Siddappa, B. & Bujurke , N.M. 1995. K.U.D Jour.Sc, 1:-253-259
4. Soundalgeker, V.M.& Thakur, M.S. 1977. Nuclear Engineering and Designs, 42: 233-236
5. Spalding , D.B & Pun,W.M. 1962. Int.,s. Heat Mass Transfer, 5: 239-250
6. Pandey G.C &kaforestual,N. 1976. As. Phy. Space, 2:397-410
7. Seth, G.S, Hussain, S.M & Sarkar, S.2015. Jou of the Egyptian Mathematical Society 23(1):197-207
8. Raptis, A.and Kafousias, N. 1982. Int. J. Energy Res; V: 241
9. Kim, Y.J. 2000. Int .J.Eng, Sci., 38: 837
10. Vidyasagar, G. and Ramana, B. 2016. IJMTST,2(8):13-20.

--------X --------