APPLICATIONS OF DIFFERENTIAL EQUATION IN HEAT GENERATED DISC BRAKES

P.GOWRI[1] , S.VINODHINI [2], G.SINDHIYA [3], P.SNEHA [4], B.MADHU GOKUL[5]
[1]Asistant Professor, Department of Mathematics
[2,3,4,5]Undergraduate Scholars, Department of Mathematics
Sri Krishna Arts and Science College,Coimbatore-641 008

Abstract : The concept used in this project is 'Applications of differential equation in heat generated disc brakes'. A disc brake is a type of brake that uses callipers to squeeze pairs of pads against a disc or "rotor" to create friction. This action retards the rotation of a shaft, such as a vehicle axle, either to reduce its rotational speed or to hold it stationary. The energy of motion is converted into waste heat which must be dispersed for the proper functioning of the vehicle. This heat generated is calculated using 'first order differential equations' of ordinary differential equations under the utilization of standard equations for 'temperature of disc brake', 'the rate at which work is done', 'cooling function', rate of change of heat of the breaking system with the constant values of ambience temperature and specific heat capacity of iron and mass of the vehicle taken under observation.

Index terms: **Disc brakes, heat generation, specific heat, disc force, temperature of the brake ,work done**

I. GENERATION OF HEAT IN A DISC BRAKE

Let Q be the heat in the braking system which obeys the equation

$$\frac{dQ}{dt} = Mav - f(T, v)$$

Where,

M is the mass of the car

a is the acceleration

v is the velocity and

f is the cooling function

T is the temperature of the brake

TEMPERATURE OF THE BRAKE

$T = T_0 + Q/mc$

Where

T_0 is ambient temperature

m is the mass of the braking system

c is the specific heat

$$\mathbf{f(T, v) = [k_1(T - T_0)(1 + k_2 v^n)]}$$

Where k_1, k_2 and n are constants

Since the rate at which a work is done is close to the rate at which heat is generated

$$\frac{dQ}{dt} = \frac{dw}{dt}$$

$$\frac{dw}{dt} = Fd*\left(\frac{ds}{dt}\right) - f(T,v)$$

Where w is work

Fd is the frictional force on disk

s is the displacement of disk at the point of brake pad contact

II. PROBLEMS

Problem 1

Find out the heat generated by the disc brake of a car with a disk force of 5N and displacement of 30tm .The ambience temperature ($T_{0\ =298.15}$) and the velocity of the car (v=60km/hr)

Temperature of the brake T=T_0+Q/mc

Where Q, heat of the braking system=309.15k

M is the mass of the car 1500 kg

c is the specific heat of iron 0.45(J/g)

T=T_0+Q/mc

=298.15+[309.15/675]

=298.15+0.458

=298.608

The rate at which work is done is

$\frac{dw}{dt}$ = Fd*($\frac{ds}{dt}$)−f (T, v)

=5*$\frac{d}{dt}$(30t)−f (T, v)

=5*30−[k_1(T− T_0)(1 + k_2 v^n)],wherek_1, k_2 and n are constants

=150−[0.7(298.608−298.15) (1+2$(60)^{0.5}$)]

=150−[(0.7*0.458) (1+15.491)]

=150−[(0.7*0.458) (16.491)]

=150−5.287

=144.713(j/hr)

Therefore the heat generated by the disk brake is 144.713 (j/hr)

PROBLEM 2:

Find the heat generated by the disc brake of a race car whose mass is 1.65 tonnes.The acceleration of the race car is 30m/s^2 and the velocity of the disc brake and race car are 10m/s and 55km/hr respectively.

