A CONNECTION BETWEEN PAIRS OF RECTANGLES AND SPHENIC NUMBERS

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Abstract : This paper aims at determining pairs of rectangles such that, in each pair, the sum of their areas is represented by a Sphenic number. Also, the number of primitive and non-primitive rectangles for each sphenic number is given.

***IndexTerms* - Pairs of rectangles, Area, Sphenic number.**

I. INTRODUCTION

Any sequence of numbers represented by a mathematical function may be considered as pattern. In fact, mathematics can be considered as a characterization of patterns. For clear understanding, any regularity that can be illustrated by a scientific theory is a pattern. In other words, a pattern is a group of numbers, shapes or objects that follow a rule. A careful observer of patterns may note that there is a one to one correspondence between the polygonal numbers and the number of sides of the polygon. Apart from the above patterns we have some more fascinating patterns of numbers namely Nasty number, Dhuruva numbers and Jarasandha numbers. For illustrations, one may refer [1- 21].

II. DEFINITION

Sphenic Number:

A Sphenic number is a positive integer which is the product of exactly 3 distinct primes.

III. METHOD OF ANALYSIS

Let $R_1(x,y)$ and $R_2(z,w)$ be two distinct rectangles whose corresponding areas are A_1, A_2.

Consider

$A_1 + A_2 = 30$, a sphenic number

That is,

$$xy + zw = 30 \quad (1)$$

Let q, r, s be three non-zero distinct positive integers and $r > s$.

Introduction of the linear transformations

$$x = s\ ,\ y = 2q + s\ ,\ z = r - s\ ,\ w = r + s \quad (2)$$

in (1) leads to

$$r^2 = 30 - 2qs \quad (3)$$

Solving (3) for q, r, s and using (2), the corresponding values of rectangles R_1 and R_2 are obtained and presented in Table:1 below:

Table: 1 Rectangles

R_1	R_2	$A_1 + A_2$	Observations	
			Primitive	Non-Primitive
(1, 15)	(3, 5)	30	R_1, R_2	
(1, 27)	(1, 3)	30	R_1, R_2	

Note that the above two pairs of rectangles are primitives as $\gcd(x,y)=1$ and $\gcd(z,w)=1$

Some other numerical examples of sphenic numbers are presented in Table: 2 below:

Table: 2 Rectangles

R_1	R_2	$A_1 + A_2$	Observations		Remarks
			Primitive	Non-Primitive	
(1, 7)	(5, 7)	42	R_1, R_2		Total number of Primitive rectangles =7 Total number of non-Primitive rectangles =1
(3, 5)	(3, 9)		R_1	R_2	
(1, 27)	(3, 5)		R_1, R_2		
(1, 39)	(1, 3)		R_1, R_2		
(1, 31)	(5, 7)	66	R_1, R_2		Total number of Primitive rectangles =9 Total number of non-Primitive rectangles =1
(3, 13)	(3, 9)		R_1	R_2	
(5, 11)	(1, 11)		R_1, R_2		
(1, 51)	(3, 5)		R_1, R_2		
(1, 63)	(1, 3)		R_1, R_2		
(1, 7)	(7, 9)	70	R_1, R_2		Total number of Primitive rectangles =11 Total number of non-Primitive rectangles =1
(3, 5)	(5, 11)		R_1, R_2		
(1, 35)	(5, 7)		R_1, R_2		
(1, 55)	(3, 5)		R_1, R_2		
(1, 67)	(1, 3)		R_1, R_2		
(3, 21)	(1, 7)		R_2	R_1	
(1, 15)	(7, 9)	78	R_1, R_2		Total number of Primitive rectangles =9 Total number of non-Primitive rectangles =1
(1, 43)	(5, 7)		R_1, R_2		
(3, 17)	(3, 9)		R_1	R_2	
(1, 63)	(3, 5)		R_1, R_2		
(1, 75)	(1, 3)		R_1, R_2		
(1, 39)	(7, 9)	102	R_1, R_2		Total number of Primitive rectangles =9 Total number of non-Primitive rectangles =1
(1,67)	(5, 7)		R_1, R_2		
(3, 25)	(3, 9)		R_1	R_2	
(1, 87)	(3, 5)		R_1, R_2		
(1, 99)	(1, 3)		R_1, R_2		
(1, 25)	(8, 10)	105	R_1	R_2	Total number of Primitive rectangles =12 Total number of non-Primitive rectangles =14
(2, 14)	(7, 11)		R_2	R_1	
(6, 10)	(3, 15)			R_1, R_2	
(3, 11)	(6, 12)		R_1	R_2	

(4, 10)	(5, 13)		R_2	R_1	
(1, 57)	(6, 8)		R_1	R_2	
(2, 30)	(5, 9)		R_2	R_1	
(4, 18)	(3, 11)		R_2	R_1	
(1, 81)	(4, 6)		R_1	R_2	
(2, 42)	(3, 7)		R_2	R_1	
(4, 24)	(1, 9)		R_2	R_1	
(1, 97)	(2,4)		R_1	R_2	
(2, 50)	(1,5)		R_2	R_1	
(1, 11)	(9, 11)	110	R_1, R_2		Total number of Primitive rectangles =11 Total number of non-Primitive rectangles =1
(5, 7)	(5, 15)		R_1	R_2	
(1, 47)	(7, 9)		R_1, R_2		
(1, 75)	(5, 7)		R_1, R_2		
(1, 95)	(3, 5)		R_1, R_2		
(1, 107)	(1, 3)		R_1, R_2		
(1, 15)	(9, 11)	114	R_1, R_2		Total number of Primitive rectangles =13 Total number of non-Primitive rectangles =1
(7, 9)	(3, 17)		R_1, R_2		
(1, 51)	(7, 9)		R_1, R_2		
(1, 79)	(5, 7)		R_1, R_2		
(3, 29)	(3, 9)		R_1	R_2	
(1, 99)	(3, 5)		R_1, R_2		
(1, 111)	(1, 3)		R_1, R_2		
(1, 39)	(9, 11)	138	R_1, R_2		Total number of Primitive rectangles =11 Total number of non-Primitive rectangles =1
(1, 75)	(7, 9)		R_1, R_2		
(1, 103)	(5, 7)		R_1, R_2		
(3, 37)	(3, 9)		R_1	R_2	
(1, 123)	(3, 5)		R_1, R_2		
(1, 135)	(1, 3)		R_1, R_2		
(1, 45)	(10, 12)	165	R_1	R_2	Total number of Primitive rectangles =12 Total number of non-Primitive rectangles =14
(2, 24)	(9, 13)		R_2	R_1	
(1, 85)	(8, 10)		R_1	R_2	

(2, 44)	(7, 11)		R_2	R_1	
(7, 19)	(2, 16)		R_1	R_2	
(6, 20)	(3, 15)			R_1, R_2	
(3, 31)	(6, 12)		R_1	R_2	
(1, 117)	(6, 8)		R_1	R_2	
(2, 60)	(5, 9)		R_2	R_1	
(1, 141)	(4, 6)		R_1	R_2	
(2, 72)	(3, 7)		R_2	R_1	
(1, 157)	(2, 4)		R_1	R_2	
(2, 80)	(1, 5)		R_2	R_1	
(1, 31)	(11, 13)	174	R_1, R_2		Total number of Primitive rectangles =18 Total number of non-Primitive rectangles =2
(3, 13)	(9, 15)		R_1	R_2	
(5, 11)	(7, 17)		R_1, R_2		
(1, 75)	(9, 11)		R_1, R_2		
(1, 111)	(7, 9)		R_1, R_2		
(5, 27)	(3, 13)		R_1, R_2		
(1, 139)	(5, 7)		R_1, R_2		
(3, 49)	(3, 9)		R_1	R_2	
(1, 159)	(3, 5)		R_1, R_2		
(1, 171)	(1,3)		R_1, R_2		
(1, 43)	(11, 13)	186	R_1, R_2		Total number of Primitive rectangles =17 Total number of non-Primitive rectangles =3
(3, 17)	(9, 15)		R_1	R_2	
(7, 13)	(5, 19)		R_1, R_2		
(1, 87)	(9, 11)		R_1, R_2		
(1, 123)	(7, 9)		R_1, R_2		
(1, 151)	(5, 7)		R_1, R_2		
(5, 35)	(1, 11)		R_2	R_1	
(3, 53)	(3, 9)		R_1	R_2	
(1, 171)	(3, 5)		R_1, R_2		
(1, 183)	(1, 3)		R_1, R_2		

IV. CONCLUSION

In this paper, an attempt has been made to obtain pairs of rectangles such that, in each pair, the sum of their areas is represented by a Sphenic number. The readers of this paper may search for pairs of rectangles other than the pairs of rectangles presented above for each sphenic number.

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