Solution:

Temperature of the disc brake

T=T_0+Q/mc

=298.15+427.15/675

=298.15+0.6328

=298.78

Where,

Q is heat of the braking system=427.15k

m is the mass of the car 1500 kg

c is the specific heat of iron 0.45(J/g)

$\frac{dQ}{dt}$= **Mav−f (T, v)**

= 1.65*30*10 −f (T,v)

= 495−[k_1 (T −T0) (1+k_2v^n)]

Where **k_1, k_2 and n** are constants

= 495 − [0.6(298.78 −298.15) (1+4(55) $^{0.7}$)]

= 495 − [0.378*67.11]

= 495 − [25.367]

= 469.64 (J/hr)

The heat generated by the disc brake is 469.64(j/hr)

PROBLEM 3 :

The work done and the displacement of the disc brake of a motor bike are 15tJ and 45tm respectively. The ambience temperature ($T_{0\,=298.15}$)and the velocity of the motor bike (v=32km/hr).Find the disc force on the disc brake.

Solution:

Temperature of the brake

$T=T_0+Q/mc$

Where,

Q is heat of the braking system=332.24k

m is the mass of the car 180 kg

c is the specific heat of iron 0.45(J/g)

$T=T_0+Q/mc$

=298.15+332.24/81

=302.25

The rate at which work is done is

$\frac{dw}{dt}$ = fd * ($\frac{ds}{dt}$) −f (T, v)

15 = fd* 45−[k_1 (T −T_0) (1+k_2v^n)]

15 = fd* 45 − [0.5(302.25 − 298.15)(1+6(32)$^{0.8}$)] where k_1,k_2 and n are constants

15 = 45fd −198. 85

213.85=45*fd

fd =4.752N

III. CONCLUSION

The heat generated out of disc brakes can also be due to the excessive usage of brakes, surface wobbles in rotor which causes uneven dissipation .Thus the heat generated out of a braking system should necessarily be calculated to maintain the proper functioning of the vehicle and to avoid any life- threatening accidents

REFERENCES

[1] Morris Tenebaum book on ‘Ordinary differential equations’ .

[2] Frank Chen book on 'Disc brake :Mechanism, Analysis, Evaluation and Reduction Prevention' .
[3] Mark G. Davidson and William A Adkins book on 'Ordinary differential equations'.
[4] Gowri.P, Abhinandana.R, Madhu Gokul.B, Saranya.B , Mrithula.R‖ A Case Study on Path of Projectile Motion and ItsApplication‖, International Journal of Engineering Science Invention Research andDevelopment, vol-4, issue-4,dec-2017.
[5] Gowri.P,Deepika. Dand Krithika.S,—A Case Study on Simple Harmonic Motionand Its Applications‖, International Journal ofLatest Engineering and Management Research (IJLEMR)
[6] Gowri.P,Asst. Prof, Department of Mathematics, Sri Krishna Arts and Science College, Coimbatore, Tamil Nadu,India. Glory Bebina.E, IIIBsc Mathematics, Sri Krishna Arts and Science College, Coimbatore, Tamil Nadu, India,—A CaseStudyon Elastic Oscillationsandits Applications in Bungee Jumping by Differential Equation**",** International Journal of Trend in Scientific Research and Development(IJTSRD).
[7] Gowri.P1,Priyadharshini.S2,Maheswari.T3,1Asst.Prof. Sri Krishna Arts and Science College,2,3Sri Krishna Arts and Science College, —A Case Study onRudgeKutta4thOrder Differential Equations and Its Application‖, Imperial Journal of Interdisciplinary Research(IJIR)
[8] S.Priyadharshini, ShajeenaM Basheer, KaushalyaM and SnehaM,‖A Study on Motion of a Free Falling Body In Kinematic Equation‖, International JournalforResearchinAppliedScience&EngineeringTechnology,Vol.6, Issue 1, 2018. ISSN 2321-9653
[9] https://www.physicsforums.com/threads/heat-dissipation-between-brake-pad-and-disc.158582
[10] https://auto.howstuffworks.com/auto-parts/brakes/brake-types/disc-brake1.htm
[11] https://www.mikesbikes.com/how-to/disc-brake-basics-pg158.htm
[12] https://www.engineersedge.com/mechanics_machines/disk-brake-calculations.htm
[13] https://www.coursera.org/learn/ordinary-differential-equations
[14] https://en.m.wikipedia.org/wiki/Examples_of_differential_equations